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On Write-Down/ Write-Up Loss Absorbing Instruments

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Abstract:

Purpose: *The article deals with banks' vulnerability to insolvency. We discuss the impact of the CoCo write-down/write-up bonds issuance on the bank solvency. Such instruments absorb losses in two ways: 1) When a bank gets in trouble, the payment of interest is ceased, and 2) If the financial standing of the bank further deteriorates, its CoCo bonds are written down. Reversing it, when the bank solvency improves, the CoCos are gradually written up, and the payment of interest is restored. The investment in the CoCo bonds is risky. That is why they offer a greater interest rate than straight bonds. Hence there is a trade-off: loss absorption versus profitability.*

Design/Methodology /Approach: *As a measure of insolvency, we consider the probability of the implementing resolution process, i.e., as it is called in actuarial sciences, the probability of ruin.*

Findings: *We show that depending on the CoCo bonds' profitability, the additional issuance of the CoCos may reduce the probability of ruin. In this respect, we propose a theoretical explanation for the optimum share of CoCos in an institution's liabilities.*

Practical implications: *Our findings may give the supervisory authorities a useful tool to determine the fair share of Additional Tier One (AT1) CoCos to fill the Pillar 2 bank capital layer. The model proves to be useful for setting the optimum size of Restricted Tier One (RT1) CoCos in the insurer's liabilities as well.*

Originality/value: *The science lacks theoretical background for CoCos' optimal size in issuers' liabilities. Besides, we provide a new measure of bank insolvency. Contrary to the typical approach with a finite time horizon, we choose the default probability at any moment in the future as a measure of insolvency.*

Keywords: *Insolvency risk, Contingent convertibles (CoCos), AT1, RT1, Lévy refracted process, Ruin probability, write-down, Common Equity Tier 1 (CET1) ratio.*

JEL codes: *C22, C60, D81, G21, G32.*

Paper Type: *Research article.*

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1. Introduction

Contingent convertibles (CoCos) are subordinated hybrid securities with an automatic conversion provision which enables issuers (banks or insurance companies) to exchange bonds for common stock, subject to a breach of a conversion trigger set forth at the inception of the issuance of the bonds (Liberadzki and Liberadzki, 2019). Unlike with regular convertibles, a conversion is not an option at the discretion of a bondholder but is forced when the bank's regulatory capital fails to meet a predetermined level. About banks, the trigger event is Common Equity Tier1 (CET1) ratio falling below 5.125% in case of so-called low triggers (and 7% or 8% in case of high triggers).

Despite their name, CoCos may present a more extreme construction of loss absorption imprinted in a write-down mechanism, where the occurrence of a predetermined stress-event automatically triggers a write-down of a bond's value without diluting shareholders. The burden of an institution's failure is imposed on the bondholders, but the bank may continue to operate, averting the financial system's disruption. If the conversion trigger is not breached, CoCos remain as ordinary subordinated debt securities to retire at the first call date (they are usually structured as callable perpetual bonds with a possible call at year five or later). The recent developments in the structuring of CoCo bonds aim to reduce the severity of an automatic write-down of the bonds' principal value, facilitating the discretionary write-up of the bonds once the issuing bank's financial situation is no longer distressing.

Thus, CoCo bonds provide buffer capital to a bank at a time of distress but before potential insolvency (i.e., loss-absorption capacity on a going-concern basis). Such an automatic 'bail-in' executed in times of distress may rescue a bank from failure without a (heavily criticized) injection of taxpayer money into large financial institutions (bail-out). One may say that this write-down mechanism is somehow similar to that embedded in so-called catastrophe bonds (or only 'CAT bonds'), which were initially designed by US reinsurers and were used to transfer the risk of losses caused by natural disasters. To some extent, this mechanism may be regarded as a cornerstone of the write-down embedded in the structure of a CoCo. When it comes to CoCos, the trigger event of a natural disaster is replaced by a capital ratio trigger. The main difference, though, is that CoCos' trigger is strictly linked to the issuer's health rather than to external occurrence.

The contingent convertibles play an essential role in the tier-based capital structure imposed by the Basel III Capital Accord package. Their most eminent virtue of contingent conversion (or write-down) allows them to be assigned to the category of the bank's Additional Tier 1 (AT1) regulatory capital. The internationally accepted principles of Basel III are translated into law worldwide. Investors in contingent capital prefer consistent and high coupon payments to equity stakes. As a rule, they anticipate that the triggers will never be reached and therefore enjoy a relatively high coupon that embodies the option to convert the bond into equity or write-down.

Apart from the risk of trigger event occurrence, holders of AT1 CoCos are exposed to coupon cancellation and call extension risks. Coupons are, like share dividends, fully discretionary, so a board could decide to cut them at any time. Interest is payable only from distributable items, which are essentially retained earnings. Besides, the regulator can restrict or prohibit coupon payments on AT1 bonds if it has concerns, among other things, about the bank's capital strength. Regarding maturity, any call option may be exercised only at the issuer's sole discretion and not at the discretion of the investor.

The depiction mentioned above may freely be addressed to insurers' Tier 1 European Union (EU) Solvency II regulatory package compliant CoCo bonds (so-called Restricted Tier 1s, RT1s). Like banks, insurers are part of the regulated industry. Much like AT1s, RT1 items present a loss absorbency mechanism on a going-concern basis, which implies that the debt could be written down or converted into equity upon the regulatory capital ratio trigger event - in this instance, breaching of the Solvency Capital Requirement (SCR). The RT1s must provide for the suspension of repayment or redemption in the event of non-compliance with the SCR.

2. Motivation

2.1 Alternative Measure of a Bank's Insolvency

In this paper, we propose a distinct approach to assess the probability of banks' insolvency. As a measure of insolvency, we consider the probability of the implementing resolution process, i.e., as it is called in actuarial sciences, the probability of ruin. We propose the analytical form of the ruin function based on the Lévy process as a function of two variables, the thickness of the AT1 layer (the share of CoCos in issuers' liabilities structure) and the cost of issuing AT1s above the cost of subordinated debt. These two underestimated factors are crucial as far as the institution's solvency is concerned. Namely, contingent convertibles deliver two counter effects: When things go wrong, CoCos are automatically converted into issuer's common equity or just wiped out: in both cases, debt is reduced. Much higher coupons of CoCos relatively to straight bonds though make CoCos issuers less resilient. What is also neglected in most studies is that our model captures coupon cancellation provisions to improve the banks' solvency. The model outputs the probability of default which can be compared depending on different issue parameters: the size of an issue, coupon level, and trigger level (either low- or high).

Our approach brings us a step further than the study (Jaworski, Liberadzki and Liberadzki, 2017). It was then proposed to measure the issuer's default risk with a VaR method given alpha significance level and Expected Shortfall. Issuing CoCos makes sense (that is: improves issuer's solvency) only if they are structured so that the probability of the triggering is more than the alpha significance level. The theorem is valid for any spread (the difference between coupon of CoCo and straight bond) less than 100%. We then concluded that whenever the probability of contingent conversion was high enough, the issuer's default risk would be reduced irrespective of the cost disadvantage of CoCo bonds in comparison to straight bonds.

Another approach to the subject the reader might find is in Jang *et al.* (2018) or Del Viva and El Hefnawy (2020). In their study, Jang *et al.* (2018) define regulatory default as the likelihood that an issuing bank fails to retain the minimum capital requirement and post-conversion risk regarding how banks' CET1 ratio breaches the regulatory default threshold the post-conversion situation.

2.2 Theoretical Model for Setting Optimum Share of CoCos in a Bank's Liabilities.

Basel III calls for banks to have the following minimums in terms of the magnitude of capital, CET1 capital, which is 4.5% of Risk-Weighted Assets (RWAs), and for Tier 1 capital, 6% of RWAs and total capital ratio (TCR) of 8% RWAs. Banks can allocate 1.5% of their RWAs to AT1 in the computation of Tier 1 capital ratio minimum.

Additionally, banks must meet a so-called combined buffer requirement (CBR), which is the sum of various capital buffers set on top of the base capital requirements. A bank may meet the CBR only with CET1 capital that is not used to meet the 8% own funds requirement. In other words, if it uses CET1 capital to meet any of the 3.5% gaps between the CET1 minimum requirement of 4.5% and the total capital requirement of 8%, that capital is not eligible to meet the CBR. Therefore, the most efficient capital structure is for banks to issue AT1 capital to cover the 1.5% point difference between CET1 and overall Tier 1 requirement and Tier 2 (T2) capital to cover the 2% point difference between Tier 1 and total capital requirements. This avoids using CET1 capital for those purposes, which would then not be available to meet the bank's CBR - a clear incentive to banks to issue both AT1 and T2 bonds.

Going beyond minimum TCR, national supervisors have the discretion to apply additional own funds requirements on an individual basis. Pillar 2 requirements (P2R) are own additional funds supposed to cover risks neither addressed by the Pillar 1 capital requirements (i.e., 4.5% RWAs of CET1, 6% of Tier 1,8% TCR) nor combined buffer requirements, such as interest rate risk in the banking book.

Then the question arises as to what extent AT1s offer real protection against losses on a going-concern basis. AT1 CoCos may fill banks' funds up to 1.5% RWAs, and their prevailing coupons range between 3.8% and 8.8% per annum (Liberadzki and Liberadzki, 2019). Pure arithmetic shows that coupon cancellation may bring a one-off relief to the issuer of merely 0.13% RWAs. This means getting paid a premium over a straight Tier 2 for little extra gain. The restrictions on coupon distributions made more sense in the light of the original Basel Committee on Banking Supervision (BCBS) proposals to build the capital buffers of AT1 instruments, increasing their admissible share in RWAs even up to 13%. The possible solution would be to allow banks fulfilling their P2R in AT1 items as well. Several jurisdictions have already confirmed that AT1 will be allowed to meet Pillar 2 requirements. The new EU CRD5 directive of 2019 sets a requirement for P2R composition to be identical to the Pillar 1 structure. That means that P2R may consist of 56.25% CET1, 18.75% AT1, and 25% T2 eligible instruments or positions. In other words, AT1s recently have been given more significant importance in the banks' capital structure. Such a development calls for a useful model for regulators and supervisory authorities to prescribe the optimal P2R composition for each bank, assessing the specific amount of AT1s to be issued within appropriate calibrated issuance parameters: trigger level the interest. In Del Viva and El Hefnawy (2020), an alternative approach to CoCo bonds or shares is discussed.

Thus, a rigorous theoretical background is needed to explain the optimal share of CoCos in an institution's liabilities (or RWAs). BCBS papers do not provide a reasonable explanation for why the regulatory eligibility for CoCos is capped at 1.5% concerning TCR. Therefore, the main regulatory question we aim to pose is if the regulatory framework of AT1 instruments reflects their role in the banks' capital structure. We feel that the researchers omit this issue. Instead, much effort is put into a quantitative approach to the existing AT1s on the market. At present, the academicians concentrate mostly on searching pricing formula for current CoCos and look into probability assessment of banks' insolvency. The most renowned of them are (i) Pennacchi (2011), analyzing contingent capital in the context of a structural credit risk model of an individual bank; and Spiegeleer and Schoutens (2012), Spiegeleer et al. (2017) with their credit- and equity derivatives as well as implied CET1 models. Positioned somehow in-between, Russo, Lagasio, Brogi, and Fabozzi (2020) studied the ability of both balance sheet items (i.e., CET1 ratio) and market data (i.e., CDS) to predict bank distress.

There is no much thought on how to reshape the existing instrument into a better one. If there is any, the researchers such as Sundaresan and Wang (2015), Vallee (2016), Pennacchi and Tchistyi (2016), Di Girolamo, Campolongo, De Spiegeleer, and Schoutens (2017), or Xiao (2019) usually concentrate on the propositions concerning changing the trigger event mechanism. We intend to go in the other direction and fill the research gap with the theoretical condition for optimum share CoCos in an institution's liabilities. This could give supervisory authorities guidance on what the P2R capital layer should be composed of to enhance stability.

The paper is organized as follows. Section 3 presents the model based on the refracted Lévy process, followed by the main results. Section 4 and 5 discuss the ruin probability of the refracted Lévy processes and provide an analytical formula for the particular case, compound Poisson process with exponential jumps. Conclusions and possible future are included in section 6.

3. Model

We consider a simplified model of bank activities. We focus on CET1 ratio, which measures bank Common Equity Tier 1 capital (roughly speaking the bank own funds and other core capital) against Risk-Weighted Assets:

$$Common\ Equity\ Tier\ 1\ Ratio = \frac{Common\ Equity\ Tier\ 1\ Capital}{Risk{-}Weighted\ Assets}. \quad (1)$$

We consider the case when the CoCo write-down / write-up bonds are not included in Common Equity Tier 1 capital. However, on instance of their write-down, CET1 ratio goes up and if it is followed by write-up then CET1 ratio diminishes. Indeed, due to the balance sheet rules the cancellation of a part of a debt raises own funds, while restoring it works the other way round. We assume that there are three thresholds th_0, th_1 and th_2, $0 \leq th_0 < th_1 < th_2$, set by the banking supervision: If the CET1 ratio falls below th_2 then both payments cease: interest on CoCo bonds and the share dividends. They are restored when CET1 ratio goes back again above th_2.

When the CET1 ratio falls below th_1 the write-down of CoCo bonds is implemented (partial or to zero). The write-up (in form of full or only partial restoration of bonds' nominal value) takes place when the CET1 ratio restoration is sufficient to bring it above th_1 level. An altrernative scenario is that writing off CoCo bonds does not make the CET1 ratio rebound above the th_0. In such an instance the bank is to be wound up in a resolution or bankruptcy procedure.

In order to make the model more feasible we assume that both: write-down and write-up are implemented in a continuous manner. Furthermore we restrict ourselves to the case of callable perpetual CoCo bonds, i.e. there is no fixed maturity, only on some fixed days (once a couple of years, no less than five after the date of issuance) an issuer has the right to call.

The management of the bank has a choice to issue the less costly straight bonds, where the interest is paid up to maturity or bank default, or the more costly loss absorbing CoCos. Neither of them is included in CET1 capital. We denote by V_L the joint volume of both types of bonds and by $\gamma \in [0,1]$ the ratio of a volume of the CoCo bonds against both (i.e. V_L). Obviously the bank management would like to have γ as low as possible, while the supervisors would like to have γ as high as possible. One of the goals of the model proposed in our study is to provide a method of finding an optimal γ.

Let a stochastic process $U = (U_t)_{t \geq 0}$, defined on a probability space $(\Omega, \mathcal{M}, \mathbb{P})$, model the surplus above a threshold th_0, of the CET1 ratio increased by a possible write down of CoCo bonds. In more details CET1 ratio is a piece-wise linear function of U_t.

$$CET1\ Ratio_t = \begin{cases} U_t + th_0 - \gamma b_L & if \quad U_t \geq th_1 - th_0 + \gamma b_L, \\ th_1 & if \quad th_1 - th_0 + \gamma b_L > U_t > th_1 - th_0, \\ U_t + th_0 & if \quad th_1 - th_0 > U_t > 0, \\ th_0 & if \quad 0 \geq U_t. \end{cases} \quad (2)$$

where b_L is the quotient of the volume V_L and Risk-Weighted Assets. Thus for $U_t \leq th_1 - th_0$ CoCo bonds are fully write down and for $th_1 - th_0 < U_t < th_1 - th_0 + \gamma b_L$ partially. Note that the write-down, write-up rule allows us to choose a Markov process as the underlying U.

We assume that U is a refracted Lévy process (see Kyprianou (2014), Kyprianou and Loeffen (2010), Lkabous *et al.* (2017), Czarna *et al.* (2019) and Albrecher *et al.* (2018)).

$$U_t = x + b + \delta_1 t + (\delta_2 - \delta_1) \int_0^t 1 - 2.5pt\ l_{\ U_s \geq b} ds - S_t, \tag{3}$$

where

- $S = (S_t)_{t \geq 0}$ is a driftless, a nondecreasing Lévy process (for example the compound Poisson process), which describes "extra" losses (compare Schoutens and Cariboni (2009));
- $b = th_2 - th_0 + \gamma b_L$ is a refraction point;
- x is the initial surplus of CET1 ratio over the threshold th_2;
- the drifts δ_1 and δ_2 are given by

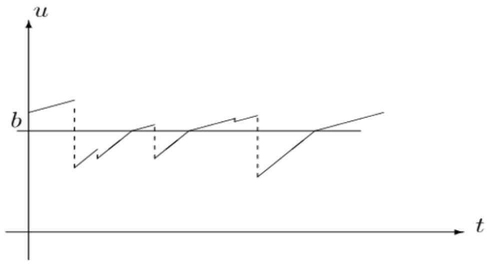
$$\delta_1 = c_1 - (1-\gamma)c_2 = c_1 - c_2 + \gamma c_2$$
and
$$\delta_2 = \delta_1 - \gamma c_3 - d = c_1 - c_2 - d - \gamma(c_3 - c_2); \tag{4}$$

- c_2 and c_3, $c_2 < c_3$, describe the decrease of CET1 ratio due to the payment of interest with respect to the mentioned above standard bonds and CoCo's i.e. the interest rate paid on standard bonds (i_S) and CoCo's (i_{Co}) multiplied by ratio $b_L = V_L/RWA$;

- d describes the fall of CET1 ratio due to the payment of the dividend, i.e. the total dividend paid during a year divided by Risk-Weighted Assets;
- c_1 describes the rise of CET1 ratio due to income from assets reduced due to the remaining costs of running the bank, i.e.divided by Risk-Weighted Assets the difference of the total income and total costs during a year;
- t denotes time in years, starting from a fixed moment (for example today);
- we assume that $\delta_1 \geq \delta_2 > \mathbb{E}(S_1)$.

On Figure 1 we draw a sample path of the prices U. Note that the proposed model is closely related to models used to determine the optimal dividend, see for example Yin and Wen (2013), Albrecher *et al.* (2018), Loeffen (2009).

Figure 1. *Sample path of a refracted Lévy process* U_t.



Source: *Own creation.*

Let a random variable κ, taking values in $(0+\infty]$, denote the moment of the default

$$\kappa(\omega) = \inf\{t > 0: U_t(\omega) < 0\}. \tag{4}$$

Note that in our model the default of U is corresponding to the implementation of the resolution or bankruptcy process. The so called ruin probability i.e. the probability of default at any time, is given by

$$P = \mathbb{P}(\kappa < \infty). \tag{5}$$

Note that the probability of default depends on real parameters x, b, δ_1 and δ_2 and the jump process $S = (S_t)_{t\geq 0}$, $x, b \geq 0$, $\delta_1 \geq \delta_2 \geq \mathbb{E}(S_1)$. We fix S and consider P as a real valued function of x, b, δ_1 and δ_2

$$P = P(x, b, \delta_1, \delta_2). \tag{6}$$

Since in our model b, δ_1 and δ_2 are linear functions of γ, $\gamma \in [0,1]$, the dependence of the ruin on γ is given by

$$P_\emptyset(\gamma) = P(x, b_0 + b_L\gamma, c_1 - c_2 + c_2\gamma, c_1 - c_2 - d - (c_3 - c_2)\gamma, S), \quad b_0 = th_2 - th_0. \tag{7}$$

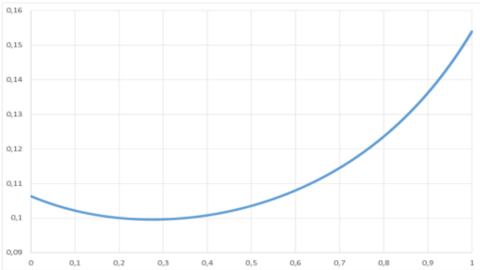
In the points of differentiability, the derivative of $P_\emptyset(\gamma)$ with respect to γ equals

$$\frac{dP_\emptyset}{d\gamma} = \frac{\partial P}{\partial b} b_L + \frac{\partial P}{\partial \delta_1} c_2 - \frac{\partial P}{\partial \delta_2}(c_3 - c_2). \tag{8}$$

It can be easily expressed in terms of the interest rates.

$$\frac{dP_\emptyset}{d\gamma} = b_L\left(\frac{\partial P}{\partial b} + \frac{\partial P}{\partial \delta_1} i_S - \frac{\partial P}{\partial \delta_2}(i_{Co} - i_S)\right). \tag{9}$$

Figure 2. *The probability of the ruin as a function of* γ



Source: *Own creation.*

Since P is decreasing with the increase of b, δ_1 or δ_2 the partial derivatives from the above formula are negative or at least non-positive. Thus if $\frac{\partial P}{\partial \delta_2}$ is nonzero then for sufficiently small rates i_{Co} the small additional issuance of CoCos is improving the probability of the ruin (default) i.e. the solvency, but for too big rates i_{Co} the small issuance may increase the probability of the ruin. Indeed we get a criterion:

Criterion 1. *Let P be differentiable with respect to γ. We denote by c^* the threshold*

$$c^* = c^*(x, b, \delta_1, \delta_2) = \frac{\frac{\partial P}{\partial b} + \frac{\partial P}{\partial \delta_1} i_S}{\frac{\partial P}{\partial \delta_2}} \tag{10}$$

If the margin between interest rates of CoCo bonds and standard bonds $(i_{Co} - i_S)$ is smaller than c^* then the small rise of γ would imply a decrease of the ruin probability P. On the other side, if the margin between interest rates of CoCo bonds and standard bonds is greater than c^* then the small increase of γ would imply an increase of P.

The above can be applied when a bank is planning its first issuance of CoCos. The threshold c^* is determining the upper bound on the margin between interest rates of CoCo bonds and alternative bonds, below which the issuance may improve the bank solvency. Similarly when the call day comes the above Criterion may help an issuer do decide whether to call or not to call.

In section 4 we provide the formulas for the ruin probability in terms of the Laplace exponent of the process $S = (S_t)_{t \geq 0}$

$$\psi(z) = \ln(\mathbb{E}(\exp(-zS_1))).$$

Next in section 5 we deal with a special case when S is a compound Poisson process process with exponentially distributed jumps with intensities respectively λ_N and λ_e. For such two parameter family of jump processes $S(\lambda_N, \lambda_e)$ we get an analytical formula. For $x \geq b \geq 0$ and $\delta_1 \geq \delta_2 \geq \frac{\lambda_N}{\lambda_e}$

$$P(x, b, \delta_1, \delta_2) = P_0(b, \delta_1, \delta_2)\exp(-(\lambda_e - \lambda_N/\delta_2)x). \quad (11)$$

P_0 is the ruin probability for process starting from the threshold b, i.e. $x = 0$.

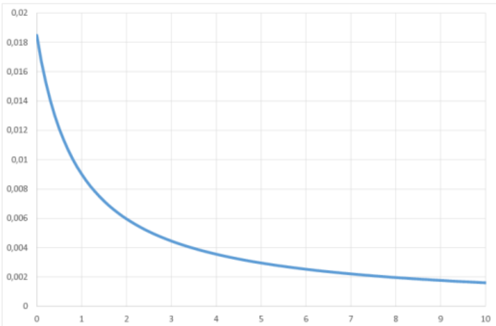
$$P_0 = P_0(b, \delta_1, \delta_2) = \frac{\lambda_N/\delta_2}{\lambda_e} \frac{(\lambda_e - \lambda_N/\delta_1)}{(\lambda_e - \lambda_N/\delta_2)\exp((\lambda_e - \lambda_N/\delta_1)b) + \lambda_N/\delta_2 - \lambda_N/\delta_1}. \quad (12)$$

On Figure 2 we show how the probability of ruin may depends on γ.The formula describing c^* is:

$$c^* = c^*(x, b, \delta_1, \delta_2) = \frac{\frac{\partial P_0}{\partial b} + \frac{\partial P_0}{\partial \delta_1}\frac{c_2}{b_L}}{\frac{\partial P_0}{\partial \delta_2} - x\frac{\lambda_N P_0}{\delta_2^2}}. \quad (13)$$

Since the partial derivatives $\frac{\partial P_0}{\partial b}$, $\frac{\partial P_0}{\partial \delta_1}$ and $\frac{\partial P_0}{\partial \delta_2}$ are negative, in the model based on Poisson compound process with exponential jumps, the threshold c^* is decreasing with respect to surplus x, see Figure 3.

Figure 3: *The threshold c^*as a function of x. Scale: multiples of b.*



Source: *Own research.*

4. Ruin Probability for Refracted Lévy Processes

By $X_\delta = (X_{\delta,t})_{t\geq 0}$, we denote the auxiliary one parameter family of Lévy processes, $\delta > \mathbb{E}(S_1)$,

$$X_{\delta,t} = \delta t - S_t, \quad (14)$$

Note that below the threshold b U is driven by X_{δ_1}, but above by X_{δ_2}. The basic characteristics are:

$$\mathbb{E}(X_{\delta,1}) = \delta - \mathbb{E}(S_1), \quad (15)$$

$$\mathbb{E}(\exp(zX_{\delta,1})) = e^{\delta z}\mathbb{E}(exp(-zS_1)), \quad Re(z) \geq 0. \quad (16)$$

In Kyprianou (2014), Kyprianou and Loeffen (2010), Czarna *et al.* (2019) the analytical formula for the ruin probability is provided in terms of so-called scale functions W_i (see also Biffis and Kyprianou (2010)). For $\mathbb{E}(S_1) < \delta_2 \leq \delta_1$ and $x \geq b$ we have

$$P(x, b, \delta_1, \delta_2, S) = \mathbb{P}(\kappa < \infty) \\ = 1 - \frac{\delta_2 - \mathbb{E}(S_1)}{1-(\delta_1-\delta_2)W_1(b)}\Big(W_1(x+b) + (\delta_1 - \delta_2)\int_0^x W_2(x - y)W_1'(y+b)dy\Big). \tag{17}$$

The scale functions are characterized in terms of the ruin and in terms of the Laplace transform

$$W_i(x) = W(x, \delta_i) \tag{18}$$

$$W(x,\delta) = \frac{\mathbb{P}(x+\inf(X_{\delta,t}:t\geq 0)\geq 0)}{\delta - \mathbb{E}(S_1)} = \frac{\mathbb{P}(-\inf(X_{\delta,t}:t\geq 0)\leq x)}{\delta - \mathbb{E}(S_1)} \tag{19}$$

For $z \in \mathbb{C}$ with positive real part

$$\int_0^{+\infty} e^{-zx}W(x,\delta)dx = \frac{1}{\delta z + \psi(z)}, \quad \psi(z) = \ln(\mathbb{E}(\exp(-zS_1))). \tag{20}$$

Note that $W_i(x)$ are nondecreasing, for $x < 0$ $W_i(x) = 0$, and moreover $\ln(W_i(x))$ are concave on the real half-line $[0, +\infty)$ (compare Kyprianou (2014) Lemma 8.2).

5. Example: Compound Poisson Process with Exponential Jumps

In this section we consider a special case when S is a compound Poisson process with exponential jumps with intensities respectively λ_N and λ_e.

$$S_t = \sum_{k=1}^{N_t} \xi_k, \tag{21}$$

where, $N = (N_t)_{t\geq 0}$ is a counting Poisson process with rate λ_N, and ξ_k $k = 1,2,\ldots$ are independent and identically distributed random variables, with distribution function $G(t) = (1 - \exp(-\lambda_e t))^+$, $\lambda_e > 0$, which are also independent of N_t, $t \geq 0$.

We recall that:

$$\mathbb{E}(\xi_1) = \frac{1}{\lambda_e} \quad \textit{and for} \quad z \in \mathbb{C}, Re\, z > -\lambda_e, \quad \mathbb{E}(\exp(-z\xi_1)) = \frac{\lambda_e}{z+\lambda_e}. \tag{22}$$

Hence:

$$\mathbb{E}(S_t) = \mathbb{E}\big(\mathbb{E}(\textstyle\sum_{k=1}^{N_t} \xi_k | N_t)\big) = \mathbb{E}(N_t\mathbb{E}(\xi_1)) = \mathbb{E}(N_t)\mathbb{E}(\xi_1) = t\frac{\lambda_N}{\lambda_e}. \tag{23}$$

It is observed that: $\mathbb{E}(S_t) = t\mathbb{E}(S_1)$. Similarly for complex z, such that $Re\ z > -\lambda_e$, we get:

$$\mathbb{E}(\exp(-zS_t)) = \mathbb{E}\left(\mathbb{E}(\prod_{k=1}^{N_t} \exp(-z\xi_k)|N_t)\right) = \mathbb{E}((\mathbb{E}(\exp(-z\xi_1)))^{N_t}) \quad (24)$$
$$= \sum_{n=0}^{\infty} \left(\frac{\lambda_e}{\lambda_e+z}\right)^n \frac{t^n\lambda_N^n}{n!} e^{-t\lambda_N} = \exp\left(\frac{-t\lambda_N z}{\lambda_e+z}\right)$$

and it is observe that $\mathbb{E}(\exp(-zS_t)) = (\mathbb{E}(\exp(-zS_1)))^t$.

Hence, the Laplace exponent for S equals

$$\psi(z) = \ln(\mathbb{E}(\exp(-zS_1))) = \frac{-\lambda_N z}{\lambda_e+z} \quad (25)$$

and

$$\frac{1}{\delta_i z+\psi(z)} = \frac{1}{\delta_i z-\frac{\lambda_N z}{\lambda_e+z}} = \frac{\lambda_e+z}{z(\delta_i\lambda_e-\lambda_N+z\delta_i)} = \frac{A_i}{z} + \frac{B_i}{z+\lambda_e-\lambda_N/\delta_i}, \quad (26)$$

Where:

$$A_i = \frac{1}{\delta_i}\frac{\lambda_e}{\lambda_e-\lambda_N/\delta_i} = \frac{1}{\delta_i-\mathbb{E}(S_1)}, \quad (27)$$
$$B_i = -\frac{1}{\delta_i}\frac{\lambda_N/\delta_i}{\lambda_e-\lambda_N/\delta_i} = -\frac{\mathbb{E}(S_1)}{\delta_i(\delta_i-\mathbb{E}(S_1))}. \quad (28)$$

Since for any $\lambda \in \mathbb{R}$ and $Re(z) > -\lambda$

$$\int_0^{\infty} e^{-zx}e^{-\lambda x}dx = -\frac{1}{z+\lambda}e^{-(z+\lambda)x}|_0^{\infty} = \frac{1}{z+\lambda}, \quad (29)$$

The following formulas are obtained for scale functions (compare Kyprianou (2014) exercise 8.3). For $x \geq 0$

$$W_i(x) = A_i + B_i\exp(-(\lambda_e - \lambda_N/\delta_i)x) \quad (30)$$
$$= \frac{1}{\delta_i}\left(1 + \frac{\lambda_N}{\delta_i\lambda_e-\lambda_N}\left(1 - \exp\left(-x\left(\lambda_e - \frac{\lambda_N}{\delta_i}\right)\right)\right)\right).$$

Basing on the above we get for $x \geq 0$

$$\int_0^x W_2(x-y)W_1'(y+b)dy = \int_0^x A_2W_1'(y+b)dy \quad (31)$$
$$+\int_0^x B_2\exp(-(\lambda_e - \lambda_N/\delta_2)(x-y))\frac{\lambda_N}{\delta_1^2}\exp(-(\lambda_e - \lambda_N/\delta_1)(y+b))dy$$
$$= A_2W_1(y+b))|_0^x$$
$$-\frac{B_2\delta_2}{\delta_1(\delta_1-\delta_2)}\exp(-(\lambda_e - \lambda_N/\delta_2)(x+b))\exp(\lambda_N(1/\delta_1 - 1/\delta_2)(y+b))|_0^x$$
$$= A_2W_1(x+b) - A_2W_1(b)$$

$$
\begin{aligned}
&-\frac{B_2\delta_2}{\delta_1(\delta_1-\delta_2)}\exp(-(\lambda_e-\lambda_N/\delta_1)(x+b))\\
&+\frac{B_2\delta_2}{\delta_1(\delta_1-\delta_2)}\exp(-(\lambda_e-\lambda_N/\delta_2)x)\exp(-(\lambda_e-\lambda_N/\delta_1)b)\\
&=A_2W_1(x+b)-A_2W_1(b)-\frac{B_2\delta_2}{\delta_1(\delta_1-\delta_2)}\frac{1}{B_1}(W_1(x+b)-A_1)\\
&+\frac{B_2\delta_2}{\delta_1(\delta_1-\delta_2)}\exp(-(\lambda_e-\lambda_N/\delta_2)x)\exp(-(\lambda_e-\lambda_N/\delta_1)b)\\
&=-\frac{1}{\delta_1-\delta_2}W_1(x+b)-A_2W_1(b)-\frac{B_2\delta_2\lambda_e}{(\delta_1-\delta_2)\lambda_N}\\
&+\frac{B_2\delta_2}{\delta_1(\delta_1-\delta_2)}\exp(-(\lambda_e-\lambda_N/\delta_2)x)\exp(-(\lambda_e-\lambda_N/\delta_1)b)
\end{aligned}
$$

Since $\delta_2-\mathbb{E}(S_1)=1/A_2$ it is finally obtain that the probability of ruin equals:

$$
\begin{aligned}
P=1&-\frac{1}{1-(\delta_1-\delta_2)W_1(b)}\frac{1}{A_2}\Big(W_1(x+b)-W_1(x+b)-(\delta_1-\delta_2)A_2W_1(b)\\
&-\frac{B_2\delta_2\lambda_e}{\lambda_N}\\
&+\frac{B_2\delta_2}{\delta_1}\exp(-(\lambda_e-\lambda_N/\delta_2)x)\exp(-(\lambda_e-\lambda_N/\delta_1)b)\Big) \qquad (32)\\
&=1-\frac{1}{1-(\delta_1-\delta_2)W_1(b)}(-(\delta_1-\delta_2)W_1(b)+1\\
&-\frac{\lambda_N}{\lambda_e\delta_1}\exp(-(\lambda_e-\lambda_N/\delta_2)x)\exp(-(\lambda_e-\lambda_N/\delta_1)b))\\
&=\frac{1}{1-(\delta_1-\delta_2)W_1(b)}\ \frac{\lambda_N}{\lambda_e\delta_1}\exp(-(\lambda_e-\lambda_N/\delta_2)x)\exp(-(\lambda_e-\lambda_N/\delta_1)b)\\
&=\frac{\exp(-(\lambda_e-\lambda_N/\delta_1)b)}{1-(\delta_1-\delta_2)(A_1+B_1\exp(-(\lambda_e-\lambda_N/\delta_1)b))}\ \frac{\lambda_N}{\lambda_e\delta_1}\exp(-(\lambda_e-\lambda_N/\delta_2)x)\\
&=\frac{(\lambda_e-\lambda_N/\delta_1)\exp(-(\lambda_e-\lambda_N/\delta_1)b)}{\delta_1(\lambda_e-\lambda_N/\delta_1)-(\delta_1-\delta_2)(\lambda_e-\lambda_N/\delta_1\exp(-(\lambda_e-\lambda_N/\delta_1)b))}\ \frac{\lambda_N}{\lambda_e}\exp(-(\lambda_e\\
&-\lambda_N/\delta_2)x)\\
&=\frac{\lambda_N/\delta_2}{\lambda_e}\ \frac{\lambda_e-\lambda_N/\delta_1}{(\lambda_e-\lambda_N/\delta_2)\exp((\lambda_e-\lambda_N/\delta_1)b)+\lambda_N/\delta_2-\lambda_N/\delta_1}\exp(-(\lambda_e\\
&-\lambda_N/\delta_2)x).
\end{aligned}
$$

Putting $P_0(b,\delta_1,\delta_2)=P(0,b,\delta_1,\delta_2)$ it is obtained:

$$
P=P_0(b,\delta_1,\delta_2)\exp(-(\lambda_e-\lambda_N/\delta_2)x). \qquad (33)
$$

6. Conclusions and Further Research

This paper aims to model the impact of the issuance of the CoCo bonds on banks' solvency. We provide a new measure of bank insolvency. In the typical approach of financial mathematics, the time horizon is assumed to be finite. That usually implies the question about the probability of a bank remaining solvable in one year (or n years). Contrary to that, as a measure of insolvency, we choose the probability of default at any moment in the future, a notion that is closely related to the so-called "ruin probability" widely applied in actuarial sciences. This means that our approach time horizon is boundless (infinite), for we base on the observation that CoCos are perpetual or very long-dated bonds.

In the proposed model, the leading characteristic of the bank's welfare is the CET1 ratio approximated by a refracted Lévy stochastic process. This leads to constructing the threshold c^* on the margin between the interest rates of CoCo bonds and straight bonds. If the margin is below the threshold, the further issuance of CoCo bonds decreases the ruin probability; if the margin is above the threshold, the further issuance makes this probability go up. This allows us to determine the CoCo bonds issuance's optimal level concerning the spread between the interest rates of CoCo bonds and straight bonds and the actual value of the CET1 ratio.

The issuance's optimal level is determined as an equilibrium between loss absorption due to possible write-down of CoCo bonds and significantly higher coupon (yield) CoCo bonds must pay compared to that of the other debt instruments with no write-down mechanism embedded. Furthermore, we show that the increase of the initial CET1 ratio reduces equilibrium c^*. When a bank is in poor condition (low CET1 ratio), then the issuance of CoCo bonds improves its solvency even for a relatively wide margin between interest rates, while when a bank is in good condition (high CET1 ratio), then the issuance of CoCo bonds improves solvency only for a relatively small margin.

There are two possible (straightforward) extensions of the results presented in our paper:

- The first one is to adopt the results concerning "Parisian ruin" (see, for example, Lkabous *et al.* (2017), Loeffen *et al.* (2018)). Indeed, there is some lapse of time between the first report that a bank is in distress and the supervisor's decision to intervene. Supervisory authorities are usually reluctant to step in before making sure that an individual bank's decline cannot be reverted by its management or shareholders on their own anymore. It would be useful to check the length of the lapse on the optimal level of CoCo bonds issuance.

- Another extension would be to compare the issuer side with the buyer (investor) side. The threshold c^* might be then considered as a selling price. It would be interesting to compare it with the buyer price obtained from the market-based pricing models for defaultable assets (see, for example, Spiegeleer and Schoutens (2012), Spiegeleer *et al.* (2017), Giang and Liang (2012), Jarrow and Turnbull (1995) Duffie and Singleton (1999), Longstaff and Schwartz (1995), Bielecki and Rutkowski (2004) and Bielecki *et al.* (2009)).

The latest EU regulation already allows for some of Pillar 2 Requirements on capital to be filled with AT1 instruments. We believe that our academic outcomes may give the supervisory authorities a reasonable tool to fill the P2R bank capital layer with AT1s so that their loss-absorbing capacity serves to maximize the purpose of extension banks' survivability. Besides, this would provide the EU legislators with some recommendations to re-design the existing CoCos legal framework – within the Basel III guidance - or even propose more radical changes to the Basel Agreement itself.

The model also proves useful in setting the optimum size of RT1 CoCos in the insurer's liabilities. AT1 and RT1 CoCos are very similar. Both present identical features: perpetuity, a 'synthetic maturity' of five years at least, and non-cumulative coupon deferral. Much like AT1s, RT1 debt presents a loss-absorbency mechanism on a going-concern basis, which implies that the debt could be written down upon the trigger event's regulatory capital ratio.

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