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An Integrated Investment Decision Analysis Procedure Combining Simulation and Utility Theory

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Abstract

In this paper we examine several methods that management could use to cope with the problem of risk in capital investment decision – making. We use the net present value model as a measure of profitability of an investment project. This measure is discussed in relation to their applicability and interpretation in risk analysis models. It is shown that it may be worth developing an integrated decision analysis procedure. The proposed procedure includes the Monte Carlo simulation to obtain a probability distribution of net present values, the calculation of the expected net present value, risk profiles analysis by stochastic dominance criterion, and as a final step the sensitivity analysis using utility functions with different levels of risk aversion. Numerical results obtained with this procedure are given. Finally, we discuss about relatively new approach of "real options" which can be used to expand our analysis methods.

Key words: Risk analysis, Monte Carlo simulation, risk profile, utility functions.

Introduction

All decision-making processes involve the future, which is subject to uncertainty. The concept of risk stems from our recognition of future uncertainty. Risk implies that a given action has more than one possible outcome. The term is usually reserved, however, for situations where the range of possible outcomes to a given action is in some way significant. Risk is therefore an important factor in the financial decision - making process and one that must be considered explicitly in all cases because:

- Such decisions often involve financial amounts, which are very significant for the firm concerned;

- Once made, such decisions are not easy to reverse, so the firm is typically committed in the long term to a particular type of finance or to a particular investment;
- It is the decision - maker's task to determine which decision will combine the lowest risk and the highest benefit.

Banks like other financial institutions are subject to both internal and external control over the amount of risk they can accept. The external controls are those imposed by government regulatory authorities and in the case of countries in the European Economic Union imposed by directives of the Council of the European Community. The importance of these directives and the way they have been implemented is their implication for risk acceptance. Perhaps the most important implication of the directives is that they represent an external influence on banks, which attempts to harmonize the magnitude of financial risk in their portfolio of risks in relation to their assets. The internal controls banks use to manage their portfolio of risks take several forms, each bank tending to develop its own methods. The procedures used by banking institutions are generally intended for confidential and strictly internal advice to the executive about the credit risks inherent proposals for loans they consider. The financial institution methods are essentially pragmatic and are only acceptable because the predictions they give agree fairly closely with experience. Now with market and credit risks being more rigorously assessed, the banks must face business risks (competitive attack, marketing errors, legal liabilities, technological change etc). Like business firms, banks may be regarded as investment agencies or intermediaries.

Central to a business firm will be decisions involved with questions of the types and quantity of finance to rise and with the choice of investments to be made. Their role is to raise funds from members of the public and from other investors and to invest those funds. In business finance, as in other aspects of life, risk and return tend to be related. In investment, investors require a minimum rate of return to induce them to invest at all but they require an increased rate of return, the addition of a risk premium, to compensate them for taking risk. Much of business finance is concerned with striking the appropriate balance between risk and return.

Accepting that maximization of shareholder wealth is the principal financial objective for all decisions in business firms, the assessment of possible investment projects should be regarded as only one step in a process of the search for opportunities. This process should be part of the routine of the firm. Business tends to be highly competitive, so opportunities overlooked by one firm will probably be taken up by another.

Staffs need to be encouraged to identify new products, new markets, new ways of supplying those markets and new approach to production. Technical help should be available to help staff to develop ideas into formal investment proposals.

The investment selection process in practical application may sometimes appear to be disorderly and imprecise. Mathematical modeling plays an important role in developing our understanding of a problem and aids in the decision making process.

When we look at the research works about how financial decisions should be made, we find a very substantial proportion of these is concerned with the improvement of the assessment risk procedures in a way that gives a decision - maker a clear understanding of what is likely to be acceptable.

In the past decades a huge literature addressing the decision under uncertainty has developed. In this literature there have been several discussions of the distinction between descriptive, normative and prescriptive modeling. In using descriptive models one seeks to understand how others do make decisions. Normative models allow one to explore the implications of certain norms or ideals of behavior. Many methods for risk assessment belong to both descriptive and normative categories.

In prescriptive approach deals with giving real people, as opposed to "fully rational" people. Prescriptive analysis can be viewed as some thoughtful guidance about how the decision - makers might wish to act in a wiser fashion in real situations.

The accumulated evidence is that utility theory has limitations as a descriptive theory of how people actually behave under uncertainty, but it remains as a valuable prescriptive approach for the inclusion of risk in decision analysis.

In this paper we examine several net present value based methods that management could use to cope with the problem of risk in capital investment decision making. It is shown that it may be worth developing an integrated decision analysis procedure that includes the Monte Carlo simulation to provide the risk profiles of the investment projects, the stochastic dominance analysis, and as final step the sensitivity analysis using utilities functions with different level of risk aversion.

The Need of Monte Carlo Simulation

In practice, the decision making process should involve the following steps: defining objectives, identifying possible investment opportunities, assembling the relevant data for an investment proposal, assessing the data and reaching a decision, monitoring the effects of the decision.

Assuming that the shareholder wealth maximization is the main objective of firms, the decision - making model should be net present value (NPV).

The NPV of an opportunity is the sum, taking account of plus and minus sign, of each of the annual cash flows discounted according to how far into the future each of one will occur, i.e.

$$\text{NPV} = \sum_{h=0}^{t} C_h \ /(1+a)^h$$

where C_h is the annual mean of the net cash flow after h years, t is the life of the opportunity in years, a is the discount rate.

But, the cash flow values selected in the net present value calculation should be their distributions, assuming that they are known, rather than their expected values. This involves the identification of the factors that will affect the cash inflows and the cash outflows. A tree diagram may be helpful, since it enables factors to be sub-divided until the decision - maker feels able to give a probability distribution for the possible values which the factor might assume. The analysis will be simplified if the factors can be identified in such a way that their probability distributions can be considered to be independent. It is possible to handle dependence, but it does add complications to the analysis.

The problem is, that having the distributions of the possible values, which each factor might assume, it must to determine their combined effect in order to obtain a probability distribution reflecting the possible range of the net present values. The number of combinations, which could affect the result, may be very large or infinite. This involves the use of a computer to generate a large number of possible combinations of circumstances, which might occur (Figure 1). When the simulation (Androecia et al. 1998, Bonini et al. 1997, Eppen et al. 1998, Luban 2000a, Luban 2000b) is performed, the more likely combination of circumstances will be generated most often, while very unlikely combinations will rarely be generated.

For each combination, the net present value is calculated and, by counting the probability with which a particular value occurred in the simulation, a decision-maker is able to estimate the risk associated to this value.

In terms of down - side risk, the decision - maker is also interested in how sensitive the advice is to changes in the estimates made about the project. The benefits of sensitivity simulation are not limited to evaluating the impact of decision - maker controlled variables on simulated NPVs. In addition, a sensitivity analysis can be run on the probability distributions, which describe uncertain variables of the model.

Expected Monetary Value

All business investment decisions have to be made on the basis of predictions of the various inputs. For example, the outlay required to undertake the project, its life, the annual cash inflows and outflows it will generate, the scrap value it will have, and even the correct rate of discount to reduce the cash flows to present values. Estimates will be made for all these factors and the project will then be appraised by calculating an expected net present value.

If for each investment project, N experiments of simulation have been carried out, *the expected monetary value and standard deviation* of that project's NPV can be calculated.

The expected E(NPV) for an investment project would be the expected value of the NPV distribution:

$$\text{E(NPV)} = \sum_{j=1}^{n} p_j \cdot NPV_i^j ,$$

where NPV^j_i is the net present value of the project *i* associated to the probability p_j , and p_j is the subjective or the simulated probability of the occurrence of the NPV^j_i.

The expected value criterion is a sensible guideline in great number of decision – making situations. If a decision-maker's attitude toward risk is not an important consideration in a decision problem, the expected value decision rule is generally preferred to other decision criteria.

It is only one that makes use of all information available, and that ensures a definite choice among the investment alternatives. The analysis can be useful for examining the value of additional information to a firm.

However, it is important to realize that it cannot be said to take into account risk, because that E(NPV) provides is a measure of the investment's expected performance, whereas risk is concerned with the likelihood that the actual performance may diverge from what is expected.

The standard deviation measures the spread of NPV distributions. A greater spread of one project distribution implies that there is more uncertainty about its NPV. Because of this, the spread of a distribution is often used as a measure of risk, which is associated with an investment project. For the standard deviation analysis to be valid, the probability distribution for NPV should be fairly close to the normal distribution.

When the decision-maker's attitude toward risk is an important consideration in the process of making decision, the dominance criteria and expected utility can be used to make quantification of preferences.

Dominance Criteria

Three methods can be employed to make a choice based on dominance: outcome dominance, event dominance and probabilistic or stochastic dominance.

The *outcome dominance* is the dominance in which the worst outcome from one action is at least as good as the best of second action. For outcome such NPV, the project i dominates the project h by outcome dominance if, for θ_{j} = state of nature 1,n:

$$\min_{j} NPV_i^j \geq \max_{j} NPV_h^j$$

The *event dominance* occurs if one action has an outcome equal to or better than that of a second action for each state of nature θ_j. Thus, the project i dominates the project h by event dominance if:

$NPV_i^j \geq NPV_h^j$, strict > for at least one θ_{j} = state of nature 1,…,n, The *probabilistic or stochastic dominance*. If there are m projects and n different states of the world, one project i probabilistically dominates a second project h if:

$$P(NPV_i^j \geq X_k) \geq P(NPV_h^j \geq X_k), \forall \theta_j, \; j = 1,...,n, \; \forall X_k, \; k = 1,...,m \times n,$$

where X is an ascending order vector of the all possible net present values NPV of the m projects for the n different states of nature θ_j, $j=1,n$.

Such probabilities as $P(NPV_i^j \geq X_k)$ are called *cumulative probabilities* or *risk profiles*, since they describe compactly the risks that the decision – maker faces.

If the project i dominates by outcome dominance, it will also dominate by event dominance and probabilistic dominance, but the reverse is not true. Also, if one project dominates a second, this implies that the expected value of the first project is greater than the expected value of the second. The reverse, however, is not true.

The main advantage is that these criteria can be useful in eliminating some alternatives and thus narrowing down the decision process. The disadvantage is that there may be no single alternative that dominates all the others.

However, the allocation of a project to particular risk class and the risk premium assigned to each class will be based on the manager's own personal attitude toward risk, and on the manager's personal perception as to the nature of risk and the reward required for accepting risk. The utility theory can be used to capture the decision – maker attitude toward risk. This attitude may be influenced by the available information about intangible factors such as future competitive advantage and future opportunities.

Utility Functions

For several years many works have been done in the field of utility theory, (Allais and Hagen 1979, Andreica et al. 1998, Fishburn 1970, French and Zhigang 1994, Keeney and Raiffa 1976, Neumann and Morgenstern 1947, Raiffa 1994). Different sets of axioms have been proposed regarding the behavior of an individual, which acts rationally and consistently. On the other side, numerous experiments have shown that people sometimes violate the basic assumptions on which the utility theory is based (for example the Allais paradox (Allais and Hagen 1979)). However, even if occasionally it may be necessary to move away from some of the axioms, it seems (French and Zhigang 1994) that utility theory provides a valuable family of models for the purposes of decision analysis.

In the utility theory, the axioms are used to construct the function of utility. An important axiom, which allows the decision - maker to assign a preferential index or utility to a set of consequences, assumes that the decision - maker is able to specify for any alternative whose results are uncertain, an exactly equivalent alternative which would be equally preferred but which would involve a certain result, i.e. for any gamble, the decision - maker is able to specify a certainty equivalent. In this way, the utility function represents the subjective attitude of a decision - maker to risk and can be used to explore the way in which an individual makes decision about risky alternatives, on the assumption that he does so in order to maximize his own expected utility index.

We will discuss two known methods for creating the utility function, which could be included into an integrated procedure. The first is a more accurate, yet tedious process because it is based on the certainty equivalents, while the second method is faster because it assumes a predetermined shape.

Creating a Utility Function with Equivalent Lottery

This approach requires the decision - maker to make a series of choices between a sure payment and a lottery. In more formal language, the decision - maker is called to create an "equivalent lottery".

Firstly, in order to express a utility function, the decision - maker will select the domain of the values of the NPVs as narrow as possible such that it contains all values that he might want to analyze. The steps of the method for creating a utility function with equivalent lottery are as follows:

Step 1. Determine two values to use as reference points. For example, to the worst NPV the decision - maker might assign utility value of 0, and to the best NPV he might assign utility value of 1. That is:

$$U(\text{worst } NPV) = 0,$$
$$U(\text{best } NPV) = 1.$$

Step 2. Choose a certainty equivalent CE, such that (worst NPV) < CE < (best NPV), and such that the decision - maker is indifferent between the following two alternatives:

Alternative 1. Receive a payment of CE for sure.

Alternative 2. Participate in a lottery that offers one-half chance at the "best NPV " and one-half chance at the "worst NPV ".

The expected utility of the *Alternative 2* is:

$U(Alternative\ 2) = 0.5U(\text{best } NPV) + 0.5U(\text{worst } NPV) = 0.5$

Decision - maker will change CE until it is equivalent to the lottery, then U(CE) = U(*Alternative 2)*.

Step3. Using the CE obtained in Step 2 as new end point, instead of "best NPV ", for example, choose a new certainty equivalent CE, such that (worst NPV) < (new CE) < (old CE) and such that the decision - maker is indifferent between the following two alternatives:

Alternative 1. Receive a payment of (new CE) for sure.

Alternative 2. Participate in a lottery that offers one-half chance at the "old CE" and one-half chance at the "worst NPV ".

The expected utility of the *Alternative 2* is:

U(*Alternative 2)* = 0.5*U(old CE)* + *0.5U(worst NPV)* = 0.25

Decision – maker will change (new CE) until it is equivalent to the lottery, then U(new CE) = U(*Alternative 2)*.

After as many steps of this process as needed and using the new CE as new end points, the decision – maker obtains utility function. In this process, certainty equivalent CE is a certain outcome as attractive as a lottery.

If the assigned certainty equivalents are less than the expected value of the end points, the decision – maker is "risk averse". The difference between the expected value and certainty equivalent is seen by the decision – maker as the compensation (risk premium) he requires to bear the risk involved with the decision alternative. Utility functions are concave if the decision – maker is risk averse.

If the assigned certainty equivalents are greater than the expected value of the alternative, the decision – maker is "risk seeker", because he is willing to pay a premium in order to be allowed to bear a risk. In this case the utility function is convex.

Similarly, if a decision – maker assigns certainty equivalents equal to the expected value of the alternative, he is termed "risk – neutral".

Creating a Utility Function based on the Arrow – Pratt Risk Aversion Coefficient

For a concave utility function U(φ), Arrow and Pratt, (Andronic 2000), derived the risk aversion coefficient:

$$S(\varphi) = -U''(\varphi)/U'(\varphi)$$

For $S(\varphi) = S = constant$, this differential equation allows general solutions of the following form:

$$U(\varphi) = \alpha - \beta e^{-S\varphi}$$

where φ is the monetary NPV that must be converted to utility, α and β are parameters that can be set to scale the function (to define the 0 and 1 points, for example). If we substitute the coefficient S by $S = 1/R$, we obtain an exponential utility function which is used to analyze many financial investment decisions and other business applications:

$$U(\varphi) = \alpha - \beta e^{-\varphi/R}$$

The only parameter to assess is the constant R. Because R has a constant value, the risk premium does not change in different part of the curve. One advantage of using such exponential utility function is that it allows one to do sensitivity analysis by varying the risk aversion parameter, R. The larger value of R, the less risk - averse is the decision - maker. Likewise, the smaller the value of R, the more risk - averse the decision - maker is.

Assessment of R

I. The decision - maker can determine the amount R such that he is indifferent between the following two choices:

   Alternative 1. A 50/50 gamble where the payoffs are a gain of R or a loss of R/2;

   Alternative 2. A zero payoff.

II. Another way to determine R comes from empirical evidence gathered by the decision analysts, (Eppen et al. 1998). A very valuable rule of thumb was found. It relates the company's net income, equity, and net sales to the degree of risk aversion R. For example, it was found that R is approximately equal to 124% of net income, 15.7% of equity, and 6.4% of net sales. Of course, these are only guidelines, but they can be very helpful and certainly indicate the trend for larger companies to have larger R-values and less aversion to risk.

Procedure for Including the Risk in Decision Analysis

While utility theory remains the most valuable prescriptive approach for the inclusion of risk in decision analysis, it is difficult to obtain accurate and consistent utility function values. It may be worth developing an integrated decision analysis procedure that includes as a final step the sensitivity analysis using utility functions with different levels of risk aversion.

For practical reasons, the procedure can be constructed in the following way:

PHASE I. *Expected monetary value analysis based on the NPV distribution obtained by Monte Carlo simulation*

Step I.1. Identify the factors that will affect the cash inflows and the cash outflows and their probability distributions.

Step I.2. Perform Monte Carlo simulation to sample a value from each distribution and calculate the NPV

Step I.3. Calculate the expected value of the NPV distribution obtained by simulation for each project.

The expected monetary value approach approximates the average outcome of a strategy. However, because of this it fails to capture what is the essence of risk, by ignoring the variability of outcome. In order to take risk into account in making decision go to Phase II.

PHASE II. *Risk profiles analysis*

Step II.1. Determine the cumulative probability distribution of the NPVs and draw the risk profile for each project.

Step II.2. Use the stochastic dominance criterion to choose among projects.

However, if the NPVs appear unreasonably large or unreasonable small, and if the decision - maker feels monetary values do not adequately reflect his true preferences for the outcomes, a utility analysis of the problem should be considered.

PHASE III. *Utility analysis*

The analysis based on the utility functions for a risk – averse decision – maker can be carried out as follows:

Step III.1. Determine the best and the worst possible NPVs in the decision problem. Assign utility values to the best and worst NPVs. Any values will work as long as the utility assigned to the best payoff is greater than the utility assigned to the worst payoff. Suppose that $U(worst\ \text{NPV}) = 0$ and $U(best\ \text{NPV}) = 1$.

Step III.2. Assessing a utility function. We propose an exponential function in order to avoid the difficulty regarding the construction of a utility function that often causes decision – makers to feel uncomfortable. It has the form:

$$U(\varphi) = \alpha - \beta\ exp(-\varphi/R^L),$$

where φ is a monetary NPV that must be converted to utility,

$\alpha=\beta \cdot exp(-(worst\ \text{NPV})/R^L)$,

$\beta =1/(exp(-(worst\ \text{NPV})/ R^L)-exp(-(best\ \text{NPV})/ R^L))$.

The risk aversion parameter R^L, can be obtained by trying different values of R such that $U(L) = 0.5 \cdot U(R^L)+0.5 \cdot U((R^L)/2) =U(0)$

Step III.3. Perform the sensitivity analysis to determine how risk averse the decision – maker would have to be toward the selected strategy.

a) Determine the certainty equivalent CE for the lottery L involving a 50/50 chance of winning R^L monetary units and losing $(R^L)/2$ monetary units, i.e.

$$CE(L)= - R^L \cdot ln((\alpha-U(L))/\beta).$$

b) If the decision – maker values the lottery L at more than $CE(L)$, he is less risk averse than R^L utility function.

c) Continue the sensitivity analysis to determine how large the risk aversion parameter R would have to be for the decision - maker to be indifferent between different alternatives vi or vh, i.e. EU(vi) = EU(vh), where the expected utility for the decision alternatives v_i is:

$$\mathrm{EU}(v_i) = \sum_{j=1}^{n} p_j\, U(\varphi_{ij}),$$

Step III.4. With risk aversion parameter R established at *Step III.3,* determine the investment project, which has the highest expected utility.

Cautions

If either of the NPV assumptions is seriously violated then the NPV will not accurately represent the decision - maker's preferences between sums of money arriving at different points in time. In this case, converting the NPVs to utilities might not lead to ranking of investment options, which reflects the decision - maker's true preferences.

Numerical Results

To demonstrate the described procedure, we consider the following example. A decision - maker which dislikes risk, has to decide between two alternative investments: machines A and B. In Table 1, the annual cash inflows and cash outflows are described with triangular distributions.

PHASE I. *The calculations involved in determining the NPV of the two potential investments are shown in Table 2. A discount rate of 15% is considered.*

It results that the expected monetary optimal investment is machine A with expected net present value of 10.13 monetary units.

PHASE II. *The risk profiles of the investment project can be obtained with @RISK package (Palisade Corporation 1997). As an "add–in"* *to Microsoft Excel, @RISK links directly to Excel to add Risk Analysis capabilities.*

With @RISK all uncertain values for cash inflows and cash outflows can be described by the @RISK distribution function.

In Figure 2 are shown the cumulative distributions for NPV for the investment options A and B. It can be seen that the NPV could be between about (–30) monetary units and 60 monetary units. There are about a 13% probability that the NPV to be negative. Also, it can be seen that for NPV greater than 15 monetary units, investment option A dominates by probabilistic dominance because its cumulative curve is everywhere the same or above the cumulative curve of B. In order to decide about the investment alternative and to take into account some intangible factors the decision – maker could use the utility analysis.

Phase III. *The NPVs obtained by Monte Carlo simulation can be converted to utilities by using the exponential utility function with only one parameter R. Figure 3 shows utility functions for different values of R obtained with EXCEL.*

For R= 14 monetary units, expected utility for investment option A is equal to expected utility for investment option B, that means that the decision – maker is indifferent between investment option A and investment option B. For R> 14 monetary units, decision – maker is less averse than the R=14 curve and the expected utility for investment option A is greater than the expected utility for investment option B.

The value of R < 14 monetary units implies a greater risk aversion. The expected utility for investment option B is greater than the expected utility for investment option A. This implies that investment B should be undertaken.

Concluding Remarks

The results show that a user – friendly spreadsheet including risk profile approach and utility analysis could be a valuable tool to add to a net present value technique. However, we should emphasize that this procedure can be useful in practice and can be included in the knowledge base of an expert system, but it is not procedure to be followed mechanically. We should not forget that the judgement of the decision - maker is still crucial inputs to models for decision analysis.

A relatively new technique, which has the potential to include management's flexibility to alter decisions as new information becomes available, is the real option approach. In most real investment opportunities there are managerial flexibility (real options) embedded into the projects. Some work in real options (Trigeorgis 1995) has generated a taxonomy that has broken down real options into six categories based upon the type of flexibility provided. The six categories are: the option to defer; the option for staged investments; the option to change scale; the option to abandon; the option to switch; and the option to grow. At this moment, it is not clear how such approach can be included to our decision analysis procedure. The first step may be to take another look at the investment projects. The manager should to look at the projects for the options they provide. Then, a project should be examined to see if it already contains flexibility embodied in the six types of real options. Alternately, the manager will see if the project can be modified to include the different types of real options thus providing the decision - maker with additional flexibility. Obviously, Monte Carlo simulation, utility analysis and decision trees still remain valuable tools for investment decision analysis.

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Table 1: *The cash inflows and cash outflows (monetary units)*

			Year				
			0	1	2	3	4
Machine A	Cash inflows	min	–	14	15	20	12
		most likely	–	28	30	30	24
		max	–	50	60	70	56
	Cash out-flows	min	60	2	2	6	8
		most likely	60	4	8	12	14
		max	60	5	16	20	24
Machine B	Cash inflows	min	–	14	18	20	16
		most likely	–	26	26	40	42
		max	–	36	58	60	70
	Cash out-flows	min	60	2	4	6	8
		most likely	60	8	8	10	10
		max	60	10	12	20	24

Table 2: *Calculating the net present values (monetary units)*

Machine A	**Year 0**	**Year 1**	**Year 2**	**Year 3**	**Year 4**
Mean of cash in-flows	0	30.67	35.00	40.00	30.67
Mean of cash out-flows	60	3.67	8.67	12.67	15.33
Net cash flow	-60	27.00	26.33	27.33	15.33
Discounting factor	1	0.87	0.76	0.66	0.57

Discounted cash flow	-60	23.48	19.91	17.97	8.77
Machine B					
Mean of cash in-flow	0	25.33	32.00	40.00	42.67
Mean of cash out-flow	60	6.67	8.00	12.00	14.00
Net cash flow	-60	18.67	24.00	28.00	28.67
Discounting factor	1	0.87	0.76	0.66	0.57
Discounted cash flow	-60	16.23	18.15	18.41	16.39
Net present value A	10.13	9.18			
Net present value B					

Figure 1: *Simulation framework*

Figure 2. *Risk profiles obtained by simulation*

Figure 3. *Utility functions for different values of R*