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The Capital Asset Pricing Model: A Review Of The Issues

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Abstract

The aim of this paper is to review the literature relating to the theoretical basis of the Capital Asset Pricing Model (CAPM). The derivation of the CAPM is presented, followed by the empirical tests of it. There exists some further research however which is also presented, that criticize the CAPM, since it has often been challenged by statistical studies that fail to verify the validity of the model, as an adequate description of the way assets are priced. (JEL G120, G100)

1. Introduction

The CAPM is a model of capital market equilibrium which attempts to measure and price risk. It was initially developed by Sharpe (1964, 1965), Linter (1965, 1969) and Treynor (1967).

The CAPM relates the expected return on an asset to its systematic risk. It merely states that in equilibrium the rates of return on all risky assets are a function of their covariance with the market portfolio.

More specifically the required expected rate of return on any asset, $E(r_i)$, equals the risk free rate of return, r_f, plus a risk premium:

$$E(r_i) = r_f + B_{im} [E(r_m) - r_f] \quad (1)$$

The risk premium can be thought of as the extra compensation, above the risk free rate, that the investors require for investing in the market portfolio. It is the product of the quantity of risk with the price of risk. The price of risk is the difference between the expected rate of return on the market portfolio and the risk free rate. The quantity of risk, usually called beta, is defined as the covariance between the returns on the risky asset and the market portfolio divided by the variance of the market portfolio:

$$B_{im} = \text{cov}(r_i, r_m) / \text{var}(r_m) \quad (2)$$

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The beta factor is the appropriate measure of risk for a single asset and has some interesting properties : When the assets returns are independent from the market then the covariance of the asset with the market is zero and beta equals zero.

Consequently the mean return on the asset equals the risk free rate of return. When the asset moves with the market then $cov(r_i,r_m) = var(r_m)$ and B=1. It follows that $E(r_i)=E(r_m)$ and the asset can be considered of average riskiness. If B>1, the asset is above average riskiness and if B<1 below average riskiness.

It is important, at this point, to make the distinction between the total risk of an asset and its systematic risk. The total risk (i.e.its variance) of an asset can be divided into two elements : The unsystematic risk of the asset which is independent of the economy (and which can be eliminated through diversification), and the systematic risk, the beta, which is the risk of the economy (and cannot be diversified away). Furthermore when the assets are combined into portfolios all we need to know in order to determine the beta of the portfolio is the individual betas of the assets, because betas have the valuable property of being linearly additive.

2. Derivation of the model

In equilibrium, the prices of all assets must adjust until they are all held by investors, i.e. there can be no excess demand, the supply of all assets must equal the demand for holding them. Consequently, in equilibrium, the market portfolio will consist of all marketable assets held in proportion to their market value weights. The equilibrium proportion off each asset in the market portfolio, $w_{i,}$ must be the market value of the individual asset divided with the market value of all assets.

A portfolio consisting of a% invested in a risky asset I and (1–a)% invested in the market portfolio will have the following mean and standard deviation:

$$E(R_p) = a\,E(R_i) + (1-a)\,E(R_m) \quad (3)$$

and

$$\sigma(R_p) = [a^2\sigma^2 + (1-a)^2\sigma_m{}^2 + 2a(1-a)\sigma_{im}\,]^{½} \quad (4)$$

where $\sigma_i{}^2$, $\sigma_m{}^2$, σ_{im}, the variance of risky asset I, the variance of the market portfolio and the covariance between the asset I and the market portfolio, respectively.

Recall that the definition of the market portfolio is that it consists of all assets held according to their market value weights.

The change in the mean and the standard deviation with respect to the percentage of the portfolio, a, invested in risky asset I is determined as:

$$dE(R_p) / da = E(R_i) - E(R_m) \quad (5)$$

and (6) the following

$$d\sigma(R_p) / da = 1/2\,[a^2\sigma^2 + (1-a)^2\sigma_m{}^2 + 2a(1-a)\sigma_{im}\,]^{1/2}\,x$$

$$x\,[2a\sigma_i{}^2 + 2\sigma_m{}^2 + 2a\sigma^2{}_m + 2\sigma_{im} - 4a\sigma_{im}]$$

But, in equilibrium the market portfolio already has the market value weight, w_i percent, invested in risky asset. Therefore the percentage a in the above equations is the excess demand for an individual risky asset, which in equilibrium must be zero.

So, when $a=0$ (5) $= E(R_i) - E(R_m)$ (7)

And (6) $= 1/2\,(\sigma_m^2)^{1/2}\,(-2\sigma_m^2+2\sigma_{im}) = \sigma_{im}-\sigma_m^2 / \sigma_m$ (8)

The slope of the risk–return trade off at the point where the market portfolio meets the opportunity set of the asset I is:

$$(7) / (8) = E(R_i) - E(R_m) / [(\sigma_{im}-\sigma_m^2) / \sigma_m] \quad (9)$$

Finally, the slope of the opportunity set that is provided by the relationship between the risky asset and the market portfolio must equal the slope of the capital market line, which is:

$$E(R_m) - R_f / \sigma_m \quad (10)$$

So, since (9)=(10), then

$$E(R_i) - E(R_m) / [(\sigma_{im}-\sigma_m^2) / \sigma_m] = E(R_m) - R_f / \sigma_m$$

and solving for

$$E(R_i)\text{: } E(R_i) = R_f + (\sigma_{im}/\sigma_m^2)\,[\,E(R_m)-R_f\,],$$

which is the CAPM.

For the derivation of the model a frictionless world is assumed, where transactions costs and taxes are absent, information is costless and equally available to all investors. Investors are also assumed to have homogeneous expectations about asset returns (which are assumed to be normally distributed), to be price takers and risk averse individuals who maximize the expected utility of their end of period wealth.

Furthermore, assets are assumed to be marketable and perfectly divisible and there also exists a risk free asset such that investors may borrow or lend unlimited amounts at the risk free rate.

Many of the assumptions appear unrealistic, the rejection of these assumptions, however, can have severe implications for the original CAPM. For example the first five assumptions are also assumptions that must hold for an efficient market to exist. If we reject those assumptions we also reject the existence of a single efficient portfolio and a unique trade off between risk and return.

Naturally, testing for the validity of the assumptions of the CAPM has drawn the attention of many researchers.

Brennan (70) has examined the effect of introducing taxes, more specifically taxes that are caused by the differential tax rate on capital gains and dividends, since none has examined the model in a world with personal and corporate taxes. He concluded that the only change required in the CAPM is the addition of an extra term in equation (1) : a dividend payout variable. So now the expected return on an asset depends not only on the systematic risk, but also on the dividend yield.

The existence of nonmarketable or perfectly divisible assets (eg.human capital) and their impact on the model was examined by Myers (72). He showed that because these nonmarketable assets will differ in riskiness investors will hold different portfolios of risky marketable assets. He developed a model similar to the CAPM where the beta measures along with the covariance with the market portfolio the payoff from the nonmarketable assets. Myers model, however, has been tested by Fama and Schwart (77) who found no evidence that it provides a better explanation of returns than the original CAPM. The assumption of homogeneous expectations about future returns has been tested by Linter (69). He demonstrated that the existence of heterogeneous expectations does not critically alter the CAPM, except that returns and covariances becomes complex weighted averages of investors diverse expectations. In this case, however, the market portfolio is not necessarily efficient, which makes the CAPM not testable.

In another study Merton (73) has derived an alternative CAPM in which trading takes place continuously over time. If the risk free rate is non–stochastic then we can extend the model in a continuous form. If the risk free rate is stochastic then the portfolio investors will hold a portfolio which will consist of three funds : the risk free asset, the market portfolio and a portfolio with returns which have a perfect negative correlation with the risk free rate. Concerning the risk free asset there are two points that bring the relevant assumption at odds with reality : First, inflation will influence the real return from that asset and make it difficult to identify and second, for most investors, in the real world, the borrowing rate does not equal the lending rate. Black (72) suggested to replace the risk free asset with a zero–beta portfolio (and by implication uncorrelated with the market portfolio) constructed by short selling. He proposed a model (known as the zero–beta CAPM) where the expected rate of return on any asset is a linear combination of the expected rate of return of the market portfolio and a unique minimum variance zero beta portfolio:

$$E(r_i) = E(_z) + B_{im} [E(r_m) - E(r_z)] \tag{11}$$

where $E(r_z)$: expected return on zero–beta portfolio.

In another study, Fama (65), examined what happens if returns are not normally distributed. He showed that as long as returns are distributed symmetrically and stable, investors can use other measures of dispression than the variance and the portfolio theory as well as the CAPM is still valid.

In the case where investors are not having a price taking behaviour Linzenberg has shown that the model is still valid only that the market price for risk (ie. the expected return on the market above the risk free rate) will be lower than usual.

As a conclusion concerning the relaxations of the assumptions of the CAPM, we can say that the results of different tests indicate that the model does not change drastically if the assumptions are violated. As Jensen (72) put it, after reviewing the studies that relax the assumptions, the theory is "reasonably robust" with regard to the violations of the assumptions.

3. CAPM : The empirical tests

The CAPM is a model that is expressed in terms of expected returns. Data, however, about investors expectations are not available. Therefore, researchers involved in tests of the model must transform it from an ex ante form to an ex post form, i.e. a form that uses observed data. In so doing they have to make an important assumption, the assumption that the rate of return on any asset is a fair game, i.e. that investors expectations have been realized, on average. If markets are efficient it is not unreasonable to assume that even though there will be a difference between the realized returns and the expected returns, this difference will not be statistically different from zero. In testing the CAPM there are usually two approaches: A two–stage cross–section approach and a time series approach.

In the former approach, first estimates of the betas are obtained from a regression of the returns for an individual i, on the returns from the market portfolio, i.e. from the market model[2]:

$$R_i = a_i + B_{im} R_{mt} + e_i \tag{1}$$

In the second stage these estimates are used in the regression equation :

$$R_{it} = c_0 + c_1 B_{it} + e_{it} \tag{2}$$

which is the way the CAPM is, usually, written when it is tested empirically.

This is done in order to get estimates of c_0 and c_1 i.e.the intercept and the slope and then compare them with their hypothesised values, the R_f and the $[R_m–R_f]$, respectively. The intercept must equal the risk free rate of return and the slope must equal the risk premium on the market portfolio, for the CAPM to be valid.

In the time–series approach an indirect test of the CAPM is conducted by comparing the model:

$$E(R_i) – R_f = B_i [E(R_m)–R_f] \tag{3}$$

with the model:

$$R_i – R_f = a_i + B_i [R_m–R_f] + e_i \tag{4}$$

in order to see whether the intercept a_i is significantly different from zero.

Many studies have been conducted on testing the CAPM.The most important ones will be reviewed below.

Jensen (1966) tested the CAPM using a two stage cross section methodology, in a sample of 115 mutual funds. He first estimated the beta coefficient of each fund, then he calculated the annual return of each fund (in a continuously compounded form) and finally he used continuously compounded annual returns of the S&P 500 as a proxy for the market portfolio. He estimated the characteristic line for each of the 115 mutual funds, then he plotted the 10 year holding period return of the funds on their systematic risk. This revealed a significant positive relationship between the realized ten year holding period returns and the systematic risk.

In addittion he examined the risk–return relationship of 56 mutual funds for the period 1945 – 1954 and for the period 1945 – 1964. He discovered a linear relationship between returns and systematic risk and also that the systematic risk of the mutual funds was stationary over time.

Douglas (1969) used a sample of 616 common stocks for the period 1926 – 1960.First he divided this period into seven 5–year subperiods and then he computed the average quarterly returns and variance of returns, for each stock during each subperiod. Then he determined the covariance of each stock return with the return on the index (the 616 sample securities). Finally he regressed the average quarterly return of the ith security on the variance of the ith security returns (i.e. the total risk) and the covariance of the ith security return with the index (i.e. the systematic risk). He found a significantly positive total risk coefficient for almost all the subperiods and a not significantly different from zero systematic risk coefficient, for all but 2 subperiods. A result inconsistent with the CAPM predictions.

The implication of his results is that returns have a linear relationship with total risk rather than systematic risk.

Linter (1970) used data for 301 common stocks for the period 1954 – 1963 and, first, he calculated the annual realized return of the securities, second, he estimated the b coefficient of each security by regressing each security's return on the average annual return of all the stocks of his sample. In addition he calculated the variances of the errors of each regression equation (a measure of unsystematic risk). Then he regressed the average returns of the securities on both risks: the systematic (beta) and the unsystematic (variance of error terms). The results revealed that average annual returns were a linear function of both risks, a result inconsistent with the CAPM theory.

Miller and scholes (1972), used the returns of 631 stocks from the NYSE for the period 1954 – 1963, to reproduce Linter's study. Their results were similar to Linter's, i.e. returns were correlated to both systematic and unsystematic risk. They argued, however, that many statistical problems can arise with respect to the methodology used, namely errors in measurement in beta, skewness in the return distributions, etc.

Black, Jensen and Scholes (1972), in an effort to reduce the bias caused by errors in measuring betas and the problem of correlation of individual stocks used portfolios of securities rather than individual securities. They estimated the systematic risk for all stocks using monthly data from the previous 5 years. Next, by ranking the securities according to their betas and dividing them into decile they were able to construct a portfolio for every decile. Then for each portfolio they estimated the following time–series regression :

$$R_{it} - R_{ft} = a_i + B_i\,(R_{mt} - R_{ft}) + e_{it}$$

Their sample was all stocks traded in the NYSE from 1926 –1966. Their findings were inconsistent with Sharpe's CAPM : The high–risk portfolios experienced lower returns than implied by the CAPM and the low–risk portfolios experienced systematically greater returns than predicted by the model. Then they did cross–sectional analysis of the data and they found a significant intercept term and a coefficient of b systematically below the average risk premium of the market portfolio. They rejected Sharpe's version of the CAPM but concluded that the results were more consistent with the Black's version of the CAPM.

Fama and Mcbeth (1973) tested the relationship between average return and risk for common stocks. The theoretical basis of their tests was the "two parameter" model.

They argued that the model had 3 testable implications : first that the relationship between risk and expected return is linear, second that the beta coefficient is a complete measure of risk and third that higher risk should be associated with higher returns. What they did was to test the following generalization of the model :

$$R_{it} = C_{0t} + C_{1t} B_i + C_{2t} B^2_i + C_{3t} S_i + n_{it}$$

In this model two terms are added : one term to measure possible nonlinearities and another term to capture possible influence of unsystematic risk. The test hypotheses were the above plus two more : that the intercept is equal to the risk–free rate (a hypothesis of the Sharpe–Linter CAPM) and that the coefficients C_{0t}–E[R_{0t}], C_{1t}– [E(R_{mt}) – E(R_{ot})], C_{2t}, C_{3t} and n_{it} are fair games, ie requirements for capital market efficiency in a two parameter model.

The data for the study was monthly percentage returns (including dividends and capital gains) for all common stocks traded in the NYSE during the period between Jan 1926 and June 1968. Their methodology was to construct 20 portfolios on the basis of ranked values of B's for individual securities. In order to avoid the regression phenomenon they formed portfolios from ranked estimates of B computed from data for one period but then using a subsequent period to obtain the portfolio B that were used to test the two parameter model. So, with fresh data within a portfolio, errors in the individual security B_i were to a large extent random across securities. Their results supported all the implications of the model: The relationship between expected return and betas was linear, no measure of risk, in addition to beta, systematically affected returns and on average there was a positive tradeoff between risk and return. The Sharpe–Linter hypothesis that the constant term is equal to the risk–free rate was not supported by the data. Finally the fair game hypothesis was supported by the data, ie. the view of efficient capital markets was supported.

Friend and Blume (1973) examined both theoretically and empirically the reasons why the market line theory does not adequately explain deferential returns on financial assets (which was the result of their previous paper). They argued that two of the reasons was the inability of investors to borrow large amounts of money at the same risk–free interest rate at which they could lend and the deficiencies in the return generating models which are required to translate ax ante expected returns and "risks" into ex post realizations. They suggested a more complicated generating process, than Jensens original model, which however could be an adequate process for a subset of all financial assets (such as common stocks) only if the minimum variance zero–beta portfolio consisting only of common stocks IS NOT Black's zero–beta portfolio. In their empirical tests they used data from the NYSE for the period 1950–1968 and the following methodology. To cope with the measurement error problem they used a grouping technique : Beta coefficients were estimated by regressing monthly investment relatives upon the corresponding Fisher Combination Link Relatives. Then 12 different portfolios of approximately 80 securities were formed on the basis of these estimates. The first portfolio consisted of the stocks with the lowest beta estimates, the second portfolio consisted of the stocks with the next lowest beta estimates, and so on.

Then monthly returns for each portfolio were calculated (for the subsequent 5 year period) under 2 different assumptions concerning the initial investment to each security: i) equal investment, ii) investment proportional to the market value of shares outstanding. Then these monthly returns (for the subsequent 5–year subperiod) were averaged for each portfolio to obtain a portfolio of monthly returns and were then regressed on the FCLR to yield an estimate of the beta for the portfolio. Finally, these arithmetic average returns were regressed on the beta coefficients in both linear and quadric forms.

Their results suggested that a linear model is a tenable approximation of the empirical relation between risk and return. The authors then argued that if the variance of the zero–beta portfolio is zero, their test is a direct test of the CAPM and that then the point estimates of the risk–free rate differ substantially from the actual.

Basu (1977) attempted to find whether the investment performance of common is related to their PE ratios. His data consisted of firms traded in the NYSE for the period 1956–1971. He first computed the PE ratio of every sample security. The numerator of the ratio was defined as the market value of common stocks (market price times number of shares outstanding) as of December 31 and the denominator as reported annual earnings (before extraordinary items) available for common stockholders. Then the securities were ranked on the basis of these ratios and 5 portfolios were formed. Basu argued that since over 90% of the firms release their financial reports within 3 months of the fiscal year–end, the PE portfolios were assumed to be purchased the following April. The monthly return of each of these portfolios were computed for the next 12 months, assuming an equal initial investment in each of their respective securities and then a buy and hold policy. The above procedure was repeated for 14 years.

Basu computed a wide range of measures of portfolio performance (annual rate of return, Treynor's reward to volatility, Sharpe's reward to variability measure, systematic risk, etc). Also he used OLS in all estimations. His results indicated that average annual rates of return decline as one moves from low PE to high PE portfolios. Furthermore, higher returns on the low PE portfolios were not associated with higher systematic risk. These results were generally true even when risk was taken into account.

Basu concluded that the results suggested a violation of the joint hypothesis that the asset pricing models have descriptive validity and that security price behaviour is consistent with the efficient market hypothesis. PE information was not "fully reflected" in security prices in as a rapid manner as suggested by the semi–strong form of efficiency.

Banz (1981) examined the empirical relationship between return and the total market value of common stocks. The data he used was all common stocks quoted on the NYSE, for at least 5 years, for the period 1926–1975, from the CRSP tape. In response to Roll's critique of asset pricing tests he used 3 different market indexes. The first two was the CRSP equally and value weighted indexes and the third was a value weighted combination of the CRSP value weighted index and return data on corporate and government bonds.

His tests were based on a generalized asset pricing model in which returns on common stocks were a function of beta risk and an additional factor which was the market value of the equity. If there is no relationship between this additional factor and expected return, then its coefficient should be zero and the model reduces to Black's zero–beta CAPM. Banz grouped securities into portfolios using the following technique : The securities were assigned to one of 25 portfolios containing similar number of securities, first to one of five on the basis of market value of the stock, then the securities in each of those 5 portfolios were in turn assigned to one of five portfolios on the basis of their beta. The first five years of data were used for the estimation of the security beta and the next 5 years of data were used for the reestimation of the portfolio betas. The portfolios were updated every year. Then the cross section of the regression described above was run in each month and the means of the resulting time–series of the coeficients were regressed once more on the excess return of the market index, (Black & Scholes, 1974). Both OLS and GLS were used for the estimation. The results revealed significantly negative estimates for the coefficients of the market value factors. This meant that shares of firms with large market values had smaller returns, on average, than similar small firms. Furthermore, the intercept was different from the risk–free rate. The analysis of the residuals returns of the 25 portfolios showed that they were not randomly distributed around zero: the residuals returns of the portfolios containing the smallest firms were all positive. Banz concluded that the CAPM is misspecified, but was unable to explain the "size effect".

Reinganum (1981) investigated empirically whether securities with different estimated betas systematically experience different average rates of return, a necessary condition for the validity of the CAPM. He called this the beta hypothesis.

To test the beta hypothesis first, in period A, he estimated individual security betas end placed securities into ten portfolios, based upon their relative rank of the beta. Then in period B, the returns of the ten beta portfolios were calculated by combining with equal weights the returns of the component securities within each portfolio. Then a multivariate statistical procedure was invoked to test whether or not the ten portfolios possessed significantly different returns.

The composition of beta portfolio was periodically updated. For example the 1964 beta portfolios were created based upon security betas estimated with 1963 daily returns. With data the beta portfolios were updated every 5 years. Also, 3 different estimates were used to compute beta estimates : a "market model" estimator, the estimator proposed by Scholes and Williams and the estimator proposed by Dimson.

For the tests with daily returns ('64–'79), the author found, that low beta portfolios experience greater average returns than high beta portfolios, a result similar for all estimation procedures.

Gibbons (1982) uses an alternative conceptual framework to avoid errors–in–variables and to increase the precision of the parameters estimates for the risk premiums, namely a non linear multivariate regression model.

He used monthly stock returns (from the CRSP tape) and the return on the CRSP equal weighted index as the return for the market portfolio. The sample period 1926–1975 was divided to 10 equal 5–year subperiods.

His strategy was to estimate B's (using 60 months of data) for all securities in the sample. With these estimates 40 groups were formed (each with the same number of securities), ranging from low to high beta values. An equally weighted portfolio was formed with all securities in a particular class. These 40 portfolios were the "left hand side " assets within this same 5–year period.

What he did was to test the parameter restriction implied by the market model on the CAPM i.e. $a_i=c_i(1-B_i)$. If the restriction is valid then the data are consistent with the CAPM. The restriction, however, was rejected for 5 out of 10 subperiods and in the remaining 5 the test statistic was marginally insignificant for 3 of them. In addition the mean–variance efficiency of the market proxy was rejected. Also graphs of the departure of the data from the theoretical model were presented. They revealed that high beta stocks tend to fall below the straight line while the reverse is also true for low beta stocks. In other words the CAPM tended to misprise all securities for some subperiods.

Ceccheti and Mark (1985) described an alternative strategy for testing asset pricing models. This strategy has 4 steps : First, a model must be specified that allowed for characterising the evolution of asset prices. Second, a set of moments is chosen in order to test the model, ie. moments of particular economic interest. Third, we estimate the model. Fourth, we test the model.

The authors specified a model in which the price of equity was a function of a parameter vector B, the discount factor, the coefficient of the relative risk aversion, a random variable and dividends. The price of the risk free asset was a function of all the above except dividends. Then, they chose the first and second moments of the equity premiums and the risk free rate as well as the variance ratio statistics computed from the equity returns, as the set of moments needed to test the model. Following that step, they estimated the moments, the parameters of the endowment process and the joint covariance matrix. They used US data for the period 1989–1987 and they found a risk–free rate of approximately 1% and an equity premium that exceeded 6%. In addition the standard deviation of the equity premium was more than 3 times that of the risk–free rate. To evaluate the models performance they examined the difference between the models' implied values for the moments of interest and the estimates from the asset price data. The model performed very well when the coefficient for the risk aversion was allowed to exceed 20, eventhough researchers traditionally have restricted their attention to values of the coefficient that were less than 10. They argued that it is useful to consider values for the coefficient of relative risk aversion that are larger than what has been considered in the past.

Jorion and Schwartz (1986) examined the issue of integration versus segmentation in the Canadian equity market relative to a global North American market. They compare the international and domestic versions of the CAPM, for both markets. Integration is defined as the situation where investors earn the same risk–adjusted expected return on similar financial instruments in different national markets.

With integration the world market index should be mean–variance efficient and as a result the only priced risk should be the systematic risk relative to the world market.

Segmentation, on the other hand, means that only domestic systematic risk should explain the pricing of assets. The authors used data from the Laval Securities tape which consisted of 749 Canadian securities and 98 NYSE–AMEX interlisted stocks, for the period 1963–1982.

In their tests, first, they defined all returns to mean excess returns. Then they defined the international (domestic) CAPM after isolating in the domestic (international) index the domestic (international) component. Finally, after correcting for thin trading effects and with one lead and one lag added, the empirical test equation for integration became

$$R_{it} = a_0\,(1-B^G{}_i) + a_2\,B^{CG}{}_i + {}^{t}{}_{k=-1}\,B^G{}_{ik}\,R_{G,t+k} + {}^{t}{}_{k=-1}\,B^{CG}ik\,V_{CG,t+k} + u_{it}$$

with a similar equation for segmentation.

The parameters were estimated jointly with a ML procedure. In addition they classified securities according to their Dimson (1979) betas, with the Canadian and World Index, into portfolios.

A test of integration vs segmentation implied that a_2 was zero against the alternative that it was positive. The null was strongly rejected for all the portfolios. The test revealed that an international CAPM was not a good description of the Canadian equities during the test period. National factors not present in the global index were an essential component of expected return in Canada.

Diacogiannis (1986) investigated the forecasting ability of the beta coefficients as well as whether beta forecasts can be improved by employing 3 alternative adjusting procedures : a method suggested by Blume (1975), the bayesian technique suggested by Vasicek (1973) and a procedure used by the Security Risk Evaluation Service, the Merrill Lynch, etc.

He used data from the London Stock Exchange from January 1955 to december 1983. Firms were included in the sample only if they had 15 years of history. Three subperiods were utilized, 60 months long each. Betas were estimated with the market model. The Mean Square Errors (MSE) between estimated and predicted betas were used to examine the forecasting ability of the betas. The data revealed that past estimates of security betas are not good predictors of the corresponding betas, but the unadjusted MSE can be reduced when an adjusting technique is employed. The Bayesian procedure achieved the largest reduction of the unadjusted MSE, with the Merril Lynch etc, method to achieve the smallest.

Rubio (1988) used a multivariate framework to study the price formation of risky assets in a "thin" capital market, such as the Spanish one. His sample consisted of 160 securities that were listed in the Spanish Stock Exchange during the period 1963–1982. Two market returns were calculated:an equally weighted market portfolio (EW) and a value weighted (VW), where the weights were the market values of each security at the end of the preceding year. The sample period was divided into 4 five–year subperiods. For each of the subperiods the number of securities with complete date was observed. These securities were ranked according to their market value and 10 equally weighted portfolios were formed where portfolio 1 contained the smallest firms and portfolio 10 the largest firms.

The market model was used to estimate the betas and then the CAPM was tested for both the Fama–McBeth and the Black, Jensen, Scholes specifications. Rubio used a statistic suggested by Shanken (1985b) to test the model. The results for both specifications were very similar. The market risk premiums were always positive but small, the mean–variance efficiency was rejected for the BJS specification. The test was then replicated for securities ranked on the basis of beta and the results were quite similar.

In an attempt to find potential seasonalities, the author, regressed a 10–vector of returns realized at month t on a 10 by 2 matrix of betas and ones, where the betas were obtained from a market model regression using the VW index. He obtained monthly estimates of the zero beta portfolio and the market risk premium for the 240 months and found a highly positive and significant risk premium in January. Algebraically when January was deleted from the sample the risk premium became close to zero.

By adding an extra (size) term to the CAPM equation he tested for the size effect. He computed the residuals of the 10 size sorted portfolios in each month and then he took the time series average of residuals for each portfolio. He found that small portfolios earn than the CAPM can predict and large portfolios earn less. He also found that nearly 47% of the size effect in the Spanish capital market is due to the month of January.

Lilian Ng (1991) examined a model in which the Sharpe–Linter CAPM was a special case. More specifically this study tested : i) whether the market proxy portfolio is on the conditional mean variance efficient frontier, ii) whether the cross sectional relationship between asset risk premia and their covariance risk is linear or proportional, and iii) whether the ratio of expected market risk premium to the conditional market variance is constant over time. The study allowed expected asset returns, conditional variances and the covariances of asset returns to vary over time.

The data used were from the CRSP tape and consisted of monthly returns on NYSE stocks for the period 1926–1987. Two grouping techniques were used.

First assets were ranked according to their market betas and divided into 10 groups. The first 5 years of data were used to estimate betas. The process was repeated after dropping the first year of data and moving forward until the final year of data was reached.

The second grouping technique was to put assets into portfolios according to their market value of equity at the end of each year. A model was then built specified as:

$$r_t = a + (d + lw'_{t-1} W_t w_{t-1}) (w'_{t-1} W_t w_{t-1})^{-1} W_t w_{t-1} + u_t$$

where a is the vector of portfolio specific intercepts, d l are the diagonal matrices with portfolio specific slope coefficients along the diagonal, w_{t-1} is the vector of value weights, W_t is the covariance matrix of the excess asset returns.

Hypothesis (i) was implied that a, l, d, are constant across assets. Hypothesis (ii) implied that a's are jointly zero. Hypothesis (iii) implied that the sum of a and d coefficients should be jointly zero. Standard likehood tests were employed and the results suggested that the residuals were well behaved and a parsimonious GARCH (1,1) provided an adequate description of monthly stock returns. The size shorted technique, however, rejected the model.

Ostermark (1991), estimated the CAPM with comparable Finish and Swedish data. He used a similar methodology to that of Fama–McBeth (1973), ie. he estimated an extended regression model with a squared beta, to capture nonlinearities, and a term to capture unsystematic influences.

He tested the hypothesis that : i) the intercept should not deviate significantly from the risk–free rate, ii) beta should be the only systematic factor to explain returns, iii) a positive trade–off between risk and return must exist.

The data consisted of daily, weekly, monthly price indexes over the period 1970–1983, (Finland), and of daily price index series for the period 1977–1987, (Sweeden). He first estimated asset betas with OLS. Then he ranked the assets based on their beta estimates from the lower to the highest beta. Based on that ranking he formed portfolios. He used 240 and 140 week data to form the portfolios, (Finish and Swedish portfolios, respectively). Then he used the subsequent 140 weeks as estimation period, where he reestimated the portfolio betas. Those betas entered the cross sectional regressions. The procedure was repeated every 5 weeks. The whole process was repeated 9 times, corresponding to the 9 formation periods. The first portfolio represents the average return and risk of those assets that have the lowest beta and the last portfolio these assets that have the highest beta risk.

He tested for normality, skewness, kurtosis with a x^2 test statistic (Bera et.al.). Significant deviation from normality was observed only for the highest risk Finish portfolio.

The results for the estimation of the extended regression equation revealed that the CAPM seems to work better with Swedish data than with Finish data.

Cadsby (1992) examined some empirical anomalies in the context of the CAPM. He used US data for the period 1963–1985. He used a methodology similar to the one used by Fama–McBeth in 1973. First he estimated the beta of the sample securities and used them to rank the securities and form portfolios on the basis of this ranking. Then he estimated the cross section CAPM equation in order to get estimates of the intercept and the slope coefficients, for all months of the year, and for each month individually. All the coefficients were positive and significant for all days between January 1963 and December 1985. January, however, was the only month in which all of the reported averages (rate of return, intercept, slope) were significant. This confirmed the validity of the CAPM and the existence of a January seasonality in the data. Then, he used a dummy variable regression where the intercept represented January and the dummy variables the other months of the year. He found that the average rates of return for months other then January are significantly lower than January. By using daily data, he was able to report that the January effect is really a turn–of–the–year effect which should include the last week of December together with the first four weeks of the new year (the average value of the CAPM risk premium was significantly positive during this period but not during the rest of the year, for the period 1963–1985). Further tests indicated that the returns were significantly higher all days of the week than on Monday.

Cadsby also reported that removing a five week period around the turn of the year causes the CAPM risk premium to loose its statistical significance. The reason for this, however, is the peculiar behaviour of stock prices on Mondays between January 29 and December 23. The CAPM works as predicted both at the turn of the year and on Tuesdays to Fridays during the rest of the year. When the two parameter model is expanded to include nonlinearities and firm–specific risk the result remains unchanged : The CAPM looks better on non Mondays.

The important result from this study was that for every calendar effect on stock returns a corresponding calendar effect was reported on the risk premium relationship, i.e. estimates of the CAPM risk premium are significant and positive during periods such as the turn of the year, etc.

The goal of a study by Fama and French (1992) was to evaluate the joint roles of market B, size, E/P, leverage and book to market equity in the cross section of average returns on NYSE, AMEX and NASDAQ stocks for the period 1963–1990.

Their asset pricing tests used the cross–sectional regression approach of Fama–McBeth (1973): each month the cross–section of returns on stocks is regressed on variables hypothesized to explain expected returns. The time series means of the monthly regression slopes then, provide standard tests of whether different explanatory variables are on average priced. In this study, however, portfolios are not used as in the Fama–McBeth regressions. The use of portfolios was because market Bs are more precise for portfolios. But size, E/P, leverage and BE/ME are measured precisely for individual stocks, so there was no reason to smear the information in these variables by using portfolios.

One of the first results, from some informal tests, was that when portfolios were formed on size alone, a strong negative relation existed between size and average returns and a positive strong relation between average return and B. The B–sorted portfolios, however, revealed no obvious relation between B and average return. For example the two extreme portfolios (high–low beta) had nearly identical returns.

Further tests, the FM regressions, i.e. the regressions of the cross–section of stock returns on size, B, leverage, BE/ME revealed that the strong negative relation with size persisted no matter which other explanatory variables were include in the regressions. In contrast, market B did not help explain average stock returns for the period 1963–1990. The average slope from the regressions on B alone was 0.15% per month and only 0.46 standard errors away from zero. When size was included the B slope became negative and only 1,21 standard errors away from zero. In addition B showed no power to explain average returns in any combination with the other variables.

The authors then formed portfolios on ranked values of BE/ME and E/P ratios. They found the familiar U–shape for the relation between average return and E/P and a strong positive relation between average return and BE/ME, a fact that confirms the importance of BE/ME in explaining the cross section of average stock returns. To investigate the impact of leverage on stock returns they used 2 leverage variables, the ratio of book assets to market equity (A/ME) and the ratio of book assets to book equity (A/BE). A/ME was interpreted as the market leverage and A/BE as the book leverage. Their results suggested that the relative distress effect captured by BE/ME could also be interpreted as an involuntary leverage effect which was captured by the difference between A/BE and A/ME.

In sort, their results suggested that B does not help to explain the cross section of average returns. Furthermore the univariate relations between average return, size, leverage, E/P and BE/ME were strong. But the combination of size and BE/ME seemed to absorb the roles of leverage and E/P.

Chen and Lakonishok (1993), under the light of some recent negative findings about beta, examined whether the very noisy and constantly changing environment generating stock returns permits strong statements about the importance of beta. They argued that even if there was no compensation for beta risk that does not mean that betas serve no use for investment decision making. As long as beta is a stable measure of exposure to market movements investors should still consider the "beta factor" of a stock.

They used CRSP data to test the CAPM for the period 1926–1991. They followed a methodology similar to Fama–McBeth (1973). The first 3 years of data were used to estimate each stock's beta and these estimates were used to rank securities and form portfolios on the basis of this ranking. The next 3 years of data were used to reestimate the beta of each stock. The portfolio beta was then a simple average of the betas of the individual stocks. In each month of the subsequent year they regressed the returns of the 10 portfolios on their estimated betas. At the end of the year they repeated the process of forming portfolios from the 3 years of data, etc. Ultimately they obtained 720 cross sectional regressions. They found a positive association between betas and average returns. In addition, they were unable to reject the null that the mean slope coefficient was equal to the market excess returns, in contrast to the Fama– French (1992) study. The positive relationship between betas and average returns varied considerably over time.

Up to 1982 there was a lot of support to the CAPM. From that time and onwards, however, the gap between the estimated compensation for beta risk and the realised market premium widened substantially. By addressing the issue of whether high beta stocks do worse than low beta stocks in periods of negative market return they were able to confirm the null that in months when the market falls (rises) investors in high beta stocks experience larger looses (gains) than investors in low beta stocks. A similar conclusion was drown by examining periods were the market experienced the largest "ups".

Their conclusion was that we do not have enough clear–cut empirical evidence to discard or support beta. As an overall conclusion on the empirical studies we can say that, eventhough Roll's arguments have theoretical grounds, most studies offer support to the linearity between risk and return, which approximates significantly the theoretical relationship predicted by the CAPM.

4. The critique of CAPM

The theory of the CAPM has often been challenged by statistical studies that fail to verify the validity of the model, as an adequate description of the way assets are priced. As we have seen, studies by Basu (1977), Banz (1981), Reinganumn (1981), and others, but most notably Fama and French (1992), have casted doubt on the most basic prediction of the model, i.e. that the systematic risk of a stock, as measured by beta, is a complete measure of risk. Variables such as the PE ratio, the market value of the firm, leverage and book to market equity, have been reported to play an important role in the procedure of asset pricing.

Other studies, however, such as Miller and Scholes (1972), Fama and McBeth (1973), Chen and Lakonishok (1993), of the most important, had found evidence in support of the model. Thus, the evidence, from the statistical tests appears mixed. The asset pricing theory's tests, however, have been criticized, most notably by R.Roll (1973)[2]. Roll argued that the theory is testable in principle but no correct and unambiguous test of the theory has appeared in the literature, and also that no such test will ever be accomplished. He argued that the only testable hypothesis, associated with the two parameter asset pricing model, is that the market portfolio is mean–variance efficient. All the other implications of the model, such as the linearity relation between return and beta, etc, follow from that hypothesis and are not independently testable.

The argument was based on the observation that for any sample of observations on individual returns, regardless of the generating process, there will always be an infinite number of ex–post mean–variance efficient portfolios. For each one of them, the sample betas calculated between it and individual assets will be exactly linearly related to the individual sample mean returns. In other words, if the betas are correlated against such a portfolio, they will satisfy the linearity relation exactly, weather or not the true market portfolio is mean variance efficient. Alternatively, Roll's basic argument was that the theory is not testable unless the exact composition of the true market portfolio is known and used in the tests, i.e. the theory is not testable unless all individual assets are included in the sample.

The empirical studies usually use a proxy for the market portfolio. Roll argues that this can be subject to two difficulties. First, the proxy itself might be mean–variance efficient even when the true market portfolio is not. Second, the proxy might turn out to be inefficient, but this implies nothing about the true markets' portfolio efficiency. In addition, most reasonable proxies will be very highly correlated with each other and with the market portfolio whether or not they are mean–variance efficient. This high correlation might make it seem that the exact composition is unimportant, but it can cause quite different inferences.

Also, Roll continued, the direct tests of the proxy's mean–variance efficiency are difficult computational because the full sample covariance matrix of individual returns must be inverted, and statistically because the sampling distribution of the efficient set is unknown. Furthermore, the widely used portfolio grouping procedure can support the theory even when it is false. The reason is that individual asset deviations from exact linearity may cancel out in the formation of the portfolios.

Roll proposed an Aitken–type procedure and a procedure that exploits asymptotic exact linearity by measuring the rate of decrease of cross sectional residual variance with respect to increasing time–series sample size, for testing linearity relations.

In another paper, Roll (1978), dealt with a problem of ambiguity when the performance of a portfolio is measured by the securities market line. He presented a simple example, to illustrate his argument, an idealized analog of professional money managers.

The example included a contest where there are 15 contestants (money managers) and a 4 asset world. The rules of the contest are as follows: every contestant had to select a portfolio, observe the returns on the assets over an interval, and declare winners and losers. Then the contestants were allowed to rebalance their portfolios and repeat the process. Finally after several intervals consistent winners were declared to be superior portfolio managers and consistent losers were declared to be inferior portfolio managers.

There are 3 judges to declare the superior and inferior money managers. They all faced the same problem: they had to develop criteria to partition winners from losers that were acceptable to participants. Those criteria should provide decisions about ability that are unambiguous to rational judges. The widely used criterion in the financial community for assessing portfolio performance was the "security market line", the linear relation between returns on assets or portfolios and betas. So, the judges just had to choose an index and calculate the betas against that index. The purpose of Roll was to expose the ambiguity in the SML criterion, i.e. that winners are above the line and losers are below the line.

The views of the 3 judges were as follows: the first judge assumed an equal weighted index of all assets, the second assumed a weighted index, with the weights proportional to the market value of the assets and the third judge assumed an index that was mean–variance efficient in the sense of Markowitz (1959), i.e. a portfolio with the smallest sample variance of return for a given level of sample mean return.

The 3 judges, therefore, ranked winners and losers differently according to their own preference of index. Most of the rankings were quite different. With this simple example, Roll, exposed the ambiguity in the SML criterion. He attributed the results to the following fact: corresponding to every index there is a beta for every individual asset (and thus for every portfolio), but these betas can be different for different indexes and will be for most. Thus, for every asset (or portfolio) judicious choice of the index can produce any desired measured "performance" (positive or negative), against the SML.

In 1987, Shanken, developed an empirical framework in which a prior belief about the correlation of the proxy and the market portfolio was explicitly incorporated. The usual notion of a proxy was extended to include a vector of variables which, together, account for much of the variation in the market portfolio return. Thus, the focus was on the multiple correlation between the proxy and the market portfolio. The empirical evidence presented suggested that either the Sharpe–Linter CAPM was invalid or the proxies used in the tests account for at most 2/3 or perhaps only 1/2 of the variation in the true market portfolio. Furthermore the results were essentially the same whether the CRSP equall weighted index was used alone or together with the Ibbotson–Siguefield long term US government bond index, in a multivariate proxy.

The analysis essentially demonstrated that it is possible to test the theory conditional on a prior belief about the proxy. Shanken argued that a good proxy need not be very highly correlated with the market portfolio. The main concern should be with the extent to which a proxy fails to capture variation in the market return that is correlated with the assets used in the test.

5. Conclusion

This paper critically reviewed the literature on the Capital Asset Pricing Model. We first outlined the theoretical concept of the model and then the derivation of it was presented. Consequently we reviewed the CAPM empirical tests and finally the critique of it. The evidence, from the statistical tests appears mixed since some studies have casted doubt on the most basic prediction of the model, i.e. that the systematic risk of a stock, as measured by beta, is a complete measure of risk. Variables such as the PE ratio, the market value of the firm, leverage and book to market equity, have been reported to play an important role in the procedure of asset pricing. On the other hand studies such as Miller and Scholes (1972), Fama and McBeth (1973), Chen and Lakonishok (1993), of the most important, had found evidence in support of the model. Finally the Roll's critique is presented, that the theory is testable in principle but no correct and unambiguous test of the theory has appeared in the literature, and also that no such test will ever be accomplished.

Notes

1. The market model is not supported by any economic theory. The market factor in the market model can be any stock market index. On the other hand the market portfolio in the CAPM must be a weighted average of all risky assets. If, however the capital markets are informational efficient,then the market model is equivalent to the CAPM.

2. See R.Roll:<u>A Critique of the Asset Pricing Theory's Tests, </u>Journal of Financial Economics 4, (1977), 129–176.

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