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The Impact of Probability Distortion On Decision-Making under Risk

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Abstract:

Purpose: *The aim of the article is to construct a behavioral model of decision-making under risk with distorted probabilities, to formulate a criterion of decision optimization in this model and to examine the impact of the probability distortion on the decision-making process.*
Design/Methodology/Approach: *A critical review of the literature is conducted, focusing on issues related to decision-making models under risk, the principles of optimal decision selection, and the issue of probability distortion in the context of cognitive distortions. A behavioral model of decision-making under risk with distorted probabilities is constructed using the BMR model. A new behavioral measure of decision value — the subjective expected relative utility on a given level of probability distortion — is defined. Based on this measure, a preference relation over admissible decisions is introduced and an optimization criterion for decision-making under risk is formulated. The study examines the strength of probability distortion's influence on choices and highlights behavioral aspects of individual decision-making under risk, including the roles of regret and satisfaction accompanying the decision process.*
Findings: *The main results of the research are: construction $BMRP_D$ model (behavioral model of decision-making under risk with distorted probabilities); defining a new behavioral measure of decision value - the subjective expected relative utility on a given level of probability distortion (SERU); formulation, based on this measure, the principle for selecting the optimal decision under risk by an individual decision-maker who, under the influence of emotions and cognitive distortions, may distort the probabilities of payoff. Additionally, the properties of the probability distortion function are described.*
The results of choosing the optimal decision in a behavioral model with probability distortions ($BMRP_D$ model) differ from the results of choosing without distortions. The strength of the distortions also influences the outcome of the choice.
Practical Implications: *The results of the research can be used to study and predict investors' behavior on the stock market, as well as to study the decisions of lottery players. They can also be used to construct a behavioral decision portfolio.*
Originality/Value: *The paper contains the author's original research results. The introduction of new behavioral measures for assessing decisions under risk allows for a better understanding of the decisions made by individual decision-makers, which may seem irrational based on classical theories such as utility maximization theory.*

Keywords: *Utility function, behavioral finance, behavioral model of decision-making under risk, weighting function, prospect theory, subjective expected utility.*

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1. Introduction

Decision-making under risk is often accompanied by various cognitive distortions. One of them is the subjective distortion of probabilities by individual decision-makers. In this paper a decision-making problem is considered, in which the decision-maker (in the context of financial decisions, the investor) aims to select a single decision that is optimal from his individual point of view, under conditions of risk. Risk, in this context, refers to a situation where the decision-maker either:

- knows the possible states of the external world, along with their corresponding probabilities (in this case, these probabilities can be considered objective), or
- does not know the probability distribution of the possible states of the world but is able to subjectively estimate them (in this case, the probabilities are considered subjective).

Savage observed that, in decision-making under uncertainty, the probabilities of certain events (such as wars, natural disasters, or economic crises) are unknown. His concept of personal probability (Savage, 1954) expresses an individual's degree of belief regarding the occurrence of a particular event (Tyszka *et al*., 2004).

These subjectively estimated probabilities may vary among different decision-makers. Even when objective probabilities are available, decision-makers may distort them during the evaluation of alternatives for various personal reasons.

Research in the literature on the subject on probability distortion shows that small probabilities, close to zero, are subjectively overestimated, and improbable outcomes are given excessive weight – this phenomenon is called the possibility effect.

On the other hand, probabilities close to unity are usually underestimated, and highly probable outcomes are given less weight than the distribution suggests – this phenomenon is called the certainty effect (Kahneman and Tversky, 1979).

The aim of the article is to construct a behavioral model of decision-making under risk with distorted probabilities, to formulate a criterion of decision optimization in this model and to examine the impact of the probability distortion on the decision-making process.

In connection with the goal defined in this way, the following research question is formulated: whether these distortions of probabilities affect the outcomes of decisions and, if so, to what extent?

In this work the behavioral model of decision-making under risk is considered (BMR model, Falkiewicz, 2025), in which the distribution of probabilities of individual payoffs is replaced by a distribution distorted by the weighting function. The resulting model is called a behavioral model of decision-making under risk with distorted probabilities ($BMRP_D$ model) and is a generalized model, i.e., with an appropriate value of the probability distortion parameter, it is the initial BMR model.

A new measure for decision evaluation is introduced: subjective expected relative utility (SERU) for a given level of distortion. This measure is then used to define the preference relation between acceptable decisions, which enables comparison of different decisions at a selected level of probability distortion. Using SERU, the rule for entering the game is stated. Finally, an optimization criterion within the $BMRP_D$ model is formulated. The application of the presented theory is illustrated with examples.

This issue is important because the growing field of behavioral finance requires measures that consider human emotions and the accompanying cognitive errors when making decisions under risk. The behavioral decision evaluation measure (SERU) introduced in the study takes psychological factors into account in the decision-making process.

In addition to probability distortion, it also considers two powerful emotions associated with decision-making: regret and satisfaction. By demonstrating the impact of cognitive distortions such as probability distortion by decision-makers, the theory presented helps to better understand human decisions that may seem irrational in the light of classical decision-making theories.

2. Literature Review

In search of an answer to the research question, literature on the subject in the field of decision-making under risk and probability distortion was reviewed. A wide array of methods for decision-making under risk have been developed and discussed in the literature. The oldest method for selecting an optimal decision under risk is the expected value maximization principle (also referred to as the expected value criterion), formulated in the 17-th century by Blaise Pascal.

According to this principle, for each decision, the expected payoff is calculated by considering the probabilities of occurrence of various possible states of the world. The recommended decision for the decision-maker is the one for which the expected payoff is the greatest.

Although the theory of expected value maximization served as a foundation for Markowitz's portfolio theory (Markowitz, 1952), it was notably criticized by D. Bernoulli, who proposed in 1738 a revolutionary (for its time) theory of expected utility maximization (Bernoulli, 1954). He introduced a new approach to evaluating gambles, replacing monetary outcomes with subjective psychological utility values.

According to Bernoulli's theory, the recommended decision is the one with the greatest expected utility. In this framework, the utility values also reflect the decision-maker's attitude toward risk. If the individual is risk-neutral, then the optimal decision under the expected utility maximization criterion coincides with the decision that maximizes the expected monetary value.

However, if the decision-maker is risk-averse, their utility function is concave, whereas, in the case of risk-seeking behavior, the utility function is convex (Dobrowolski, 2014). The classical formulation of the expected utility maximization theory can be found in the seminal monograph by John von Neumann and Oskar Morgenstern (von Neumann and Morgenstern, 1944), in which the authors formulated axioms that must be satisfied by investor preferences for them to be compared and described using indifference curves. These axioms are comparability, transitivity, independence, constancy and continuity. In practice, however, they are not always satisfied (Szyszka, 2009; Forlicz, 2001).

Despite its foundational importance, the theory of expected utility maximization has also been criticized. One of the first economists to demonstrate, based on an experiment at a conference of economists in Paris in 1952, the contradiction between investors' preferences and the assumptions of utility maximization theory was Maurice Allais, later winner of the Nobel Prize in Economics (Allais paradox).

In subsequent years, new concepts emerged aimed at identifying the optimal decision under conditions of risk, with increasing emphasis on psychological factors, thereby laying the groundwork for the development of behavioral economics. A notable example is the subjective expected utility theory proposed by Savage in 1954 (Savage, 1954), in which the decision-maker, lacking knowledge of the objective probabilities of various states of the world, instead forms subjective probability estimates (known as personal probability).

Savage's theory was later challenged by D. Ellsberg, who argued that when modeling risk with psychological considerations, it is not only the expected utility that matters but also the distribution of the remaining utility values around the mean (Ellsberg, 1961, 2001).

After Leonard Savage introduced the concept of personal probability in 1954, research began to develop on the subjective perception of probabilities when making risky decisions (Edwards, 1954a; 1954b; 1954c). A particularly significant development—both in terms of capturing the decision-maker's individual attitude toward risk and in anchoring the mainstream of behavioral economics—is the prospect theory proposed (with criticism of the theory of utility maximization, Kahneman and Tversky, 1979), for which Daniel Kahneman was awarded the Nobel Prize in 2002.

The theory is grounded in the assumption that human choices are context-dependent and vary significantly depending on whether decisions are made in the domain of losses or gains. The phenomenon of probability distortion as one of the cognitive biases was studied and described by the authors of the prospect theory (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992; Kahneman, 2011).

The results of their research and that of their successors consistently show that people attach greater importance than their actual value to low probabilities (they overestimate them) and less importance to medium and high probabilities, underestimating them. They introduced the so-called probability weighting function. An expanded version of the prospect theory is the cumulative prospect theory (Tversky and Kahneman, 1992), which is also applicable in conditions of uncertainty.

Research on probability distortion using various types of probability weighting functions has also been conducted by Lattimore *et al.*, 1992; Camerer and Ho, 1994; Fox and Tversky, 1998; Prelec, 1998; Gonzales and Wu, 1999; Abdallaoui, 2000; Bleichrodt and Pinto, 2000; Domurat, 2008; Idzikowska, 2013.

As research shows, the degree of probability distortion at the perception level is influenced by the way probability information is presented – greater distortion occurs when the information is presented numerically, i.e., in the form of a distribution of potential payout values with corresponding numerical probabilities (the so-called descriptive probability format), while probabilities are less distorted when people learn about lottery distributions through experience (the so-called experiential probability format), (Hertwig *et al.*, 2004;, Gottlieb *et al.*, 2007; Camilleri and Newell, 2009; Ungemach *et al.*, 2009; Hau *et al.*, 2010; Hertwig, 2012; Idzikowska, 2013).

Another key behavioral approach is the regret theory (Bell, 1982; Loomes and Sugden, 1982), which was later extended (Quiggin, 1994) and dual theory under risk (Yaari, 1987), revealed preference theory of temptation and self-control (Gul and Pesendorfer, 2001; 2004).

Research findings on the behavioral aspects of financial decision-making can be found in, among others: Zaleśkiewicz, 2003; Zielonka, 2004; Zaleśkiewicz, 2011; Zielonka, 2018; Czerwonka and Lang, 2024; Falkiewicz, 2025.

In this study the behavioral model of decision-making under risk (BMR model) is used (Falkiewicz, 2025), which refers to the prospect theory (by using a utility function that satisfies the assumptions of the prospect theory and the weighting function to distort probabilities) and the theory of regret by incorporating the emotions of regret and satisfaction. A model with distorted probabilities ($BMRP_D$ model) is constructed based on the BMR.

3. Methodology

The behavioral decision-making model under risk (Falkiewicz, 2025) serves as the foundation for the theory developed in this paper, in which the original BMR model is extended to probability distortion. This work presents a formal extension of that model. The main concepts of the BMR model are outlined below. Let $(\mathcal{M}, \mathcal{S}, P, U_w)$ be the behavioral decision-making model under risk (BMR model), where:

- $\mathcal{M} = \{d_1, \dots, d_k\}$ is a set of acceptable decisions,
- $\mathcal{S} = \{s_1, \dots, s_n\}$ is a set of external world states,
- $P = (p_1, \dots, p_n)$ is a probability distribution corresponding to world states, where p_j is a probability of the world state s_j occurring for $j = 1, \dots, n$, and $p_j > 0, \sum_{j=1}^{n} p_j = 1$,
- $U_w = [(u_w)_{ij}]_{i=1,\dots,k; j=1,\dots,n}$ is the relative utility matrix described by the formula:

$$U_w = \left[(u_w)_{ij}\right]_{i=1,\dots,k; j=1,\dots,n} = M + \alpha M_r + (1 - \alpha) M_s, \tag{1}$$

Where

$M = [u_{ij}]_{i=1,\dots,k; j=1,\dots,n}$ - the utility matrix (u_{ij} – the utility of making the i-th decision in the j-th state of the world),
$M_r = [r_{ij}]_{i=1,\dots,k; j=1,\dots,n} = [u_{ij} - \max_{1 \le i \le k} u_{ij}]_{i=1,\dots,k; j=1,\dots,n}$ - the regret matrix,
$M_s = [s_{ij}]_{i=1,\dots,k; j=1,\dots,n} = [u_{ij} - \min_{1 \le i \le k} u_{ij}]_{i=1,\dots,k; j=1,\dots,n}$ - the satisfaction matrix,
$0 \le \alpha \le 1$ - regret coefficient.

The terms u_{ij} of the utility matrix M are values of the utility function, which meets the assumptions for the so-called value function (Kahneman and Tversky, 1979) – it is increasing across the entire domain, negative and convex for losses, positive and concave for gains, and steeper on the loss side than on the gain side.

The terms $(u_w)_{ij}$ of the relative utility matrix U_w denote relative utility of the i-th decision d_i, in the j-th world state s_j, for $i = 1, \dots, k$, $j = 1, \dots, n$, and can by express by the formula:

$$(u_w)_{ij} = u_{ij} + \alpha \cdot r_{ij} + (1 - \alpha) \cdot s_{ij}. \tag{2}$$

Based further on the definition of the expected relative utility of the i-th decision d_i in the j-th world state s_j (Falkiewicz, 2024), which is

$$Eu_w(d_i) = \sum_{j=1}^{n} (u_w)_{ij} \cdot p_j, \tag{3}$$

the vector of the expected relative utilities of decisions $d_1, \ldots, d_k$ is of the form:

$$E_w = U_w \cdot p = (M + \alpha M_r + (1-\alpha) M_s) \cdot p, \tag{4}$$

where $p \in \mathbb{R}^n$ is the vector of the probabilities $p_1, \ldots, p_n$ of the world states $s_1, \ldots, s_n$.

Next, taking into account the conclusions from the prospect theory concerning the subjective assessment of the probabilities of particular states of the external world occurring, and thus the usefulness of particular decision outcomes, instead of the probability distribution $p_1, \ldots, p_n$ of the occurrence of states $s_1, \ldots, s_n$, wherein $p_j = P(s_j)$, $j = 1, \ldots, n$, in the BMR model a probability transformation function is used:

$$\pi_b \colon [0;1] \to [0;1],$$

$$\pi_b(p_j) = \frac{p_j^b}{\sum_{j=1}^{n} p_j^b}, \; 0 \leq b \leq 1, \tag{5}$$

which is called the weighting function (Jajuga, 2008). The values of this function are consistent with the results of observations concerning the distortion of probabilities in risky decisions, i.e. low probabilities, close to zero, are usually overestimated, while high probabilities, close to one, are underestimated.

Both effects – possibilities and certainties – are clearly visible after applying function (5). The parameter *b* in the weighting function serves as a subjective probability distortion. The closer the parameter *b* is to 0, the greater the distortion of probability; the closer it is to 1, the smaller the distortion.

Function π_b, unlike other probability-distorting functions considered in literature, does not cause probability disturbances that would invalidate the entire distribution, i.e. probabilities transformed by function (5) still form a distribution from a mathematical point of view (for any probability distribution, the values of this function are positive and add up to 1). For this very reason, this function, and not another, is used in the presented decision model.

Remark 1:

1. *For $b = 0$ the weighting function (5) distorts the probability distribution to a uniform distribution, i.e. for any probability distribution $p_1, \ldots, p_n$,*

$$\forall_{j=1,\ldots,n} \; \pi_0(p_j) = \frac{1}{n}.$$

2. *For $b = 1$ the probability distribution $p_1, \ldots, p_n$ remains unchanged,*

$$\forall_{j=1,\dots,n} \quad \pi_1(p_j) = p_j.$$

Example 1. *Table 1 shows the probability of:*

(a) a certain asymmetric distribution – one probability significantly greater than the other two: $p_1=0.8$, $p_2=0.15$, $p_3=0.05$,
(b) a distribution that is more uniform than the previous one: $p_1=0.4$, $p_2=0.35$, $p_3=0.25$,
transformed by the weighting function (5) for different values of the parameter $b \in [0;1]$.

Table 1. *Probabilities transformed by the weighting function, (a) columns 2-4, (b) last three columns.*

b	$\pi_b(0.8)$	$\pi_b(0.15)$	$\pi_b(0.05)$	$\pi_b(0.4)$	$\pi_b(0.35)$	$\pi_b(0.25)$
1	0.8	0.15	0.05	0.4	0.35	0.25
0.99	0.7969	0.1519	0.0512	0.3993	0.3499	0.2508
0.9	0.7668	0.1700	0.0632	0.3934	0.3489	0.2577
0.8	0.7295	0.1912	0.0794	0.3868	0.3476	0.2656
0.75	0.7061	0.2037	0.0901	0.3830	0.3468	0.2702
0.5	0.5279	0.2835	0.1816	0.3582	0.3409	0.3009
0.01	0.3352	0.3331	0.3317	0.3336	0.3334	0.3330
0	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333

Source: *Author's calculations.*

Table 1 shows that as the parameter *b* decreases from 1 to 0, the probabilities become increasingly distorted – from the initial distribution for *b*=1 to a uniform discrete distribution for *b*=0. It can also be observed that high probabilities, close to 1, are underestimated, while low probabilities, close to 0, are overestimated. This is consistent with the observations and research results presented in the prospect theory by Kahneman and Tversky, 1979.

The effects of certainty and possibility are already visible with a slight distortion of probabilities for b=0.99. The stronger the distortion (with a decrease in the b parameter), the more visible these effects become. It can be also seen that the probabilities in (a), whose distribution is more asymmetrical than in (b), are exposed to stronger distortions.

From the analysis of cases (a) and (b) in example 1, the following conclusion can be drawn:

Corollary 1: *The more asymmetrical the initial probability distribution is, the more it is exposed to stronger distortions. The more the initial distribution is close to uniform distribution, the smaller the probability distortions are.*

Remark 2: *The weighting function (5) does not change the uniform distribution, i.e.,* *when the probabilities of each state of the world occurring are the same: if for any* $j = 1, \dots, n$, $p_j = \frac{1}{n}$, *then* $\pi_b(p_j) = \frac{1}{n}$ *for any probability distortion parameter* $b \in [0; 1]$.

Remark 3: *The weighting function (5) does not change the probability of impossible and certain events, i.e.:*

$$\pi_b(0) = 0, \quad \pi_b(1) = 1$$

for any probability distortion parameter $b \in [0; 1]$.

This means that probabilities are not subjectively distorted when there is zero chance of a certain outcome occurring and when the outcome is certain.

Based on Table 1, another interesting property of distorted probabilities can be observed, namely that no matter what the distortion parameter *b* is, the sum of the deviations of the distorted probabilities from their original values is always zero.

Proposition 1: *For any probability distortion parameter* $b \in [0; 1]$ *the following property applies:*

$$\sum_{j=1}^{n} (\pi_b(p_j) - p_j) = 0.$$

Proof. Using the fact that $p_1, \dots, p_n$ is probability distribution, for any $b \in [0; 1]$ is:

$$\sum_{j=1}^{n} (\pi_b(p_j) - p_j) = \sum_{j=1}^{n} \left(\frac{p_j^b}{\sum_{j=1}^{n} p_j^b} - p_j \right) = \sum_{j=1}^{n} \frac{p_j^b}{\sum_{j=1}^{n} p_j^b} - \sum_{j=1}^{n} p_j = 1 - 1 = 0. \quad \blacksquare$$

Definition 1: *The subjective expected payoff at distortion level* $b \in [0; 1]$ *is called the expected payoff with probabilities distorted by the function* π_b:

$$S_b E(X) = \sum_{j=1}^{n} x_j \pi_b(p_j), \tag{6}$$

where x_j *is payoff value with probability* p_j, $j = 1, \dots, n$.

Based on the concepts introduced above, it is possible to formulate the principle of entering the game, i.e. the profitability of participating in the lottery from the player's subjective point of view, with a distortion of probabilities.

Criterion for entering the game with distorted probabilities:
If X is a random variable denoting the payout value in a lottery with distribution $p_j = P(X = x_j)$, $j = 1, \dots, n$, and R is a certain fixed reference level, then from the point of view of a game model with probabilities distorted by the weighting function (5) at distortion level $b \in [0; 1]$, it is profitable for the player to participate in the random game described by variable X if the subjective expected value of the lottery payout $S_b E(X)$ at the level b is greater than the reference level R:

$$S_b E(X) > R. \tag{7}$$

After applying formula (6) and equivalent transformations of inequality (7), this condition can be written as

$$(x_1 - R)p_1^b + (x_2 - R)p_2^b + \dots + (x_n - R)p_n^b > 0,$$

or, in short,

$$\sum_{j=1}^{n} (x_j - R)p_j^b > 0.$$

The reference level R in the above formulas can take different values – it can be zero when the alternative to the game is doing nothing, it can be equal to the classic expected value EX, it can be equal to the rate of return on an alternative action, or it can be any other value determined individually by the decision maker.

4. Research Results and Discussion

The issue of probability distortion presented in the previous chapter shows that it is reasonable to take them into account in the decision-making process under risk. Taking the BMR model $(\mathcal{M}, \mathcal{S}, P, U_w)$ as a starting point, instead of the probability distribution $P = (p_1, \dots, p_n)$, we consider the probability distribution distorted by the weighting function (5): $\pi_b(P) = (\pi_b(p_1), \dots, \pi_b(p_n))$, which will be denoted by P_D.

Definition 2: *The subjective expected relative utility (SERU) of decision $d_i, i = 1, \dots, k$, at distortion level $b \in [0; 1]$, is called the expression*

$$S_b E u_w(d_i) = \sum_{j=1}^{n} u_w\left(x_{ij}\right) \cdot \pi_b\left(p_j\right), \tag{8}$$

where $(u_w)_{ij}$ is relative utility of the i-th decision d_i in the j-th state of the world s_j, $i = 1, \dots, k$, $j = 1, \dots, n$, given by the formula (2) and $\pi_b(p_j)$ is transformed probability of the occurrence of the j-th state of the world s_j.

It should be noted that to calculate the SERU of a given decision, it is first necessary to determine the level $b \in [0;1]$ of probability distortion.

Definition 3: *The vector of subjective expected relative utilities of decisions $d_1, \dots, d_k$ at distortion level $b \in [0;1]$, is called the expression*

$$S_bE_w = [S_bEu_w(d_i)]_{i=1,\dots,k} = E_w \cdot \pi_b(p) \in \mathbb{R}^k, \tag{9}$$

where $\pi_b(p) = [\pi_b(p_j)]_{j=1,\dots,n} \in \mathbb{R}^n$ is a vector of probabilities of world states $s_1, \dots, s_n$ transformed by the weighting function (5) for a certain parameter $b \in [0;1]$.

In the set of admissible decisions $\mathcal{M} = \{d_1, \dots, d_k\}$, the relation of preference in terms of the subjective expected relative utility is defined.

Definition 4: *Decision d_j is not worse (or not less preferred) than decision d_i in terms of the subjective expected relative utility (SERU) at the distortion level $b \in [0;1]$, what is denoted by $d_i \leq_{S_bEu_w} d_j$, if and only if the SERU of decision d_j is not less than the SERU of decision d_i at the distortion level $b \in [0;1]$, that is,*

$$d_i \leq_{S_bEu_w} d_j \leftrightarrow S_bEu_w(d_i) \leq S_bEu_w(d_j). \tag{10}$$

The relation "$\leq_{S_bEu_w}$" is called the partial preference relation in terms of the subjective expected relative utility (SERU) on the set of admissible decisions $\mathcal{M}$. It is a partial order relation on the set $\mathcal{M}$.

Remark 4: *In the case of two different decisions $d_1 \neq d_2$, $d_1, d_2 \in \mathcal{M}$, if the condition for the equality of the subjective expected relative utilities exists, at the fixed distortion level b,*

$$S_bEu_w(d_1) = S_bEu_w(d_2),$$

then the decisions d_1 and d_2 are equivalent in terms of the subjective expected relative utility SERU:

$$d_1 \sim_{S_bEu_w} d_2.$$

Definition 5: *The behavioral model under risk with distorted probabilities ($BMRP_D$ model) on some distortion level $b \in [0;1]$ is called the decision-making system $(\mathcal{M}, \mathcal{S}, P_D, U_w, \leq_{S_bEu_w})$ in which:*

- $\mathcal{M} = \{d_1, \dots, d_k\}$ *- a set of acceptable decisions,*
- $\mathcal{S} = \{s_1, \dots, s_n\}$ *- a set of external world states,*
- $P_D = (\pi_b(p_1), \dots, \pi_b(p_n))$ *– a distribution of distorted probabilities, where*

$\pi_b(p_j) = P_D(s_j) > 0$ *for* $j = 1, \ldots, n$, $\sum_{j=1}^{n} \pi_b(p_j) = 1$,

- $U_w = [(u_w)_{ij}]_{i=1,\ldots,k,\ j=1,\ldots,n}$ - *a matrix of relative utility described below by the formula (1),*
- $\leq_{S_b E u_w}$ - *the partial preference relation given by the formula (10).*

Thus, the BMRP_D model consists of the behavioral decision-making model under risk BMR, enriched with the preference relation $\leq_{S_b E u_w}$ *in terms of the subjective expected relative utility with the system of probabilities distorted by the function (5) on some distortion level* $b \in [0; 1]$.

Remark 5: *For* $b = 1$ *(without any distortion) BMRP*$_D$ *model is BMR model.*

The objective of the decision-maker is to select the optimal decision, from his point of view, from the set $\mathcal{M} = \{d_1, \ldots, d_k\}$ of acceptable decisions, given the probabilistic states of the world $s_1, \ldots, s_n$.

The criterion of decision optimization in the BMRP$_D$ *model:*
Let the *BMRP*$_D$ model be given as the decision system $(\mathcal{M}, \mathcal{S}, P_D, U_w, \leq_{S_b E u_w})$. For each admissible decision $d_i \in \mathcal{M}$, $i = 1, \ldots, k$, at some distortion level $b \in [0; 1]$, the subjective expected relative utility $S_b E u_w(d_i)$ can be calculated (formula (8)). The recommended decision for an individual decision-maker, from the perspective of the partial preference relation $\leq_{S_b E u_w}$, is the decision d^* for which the subjective expected relative utility at level b is maximal:

$$E^* = \max_{i=1,\ldots,k} S_b E u_w(d_i). \tag{11}$$

The above criterion, in addition to rational factors, which are represented in relative utility u_w by utility in the classical sense, also considers behavioral factors, represented by the values of the regret function and the satisfaction function, as well as by the weighting function (5) distorting probabilities.

It becomes clear, if in formula (8) we put the form of relative utility $u_w(x_{ij})$ of i-th decision in j-th state of the world described by formula (2), then we can see that the optimal decision is the decision d^* for which

$$E^* = \max_{i=1,\ldots,k} \sum_{j=1}^{n} u_w(x_{ij}) \cdot \pi_b(p_j) = \max_{i=1,\ldots,k} \sum_{j=1}^{n} (u_{ij} + \alpha \cdot r_{ij} + (1-\alpha) \cdot s_{ij}) \cdot \pi_b(p_j).$$

The optimization condition (11) can also be formulated using the vector of subjective expected relative utilities (9): the optimal decision d^* at the probability distortion level b is indicated by the highest value of this vector's coordinate.

Example 2: *Let a set of acceptable decisions* $\mathcal{M} = \{d_1, d_2, d_3, d_4\}$, *a set of external world states* $S = \{s_1, s_2, s_3\}$ *and their corresponding probabilities* $p_1 = 0.8, p_2 = 0.15, p_3 = 0.05$, *and a matrix of utility of making the particular decisions in the subsequent world states be given:*

Table 2. *Utility matrix for example 2*

Probabilities	$p_1 = 0.8$	$p_2 = 0.15$	$p_3 = 0.05$
Decisions\ States	s_1	s_2	s_3
d_1	8	14	20
d_2	10	10	10
d_3	15	0	-40
d_4	-10	80	100

Source: *Falkiewicz, 2025.*

We will determine the optimal decision for a decision maker considering one of the decisions belonging to the set $\mathcal{M} = \{d_1, d_2, d_3, d_4\}$, if the probabilities p_1, p_2 and p_3 may be subjectively distorted by function (5). For this purpose, we will use the $BMRP_D$ model and the optimization criterion (11).

The relative utility matrix U_w calculated using formula (1) is equal to:

$$U_w = M + \alpha M_r + (1-\alpha) M_{s=} \begin{bmatrix} 8 & 14 & 20 \\ 10 & 10 & 10 \\ 15 & 0 & -40 \\ -10 & 80 & 100 \end{bmatrix} + \alpha \begin{bmatrix} -7 & -66 & -80 \\ -5 & -70 & -90 \\ 0 & -80 & -140 \\ -25 & 0 & 0 \end{bmatrix} + (1-\alpha) \begin{bmatrix} 18 & 14 & 60 \\ 20 & 10 & 50 \\ 25 & 0 & 0 \\ 0 & 80 & 140 \end{bmatrix},$$

where $0 \leq \alpha \leq 1$. Depending on the value of parameter α, which in turn depends on the ratio of regret to satisfaction of a given decision maker, different relative utility matrices U_w are obtained. According to Kahneman (2011), the ratio of regret to satisfaction is usually 2:1 (here α: $(1-\alpha)$).

Therefore, we can assume that α =0.66 and 1- α =0.34. For a chosen α the vector $S_b E_w$ of subjective relative utilities is then calculated (formula (9)) and, in accordance with the optimisation criterion (11), the optimal decision is selected, which is indicated by the largest coordinate of this vector.

Depending on the parameter $b \in [0; 1]$, different systems of distorted probabilities $P_D = (\pi_b(p_1), \ldots, \pi_b(p_n))$ are obtained – they are presented in Table 1 (columns 2-4).

Table 3. *Subjective expected relative utilities (SERU) of decisions d_1, d_2, d_3, d_4 for different values of the probability distortion parameter b, for the regret parameter α=0.66, and the results of decision selection based on the criterion (11)*

SERU \ b	1.00	0.99	0.90	0.80	0.75	…	0.01	0.00
$S_bEu_w(d_1)$	3.26	3.16855	2.28492	1.20393	0.53895	...	-9.18956	-9.23241
$S_bEu_w(d_2)$	**4.26**	**4.11695**	2.72812	1.00433	-0.06825	...	-17.14756	-17.23161
$S_bEu_w(d_3)$	**4.26**	3.92795	0.67612	-3.46467	-6.09125	...	-53.62756	-53.89461
$S_bEu_w(d_4)$	2.26	2.72295	**7.23212**	12.88433	**16.42375**	...	**75.78444**	**76.09239**
Result	d_2 or d_3	d_2	d_4	d_4	d_4	...	d_4	d_4

Source: *Author's computation.*

Analyzing the results in Table 3, it can be seen that for $b = 1$, i.e. in the absence of probability distortion, the highest value of subjective relative expected utility is 4.26 and indicates the equivalence of decisions d_2 and d_3, in the sense of subjective expected relative utility $d_2 \sim_{S_{b=1}Eu_w} d_3$.

This is the same conclusion as in Example 3 (Falkiewicz, 2025), because according to Remark 5, for b=1, the $BMRP_D$ model is the BMR model. With a slight distortion of probabilities, for $b = 0.99$, the most advantageous decision from the subjective relative expected utility point of view is d_2 - for it, SERU is the highest and amounts to 4.11695.

However, with increasing distortion of the probabilities of world states - from $b = 0.9$ to $b = 0$ – decision d_4 comes out on top. Already with distortion for $b = 0.9$, SERU for this decision is significantly higher than the other values and amounts to 7.23212. With further distortion of probabilities by parameter b, up to a uniform distribution (for $b = 0$), the subjective expected relative utility of this decision significantly exceeds the other values: SERU for d_4 is at 76.09239 with negative values for the other decisions.

Similar calculations were performed for other values of the regret parameter $\alpha \in [0; 1]$. All of them confirmed the same recommended decisions as for $\alpha = 0.66$.

5. Conclusions, Proposals, Recommendations

The presented research results prove the thesis that distorting probabilities in the decision-making process under conditions of risk affects the outcomes of those decisions. The analysis conducted using the $BMRP_D$ relative utility model showed that decision outcomes may vary depending on the degree of probability distortion. After applying the optimization criterion based on subjective expected relative utility, differences in the recommendation for the optimal decision are already visible with slight distortions of probabilities by function π_b.

The analyzed examples also reveal that, above a certain level of probability distortion parameter b, the same decision is recommended as for maximum distortion (for $b =$

0), i.e. as for a uniform distribution. This can be explained as follows: with progressive distortion, the probability distribution becomes increasingly flattened, tending towards uniform distribution. This is accompanied by a change in the recommended decision to one that is optimal for uniform distribution. It can also be seen that the more asymmetrical the original distribution is, the more susceptible it is to distortion.

A certain weakness of the presented approach is the requirement to determine in advance the parameter b distorting probability – it must be determined individually for a given decision-maker and decision-making situation. Nevertheless, there are studies in literature (Idzikowska, 2013) according to which the level of probability distortion can be determined.

It is also possible, for a given decision-maker and decision-making situation, to determine in advance a certain value of the distorting parameter b and to conduct further analysis for it in the $BMRP_D$ model. However, this weakness can also be seen as an advantage – the individual selection of the parameter b results in individualization, the adaptation of the $BMRP_D$ model to a given decision-maker and decision-making situation.

The presented approach, which takes behavioral factors into account, explains better than the classical approach the behavior of decision-makers operating in conditions of risk, which from the point of view of expected value maximization or expected utility maximization may often seem irrational. The presented results therefore contribute to the development of behavioral economics methods. They also provide a basis for further research on decisions, particularly financial ones of investor behavior on the stock market or lottery players.

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