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# Cash Limits and the Control of Public Expenditure in the United Kingdom

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## Abstract

*This paper uses a generalisation of intervention analysis to examine quarterly data on UK public expenditure from 1963:1 to 1992:4. The aim is to test whether government policy, and particularly the introduction of cash limits, had any significant effect upon the level of real public expenditure. Four series are examined. These are for (i) central government consumption, (ii) local authority consumption, (iii) general government capital formation and (iv) transfer payments. No evidence is found of intervention effects that can be attributed to cash limits.*

## 1. Introduction

In the period since the early nineteen - sixties there has been a significant increase in the size of the UK's public sector, whether this is measured by the share of general government expenditure in Gross Domestic Product or by the absolute level of expenditure itself.<sup>1</sup>

After the first oil crisis of 1973 - 4, the size of public expenditure began to be seen by different governments, for different reasons, as an obstacle to economic growth and prosperity. The Labour Government was under great pressure after 1976, from both the Treasury and the IMF, to reduce government spending so as to restore external equilibrium ( Healey, 1989, pp. 429 - 30. ). The 1979 election saw a new Conservative Government committed to rolling back the state, reducing the burden of taxation and restoring incentives. Reducing the level of public expenditure was seen as essential to this policy.<sup>2</sup>

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1. Of course, this is only part of a much longer time trend that has existed since at least the end of the last century. See Brown and Jackson ( 1990, T 6.1, p. 164 ). It is also a phenomenon that has been common in many other countries. See Mueller ( 1993, T 17.2, p. 322 ). The explanation of this occurrence has spawned a vast literature, but is not the main focus of this paper.

2. The 1979 Conservative Party Manifesto argued that "The State takes too much of the nation's income; its share must be reduced." ( quoted in Lawson, 1993, p. 103 ). Lawson (1993) also says "We ( the new Conservative Government ) ... resolved that our first Budget should make a decisive start to the process of reducing the deficit, and to do so entirely by cutting government spending." ( p. 31. ).

The objective of this paper is to determine whether Government policies, and particularly cash limits, were effective in controlling the level of public expenditure in the UK. Cash limits were introduced into the public expenditure planning process in an experimental manner in 1974 - 5 in an attempt to control spending on various central and local government building projects, following significant increases in tender prices in this area.<sup>3</sup> However, in 1976 - 7 they were rather more widely adopted as a control device running alongside the more traditional volume planning of public spending, which had been introduced as a result of the Report of the Plowden Committee in 1961. Between 1976 and 1982 cash limits operated in tandem with volume planning, although cash limits overrode volume decisions where there was any conflict ( Likierman, 1988 ). From 1982 onwards cash limits were used as the sole basis for public expenditure planning in the UK.

It has been argued that cash limits were introduced in part because the system of volume planning was extremely difficult to understand. For example, it was almost impossible to relate expenditure in so - called "survey" prices to actual outlays.<sup>4</sup> Indeed, estimates of expenditure in survey prices were often referred to as "funny money", even by Whitehall civil servants. ( Likierman, 1988 ). Yet, it has been also claimed by a former Chancellor of the Exchequer that Government Departments understood only too well how the system operated and used the confusion between survey prices and current prices to increase their spending beyond the planned magnitudes.( Healey, 1989 ).<sup>5</sup>

Cash limits were meant to introduce greater financial discipline into the planning and control of the overall level of public expenditure ( see HMSO, 1976, para 3. ). In 1976 the Labour government was under considerable pressure from the IMF to prune public expenditure ( Treasury Bulletin, 1993, p. 20 ). Cash limits, which are unlikely to be relaxed even if there are cost increases which are beyond the control of public sector managers, act as a strong incentive to civil servants to keep within their budgets, although it is rather unclear what the punishments would be if Departments were to persist in overspending.

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3. See Cash Limits on Public Expenditure, Cmnd 6440, HMSO (1976). Cash limits represent a planned limit on cash outlays for the coming financial year ( HMSO, 1976, para 7 ).

4. "Survey" prices refer to prices prevailing at the time at which the Public Expenditure Survey was undertaken, which was normally in the November of the year preceding the first year of the planning period.

5. Dennis Healey, who was Chancellor of the Exchequer in 1976, in referring to public expenditure planning and control, says "...my task was complicated by the Treasury's inability to know exactly what was happening, or to control it ... This was one of the reasons why I decided to fix cash limits on spending..., since Departments tended to use inflation as a cover for increasing their spending in real terms." ( Healey, 1989, p. 401. ). This is not necessarily a contradiction, if the spending Departments understood how the system worked, but the controlling Department ( the Treasury ) did not.

The election of a Conservative Government in 1979, committed to reducing both the share and the absolute size of the government sector was expected to lead to a further tightening of the controls on public expenditure ( Lawson, 1993, p. 34 ). However, the structure of the process was left essentially unaltered. Instead the government acted to close various loopholes in the cash control system and to supplement cash controls with limits on borrowing and manpower restrictions in the civil service. It soon became obvious that manpower targets were unnecessary and so they were dropped.<sup>6</sup>

Of course, not all public expenditure has been, or indeed can be, cash limited. The most notable exceptions to the operation of cash limits have been (i) expenditure on social security, which must, of necessity, be demand determined<sup>7</sup> and (ii) local government current expenditure. In effect most transfer payments lie outside the cash limits system, the only exception being payments from the Social Fund, which was introduced in 1988 in order to reduce the amount of discretion given to local benefit offices. In practice about forty percent of public expenditure is cash limited ( or about two - thirds of supply expenditure )<sup>8</sup>, a proportion that has changed little since the system was introduced ( Likierman, 1988 ).

Whilst cash limiting various central government expenditure programmes might be expected to be quite successful, the control of local government expenditure has proved altogether more problematical. The main problem confronting central government control of local government expenditure was the fact that local governments have their own sources of income, such as local taxes and receipts from the sale of local services, which can be used to thwart central government controls. Also, local governments could borrow to finance expenditure, so that reducing central government grants to local government may not have a direct impact upon local government expenditure. In the nineteen - eighties the biggest problem confronting central government's attempts to control public spending was its inability to control local government spending directly. Various attempts were made to tighten controls over local government expenditure, the most important of which were the introduction of rate - capping of selected local authorities in 1984 - 5 and the

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6. The government did set up an Efficiency Unit in the Cabinet Office, but this was not expected to achieve expenditure savings on the scale required. ( see Lawson, 1993, p. 34 ).

7. Once qualification criteria and rates of payment are determined expenditure depends solely upon the number of qualified claimants coming forward. A government could not turn away unemployed people on the grounds that the budget for unemployment benefit had been exhausted. As a result the whole of the social security budget was excluded from the cash limit process. Other major exclusions included housing subsidies, agricultural support and family practitioner services. Whilst cash limiting local government current expenditure was not possible, the financial assistance to local government by central government was cash limited.

8. Supply expenditure consists of expenditure by central government that is financed by funds made available by Parliament. For an explanation of its relationship to total public expenditure see HM Treasury ( 1988, chart 6 ).

expenditure capping of all local authorities from 1992 - 3 onwards. ( Audit Commission, 1993 )

The other main problem area for control purposes was the National Health Service ( NHS ). Here the problems were related to the cost of drugs and to wage costs. To deal with the former, in 1985 the government introduced a 'limited list' of drugs from which doctors were expected to prescribe. Pay in the NHS was largely outside the control of Central Government, being determined by independent pay review bodies. Often these granted pay increases above those implicit in the cash limit for the NHS. Whilst the government often implemented NHS pay awards in a staged manner, they were frequently forced to raise the cash limit for the NHS. (Harrison, 1989).

Cash limits were not meant to be set in stone. They could be exceeded, but the intention was that they should not normally be changed. The two main reasons for relaxing them were (i) a change in policy made during the course of a financial year, and (ii) a change in circumstances which the government decides to accommodate, such as a rise in costs caused by a fall in the exchange rate. In fact, according to one commentator ( Likierman, 1988 ), cash limited items tended to be underspent, a tendency that was possibly accentuated by the increased flexibility that was granted to central government departments to carry forward any underspending on their capital allocation into the next financial year. This scheme was first introduced in 1983 - 4 ( HM Treasury, 1988, para 51 ). ( Appendix 2 contains information on cash limits and outturn for broad aggregates of public expenditure. )

Concern in central government over the control of public expenditure has led most recently to the announcement of a new approach to the control of public expenditure ( Treasury Bulletin, 1992 ). However, this change relates primarily to the definition of the public expenditure total to be controlled rather than to a change in the cash limit system, which will remain at the heart of the process.<sup>9</sup>

In analysing cash limits there are essentially two issues, which need to be kept apart. The first is whether their introduction was successful in keeping government spending within planned limits and whether cash limits were more successful at doing this than previous control devices.<sup>10</sup> This is a relatively uninteresting question. As there was no real control mechanism before the introduction of cash limits almost anything was bound to be more successful than anything that had gone be-

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9. The new control total, as it is called, excludes (i) privatisation proceeds and (ii) social security spending related to unemployment and includes local authority self - financed expenditure.

10. This was certainly the original intention of cash limits. The 1976 White Paper clearly shows that the government intended to use cash limits as a control device for the current financial year and medium term planning was undertaken in constant prices. However, there was a gradual shift of emphasis away from planning in real terms, so that by 1982 it was abandoned altogether.

fore. It is also fairly easy to show ( see Appendix 2 to this paper ) that after the introduction of cash limits, government departments almost universally underspent their budgets. However, this could arise if generous cash limits had been set in the first place. In a sense seeing that central government departments stayed within their cash limit tells us little that is interesting.

The second, and much more interesting, question is whether the introduction of cash limits led to a reduction in the level or rate of growth of government spending in either real or nominal terms. In other words, did the introduction of cash limits represent the main vehicle by which government expenditure cuts were delivered. It might be argued that one should look for evidence of expenditure cuts in terms of what happened to planned levels of government spending. However, as Dennis Healey has argued, when planning was undertaken in real terms departments were often able to mislead the Treasury about real spending. In any case what is important from a policy viewpoint is what actually happened to expenditure, not what was intended to happen. In effect without cash planning cutting public expenditure would have been extremely difficult if not impossible. However, whilst cash planning may be necessary in bringing about a reduction in government spending it is not sufficient in its own right. The limits need to be tight. The purpose of this paper is to test statistically if the introduction of cash limits, which supposedly represented a significant change in regime and attitude, led to a significant change in the level/rate of growth of public expenditure.

## **2. Methodology and data.**

In order to determine whether Government policies, particularly cash limits, were successful in controlling the overall level of public expenditure, we have analysed various time - series of public expenditure using a development of intervention analysis proposed by Tsay ( 1986, 1988 ). Recent applications of this method include analyses of outliers in agricultural land values ( Lloyd, 1993 ) and in macroeconomic time series ( Blake and Formby, 1994 ). An account of the method is given in Mills (1990). Traditional intervention analysis ( Box and Tiao, 1975 ) assumes a specific known point at which a structural break may have occurred. Although the dates at which alterations to policy on cash limits were undertaken are known ( see the introduction to this paper ), the actual timing of the effects (if any) of such policy changes are much less certain. In traditional intervention analysis, an ARIMA model is fitted to that part of the data set prior to the hypothesised structural break and it is assumed that the model appropriate to the 'pre-intervention' period continues to be appropriate for the 'post-intervention' period, subject only to the effects of the intervention. It is not normal to use the full data sample in order to select the initial ARIMA model, as the presence of any intervention effect in the post - intervention sample period may distort the model selection process. Although a particular form or effect of the intervention is prespecified in traditional analysis, different forms can be entertained and a choice made between these forms on the basis of

standard significance tests.

In contrast to the traditional approach, the method used in this paper starts the ARIMA modelling on the assumption that there have been no intervention effects over the whole sample period, so that the initial ARIMA model is chosen using the full data set. Tests are then undertaken using this model to identify aberrant behaviour in the residuals which may indicate some model misspecification. The form of intervention effects thus evolves from a sequence of tests performed in an iterative way on a succession of estimated ARIMA models where, at each iteration, the data set is transformed to allow for the effect of the most significant identified structural change.

This approach (which is essentially a data screening approach) seems particularly appropriate for this study, where neither the dates nor the forms of any intervention effects are clear. In particular, if one reaction to the introduction of cash limits was a rescheduling of, rather than a reduction in, expenditures, then this could appear as a change in the variance of the series. For example, the seasonal pattern of the series might be changed if budget holders need to curb spending in the final quarter of the year in order to remain within their budget constraints. Such a variance change could mask other effects, such as changes in the level of expenditures. The iterative method proposed by Tsay (1986) is designed to address such a problem.

In this paper we have examined a number of time series of government expenditure. The public expenditure series we have considered are (i) central government consumption expenditure, (ii) local government consumption expenditure, (iii) general government capital expenditure and (iv) general government current transfer payments. All four series have been examined in real terms.<sup>11</sup> All of the series are quarterly and cover the period from 1963:1 to 1992:4. Of these series, only that for transfer payments was not at any time cash limited ( see the discussion above ). This series is included in the analysis as a control series. There should not be any evidence of a structural break in this series that could be attributed to the introduction of cash limits.

Whether the series for local government consumption shows any evidence of an intervention effect, depends upon whether local governments were able to circumvent the attempts of central government to control their spending ( recall the discussion above ). The series for central government consumption and capital spend-

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<sup>11</sup> It is possible that the influence of cash limits could be detected in either the nominal or real series, although it is more likely to be detected in the real series. However, if government expenditure cash limits included inflation adjustments that were lower than recent inflation rates this would reveal a structural break in the nominal series after the introduction of cash limits. The problem with the nominal series is that a fall in the inflation rate will also show a break in the series and this would not necessarily be related to the introduction of cash limits. It is also clear from ministerial statements that it is the series for real spending that are paramount.

ing are, a priori, much more likely to show evidence of an intervention effect attributable to the introduction of cash limits. However, the series for capital expenditures might be subjected to other influences, such as the introduction of the end of year flexibility scheme in 1983 - 4. This raises the question as to whether cash limits have affected the seasonal pattern of expenditures.

In situations where agencies can defer expenditures into the next financial year, one might find that spending in the fourth quarter is reduced, while expenditure in the first quarter of the following year is higher, compared with a system without such a carry over facility. Without deferment there will be strong incentive for budget holders to spend any remaining balances in the final quarter of the year. Also, where there is no flexibility in the cash limit departments may adopt a cautious approach to spending in the early quarters of the financial year. The series for capital spending might be affected more than current expenditure in this respect, because of the increased opportunities to defer capital spending resulting from the introduction of greater flexibility in capital spending. As a consequence, the imposition of cash limits and allowing greater flexibility in spending may have increased the seasonal element in capital spending, in two, possibly conflicting, ways.

### 3. An iterative approach to the detection of variance changes, outliers and level shifts.

The basic model is a standard ARIMA model, which can be written as:

$$\Phi(B)z_t = \Theta_0 + \Theta(B)a_t \quad (1)$$

where  $\Phi(B) = 1 - \Phi_1 B - \Phi_2 B^2 - \Phi_3 B^3 \dots - \Phi_p B^p$

and  $\Theta(B) = 1 - \Theta_1 B - \Theta_2 B^2 - \dots - \Theta_q B^q$ ,

and where  $t$  is the explanatory variable ( government expenditure ),  $a_t$  is a stochastic variable, taken as white noise, and  $B$  is the backshift operator such that

$$B^j z_t = z_{t-j}.$$

Note that  $\Phi(B)$  could include unit root terms of the form  $(1-B)$  or  $(1-B^d)$  to take account of non-stationarity in the underlying series.

When there are outliers or structural changes present,  $z_t$  is unobservable. The observed series,  $Y_t$ , is assumed to follow the model:

$$Y_t = f(t) + z_t \quad (2)$$

where  $f(t)$  is specified as either a deterministic function to take account of outliers or changes in mean level, i.e.

$$f(t) = \omega_0 \frac{\omega(B)}{\delta(B)} \xi_t^{(d)} \quad (3)$$

where  $\xi_t^{(d)} = 1$  if  $t = d$

and  $\xi_t^{(d)} = 0$  if  $t \neq d$ ,

or as a stochastic function in order to capture changes in variance, i.e.

$$f(t) = \omega_0 \frac{\omega(B)}{\delta(B)} e_t^{(d)} \quad (4)$$

where  $e_t^{(d)} = 0$  if  $t < d$  and  $e_t^{(d)}$  is *NID*  $(0, \sigma^2)$  for  $t \geq d$ .

Unless there is an obvious structural break in the observed series which can be taken into account immediately (a single extreme value or outlier, for example), it is generally preferable to test for variance changes using equation (4) before conducting the other tests on equation (3), because variance changes may mask these other effects. To conduct the test, an ARIMA model is fitted to the whole data sample assuming no intervention effects. The residuals,  $\varepsilon_t$ , are obtained from this model, and the statistic:

$$r_d = \frac{(d-1) \sum_{t=d}^n \varepsilon_t^2}{(n-d+1) \sum_{t=1}^{d-1} \varepsilon_t^2},$$

is computed for all  $h \leq d \leq n-h$ , where  $h$  is sufficient to allow variances to be reasonably computed at each end of the data sample ( $h$  is taken as 25 in this study).

Both minimum and maximum values of  $r_d$  over the range of  $d$  are computed, and



the larger (denoted below as  $\lambda$ ) of the maximum and the inverse of the minimum is selected. This determines  $d$ , the point of a possible variance change. The maximum will be selected if there has been an increase in the variance of the series after  $t = d$ , and the inverse of the minimum if there has been a reduction in the variance at this point. A significant alteration in variance is identified if the value of  $\lambda$  is greater than the appropriate critical value. Exact critical values are unknown, but are normally taken as 2.5 (10%), 3.0 (5%) and 3.5 (1%), based on the simulation results computed by Tsay (1988). In this study, we have taken the critical value to be 3.0 in all cases. Should an increase in variance be detected, the data is adjusted as:

$$Y_t^* = Y_t \quad \text{for } t < d$$

$$Y_t^* = \bar{Y} + \lambda^{-0.5} (Y_t - \bar{Y}) \quad \text{for } t \geq d$$

where  $\bar{Y}$  is the sample mean of the data.

A straightforward reversal of the inequalities above provides the data transformation for the case of a variance reduction. A new *ARIMA* model is fitted to this adjusted data set, and a further variance change test is made. This process is repeated until no such changes are detected. The final adjusted series is then used to test for intervention effects other than variance changes.

Several kinds of intervention effect are possible. These are level changes, transient changes, innovational outliers and additive outliers. Level changes and transient changes are familiar from standard intervention analysis. Level change models allow for permanent step changes in the series being modelled, whilst transient change models allow the intervention effect to die away over time. An additive outlier model allows a one period "pulse" in the data and would be appropriate in the case of a recording error in the data series, for example. The effect of an innovational outlier depends upon the particular model which has generated the  $z_t$  values. That is, it depends upon the particular parameter values of the relevant stochastic model. The effect of the outlier may be permanent or temporary, as in general "...it influences the process  $z_t$  on  $z_t, z_{t-1}, \dots$  through the dynamic structure  $\theta(B) / \phi(B)$ ." (Tsay, 1986, p. 132). Tsay (1988, p. 3) notes that "In practice, an IO often represents the onset of an external cause."

Thus, in the **innovational outlier (IO)** model, (2) is written as:

$$Y_t = z_t + \frac{\theta(B)}{\phi(B)} \omega_t \xi_t^{(d)} \quad (5)$$

so that a shock,  $\omega_t$ , at time  $d$  is transmitted to  $Y_d, Y_{d+1}, \dots$ , through the mem-

ory of the model captured in  $\frac{\theta(B)}{\phi(B)}$ . An ordinary least squares estimate of  $\omega_t$

is given by  $\varepsilon_d$ , the value of the residual from the fitted ARIMA model at  $t = d$ .

A test for the presence of an innovational outlier is based on the computation of

the maximum value of the statistic  $\lambda_t = \frac{\omega_{t,d}}{\sigma}$  over all  $1 < d < n$  where  $\sigma$  is the

estimated standard error of the residuals. If this statistic is greater than an appropriate critical value (computed by Tsay (1986) and Chang (1988) as 3.0 (10%), 3.5 (5%) and 4.0 (1%)) and is greater than the corresponding statistics obtained for testing for the presence of additive outlier, transient change and level change models, then an *IO* model shock has been identified at time  $d$ . The data set is then modified to take account of this innovational outlier as follows:

$$Y_t^* = \text{for all } t = 1, 2, \dots, d-1$$

$$Y_t^* = Y_t - \psi_{t-d} \omega_t \text{ for } d \leq t \leq n \quad (\psi_0 = 1)$$

$$\text{where } \psi(B) = \frac{\theta(B)}{\phi(B)} = 1 - \psi_1 B - \psi_2 B^2 - \dots$$

Values of  $\psi_i$  can be obtained from the estimated polynomials for  $\theta(B)$  and  $\phi(B)$  by the method of equating coefficients.

In the **additive outlier (AO)** model, equation (2) is taken to be:

$$Y_t = z_t + \omega_A \xi_t^d \quad (6)$$

The ordinary least squares estimate of  $\omega_A$  (the additive shock at  $t = d$ ), is computed for all  $1 < d < n$  as:

$$\omega_{A,d} = \rho^2_{A,d} \left( \varepsilon_d - \sum_{i=1}^{n-d} \pi_i \varepsilon_{d+i} \right)$$

where  $\pi(B) = \frac{\phi(B)}{\theta(B)} = 1 - \pi_1 B - \pi_2 B^2 - \dots$  and  $\rho_{A,d}^2 = \left(1 + \pi_1^2 + \pi_2^2 + \dots + \pi_{n-d}^2\right)^{-1}$

The test statistic for the *AO* case is computed as the maximum value of

$$\lambda_{A,d} = \frac{\omega_{A,d}}{\sigma \rho_{A,d}} \text{ over all } 1 < d < n. \text{ If the test statistic is greater than the critical}$$

value for the test ( again taken as between 3.0 and 4.0 ), and is the highest of the test statistics calculated for the four potential types of structural break, then an additive outlier is identified at  $t = d$ . The data are then simply transformed as:

$$Y_t^* = Y_t \text{ for } t \neq d$$

$$Y_t^* = Y_t - \omega_{A,d} \text{ for } t = d.$$

In the **level change (LC)** model, equation (2) becomes:

$$Y_t = z_t \text{ for } t < d$$

$$Y_t = z_t + \omega_{L,d} \text{ for } t \geq d \tag{7}$$

An ordinary least squares of  $\omega_{L,d}$  is calculated from:

$$\omega_{L,d} = \rho_{L,d}^2 \left( \varepsilon_d - \sum_{i=1}^{n-d} \eta_i \varepsilon_{d+i} \right) \text{ for all } 1 < d < n, \text{ and where } \eta_i \text{ is}$$

the coefficient on  $B^i$  in the polynomial  $\eta(B) = \eta_0 - \eta_1 B - \eta_2 B^2 - \dots = \frac{\pi(B)}{1 - B}$ ,

and where  $\rho_{L,d}^2 = \left(1 + \eta_1^2 + \eta_2^2 + \dots + \eta_{n-d}^2\right)^{-1}$ .

The test statistic  $\lambda_{L,d} = \frac{\omega_{L,d}}{\sigma \rho_{L,d}}$  is computed for all  $1 < d < n$  and its maximum located. If this value is both greater than the selected critical value (as above) and is the largest of the four test statistics at this point in the calculations, a level change at  $t = d$  is located. For the next round of computations, the data are adjusted as:

$$Y_t^* = Y_t \quad \text{for } 1 < t < d$$

$$Y_t^* = Y_t - \omega_{L,d} \quad \text{for } t \geq d.$$

Finally, in a **transient change (TC)** model equation (2) is specified as:

$$Y_t = z_t \quad \text{for } t < d$$

$$Y_t = z_t + \delta^{t-d} \omega_{T,d} \quad \text{for } t \geq d \quad (8)$$

so that the effect of the intervention ( $\omega_{T,d}$ ) declines over time ( $\delta$  is a pre-signed value between zero and one; in this study a value of 0.7 was adopted). An ordinary least squares estimator of  $\omega_{T,d}$  is given by:

$$\omega_{T,d} = \rho_{T,d}^2 \left( \varepsilon_d - \sum_{i=1}^{n-d} \beta_i \varepsilon_{d+i} \right) \quad \text{for all } 1 < d < n$$

where  $\beta(B) = \beta_0 - \beta_1 B - \beta_2 B^2 - \dots = \frac{\pi(B)}{1 - \delta B}$

and where  $\rho_{T,d}^2 = \left( 1 + \beta_1^2 + \beta_2^2 + \dots + \beta_{n-d}^2 \right)^{-1}$ .

A test statistic  $\lambda_{T,d} = \frac{\omega_{T,d}}{\sigma \rho_{T,d}}$  is computed for all  $1 < t < n$  and the maximum

value located. If this is both greater than the selected critical value (as above) and the largest of the four test statistics calculated at this point in the computations, a transient change is detected at this time period. The data are then transformed as:

$$Y_t^* = Y_t \quad \text{for } t < d$$

$$Y_t^* = Y_t - \delta^{t-d} \omega_{T,d} \quad \text{for } d \leq t \leq n$$

After any of the four data transformations above have been performed, the original *ARIMA* model is then used with this new data to compute new residuals, and the test statistics are recomputed. In this way inner iterations are performed using the original *ARIMA* model but with data adjustments at each iteration which try to correct for the most significant intervention located in the previous round. Only when no test statistic is significant is a new *ARIMA* model estimated, using

the last computed adjusted series as input. This new model becomes the first stage of the next outer iteration. This procedure stops when no significant interventions are located for such an iteration.

#### 4. Empirical Results

In the following section we report empirical results obtained by using the estimating procedure, described in the last section, on quarterly time - series data for government expenditure, which have been obtained from Economic Trends. For each of the series considered, the sample period was from 1963:1 to 1992:4 and, where necessary, series were converted into real terms by the application of a price deflator. Gross government domestic capital formation ( i.e. central plus local government investment ) was deflated by a price index for all capital goods. This index was obtained by dividing a series for all capital formation, seasonally adjusted and in current prices, by the same series expressed in 1985 prices. These series are also available from Economic Trends. Seasonally unadjusted consumption expenditure in both current and 1985 prices was used to construct a price index for consumption expenditure used in deflating the series on transfer payments. Economic Trends also publishes series in 1985 prices for local authorities' final consumption and central government final consumption expenditures. All computations were made using a combination of the BMDP computer package for the ARIMA modelling and GAUSS for the computation of the test statistics.

##### (i) Real Government Capital Formation, Seasonally Unadjusted.

Figure 1 illustrates the general development of this series over the sample period considered. 'Eyeballing' this series suggests a possible increase in variance, which is perhaps connected to a changing seasonal pattern. As the methodology adopted presumes that this series will be modelled by an ARIMA process assuming no structural breaks, the normal convention of taking natural logarithms of the series to attempt to model the increasing variance was followed. The resulting series is displayed in Figure 2. A variance increase in this series is still suggested.

A satisfactory ARIMA model for this series, based on the statistical significance of coefficients, the 'randomness' of residuals judged from the autocorrelation and partial autocorrelation functions and the Ljung-Box Q statistic of the residuals, and the under and over fitting of parameters, was a seasonal multiplicative ARIMA model, written in the usual notation as  $(1,0,0) \times (1,1,0)_4$ . This represents a model with both regular and seasonal autoregressive terms, and no moving average parameters. This initial model was estimated as: ( results are given in full in row 1 of Table 1)

$$(1 - 0.743B)(1 + 0.324B^4)(1 - B^4)Y_t = a_t$$

A plot of the residuals from this model indicated that the combination of seasonal differencing and taking logarithms may be insufficient to model the changing variance of this series. The test described above in relation to equation (4) was computed using the residuals from the initial fitted model. A variance change (increase) was detected at  $t = 41$  (1973:1), and a modified series computed. A new *ARIMA* model was estimated for this modified series, which again proved to be adequately expressed as a multiplicative seasonal autoregressive process, that is a  $(1,0,0) \times (1,1,0)_4$  model.

This model was estimated as:

$$(1 - 0.747B)(1 + 0.311B^4)(1 - B^4)Y_t = a_t$$

The residuals from this model did not indicate a variance change, so that this model with its residuals could be used to test for the other structural breaks. Following Lloyd (1993, p.449), a critical value of 3.0 was adopted for 'high sensitivity'. Using this critical value, a variety of significant interventions were identified between  $t = 103$  (1988:3) and  $t = 111$  (1990:3). Such a bunching of effects is normally taken as being indicative of a general model change for the series at around these periods (e.g. see Tsay (1988, p.15)). The series was accordingly modified to take account of these breaks within this inner iteration. Specifically, as the first 'most significant' intervention on the initial inner iteration was identified to be a transient change at  $t = 107$ , equation (8) was used to modify the series as follows:

$$Y_{t^*} = Y_t \quad \text{for } t < 107$$

$$Y_t^* = Y_t - \delta^{t-107} \omega_{107} \quad \text{for } 107 \leq t \leq 120,$$

$$\text{where } \omega_{107} = \rho^2 \left( \varepsilon_{107} - \sum_{i=1}^{13} \beta_i \varepsilon_{107+i} \right),$$

$$\rho^2 = (1 + \beta_1^2 + \dots + \beta_{13}^2) \quad \text{and the } \beta_i \text{ are obtained from}$$

$$\beta(B) = \beta_0 - \beta_1 B - \beta_2 B^2 - \dots = \frac{\pi(B)}{1 - \delta B}$$

In this case,

$$\pi(B) = (1 - 0.747B)(1 + 0.311B^4)(1 - B^4) = 1 - \pi_1 B - \pi_2 B^2 - \dots$$

and the  $\pi_i$  values obtained by multiplying out and comparing coefficients.

The  $\beta_i$  coefficients were obtained analogously. The resulting series was then used along with the parameter values established for this inner iteration (0.747 and -0.311) to derive a new set of residuals from which four new test statistics are computed and the largest identified. In this case, another transient change is found at  $t = 111$  and the process above is repeated to derive a further modified series, and so on. For this inner iteration, the process stops after the series has been modified for an additive outlier at (again)  $t = 111$  ( see Table 1 ) as no significant intervention effects are identified for this series. This modified series is then modelled as a new ARIMA process, forming the model for a next set of inner iterations. In this case, the new model was estimated as:

$$(1 - B^4)(1 - 0.758B)(1 + 0.396B^4)Y_t = a_t$$

No significant interventions were identified for this model. It should be clear that there are substantial computations involved in the calculation of the coefficients (which are model specific) necessary to compute each test statistic within any inner iteration, in addition to the estimation of a sequence of ARIMA models.

From the above, we conclude that real government capital expenditure was subject to two interventions over the sample period. The series displays an increase in variance in 1973:2 ( period 41 ) and a change in structure indicated by the bunching of intervention effects from around 1988:3 ( period 103 ), neither of which can be easily and directly associated with the introduction and operation of cash limits.

Table 1.

Outer Iteration	Inner Iteration	ARIMA parameters		Type	Time	Size	RMS	LBQ	$\lambda$
		$\phi_1$	$\phi_2$						
I		0.743	-0.324				0.017	14	
	1	(11.31)	(-3.43)	V	41				5
II		0.747	-0.311				0.004	10	
		(11.98)	(-3.41)						
	1			TC	107	0.18			3
	2			TC	111	0.18			3
	3			IO	103	-0.19			3
	4			AO	107	-0.17			3
	5			IO	105	0.20			3
6			AO	111	-0.14			3	
III		0.758	-0.396				0.003	9.6	
		(12.59)	(-4.50)						

**(ii) Real Central Government Consumption Expenditure, Seasonally Unadjusted.**

Figure 3 illustrates the form of the (logged) series over the sample period. Results of model estimation are shown in Table 2. A satisfactory initial ARIMA model for this series was found to be a multiplicative seasonal model of the form

$(1,0,0) \times (1,1,0)_4$ . The residuals from the fitted model displayed no apparent variance alterations, and appeared random from the associated autocorrelation and partial autocorrelation functions, and from the computed LBQ statistic. The adopted procedure failed to locate any intervention effects for this series, either of a variance (stochastic) type or of deterministic types, if a critical value of 3.5 (suggested by Tsay (1988) as an appropriate 5% significance value) was adopted for the tests (see Table 2). However, if the approximate 10% value of 3 suggested by Chang *et al* (1988) for 'high sensitivity' is adopted, then the interventions shown in Table 2 were located. Following the data transformations implied by the initial chosen model and interventions, a second ARIMA model was estimated in the form:

$$(1 - B^4)(1 - \phi_1 B)(1 - \phi_2 B^4)Y_t = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-8}$$

As can be seen, as with the initial model no significant interventions would be identified using a critical value of 3.5, and only two (one at each end of the series) using the value of 3.0. The second model had no significant spikes in either the autocorrelation or partial autocorrelation function of the residuals, and the residuals appeared random. The third ARIMA model based on the data transformations stemming from the second model was of the form:

$$(1 - B)(1 - B^4)(1 - \phi_1 B^4)Y_t = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-3} - \theta_3 a_{t-8}$$

which indicated two further interventions, both towards the start of the sample period. The final model based on the data transformations stemming from the interventions of the third model was of the same form as the third model.

No further intervention effects were located in model IV. Our conclusion is that unless a highly sensitive critical value is adopted, no significant intervention effects seem present for this variable. The adoption of the more sensitive critical value locates some effects at each end of the data sample, but there are no convincing intervention effects located at periods that can be easily linked with either the imposition or the operation of cash limits.



Table 2.

		ARIMA parameters											
Outer Iteration	Inner Iteration	$\theta_1$	$\theta_2$	$\theta_3$	$\phi_1$	$\phi_2$	Type	Time	Size	RMS	LBQ	$\lambda$	
I					0.734 (10.99)	-0.285 (-3.00)				0.0005	17		
	1						IO	10	0.067			3.19	
	2						AO	89	0.051			3.28	
	3						IO	93	0.061			3.10	
	4						IO	17	0.060			3.24	
II	0.291 (2.69)	0.276 (2.69)		0.951 (21.68)	-0.581 (-5.64)					0.0004	17		
	1						IO	114	0.063			3.27	
	2						IO	18	0.056			3.05	
III		0.466 (8.22)	-0.207 (-3.01)	0.560 (7.90)	-0.89 (-12.1)					0.0005	23		
	1						AO	46	-0.04			3.47	
	2						LC	19	0.04			3.57	
IV		0.524 (9.24)	-0.273 (-4.30)	0.564 (9.57)	-0.935 (-17.10)					0.0004	21		

**(iii) Real Local Authority Consumption Expenditure (Seasonally Unadjusted).**

Figure 4 illustrates the (logged) series for this variable over the sample period. 'Eyeballing' this graph would suggest a possible outlier at  $t = 49$  (1975:1). The series appears to grow faster before this time than after. The initial ARIMA model fitted to this series was a multiplicative seasonal ARIMA model of the form .

$(1,1,0) \times (01,1)4$  From the results reported in Table 3, it appears that there was a change of variance at period 77 (i.e. 1982:1). In this case this variance alteration represents a reduction in the variance of the series, which might represent an alteration in the expenditure behaviour of local authorities at about this time (e.g. by a change in the seasonal pattern of expenditures). The variance of the first 76 observations was reduced by transforming these observations by the method described above. The model identified and estimated using this revised data was of the same form as the first model.

It appears that the variance change at time period 77 may in fact indicate a more basic model change at around this time as further interventions are significant here. The evident additive outlier at time period 49 is also identified, along with a transient change in mean (a reduction) shortly before this in 1974:2. A level change (upwards) is identified towards the end of the series (1989:2), which also seems reasonable from the time series plot of the data in Figure 4. The final ARIMA model for which no significant interventions were detected was estimated from the last set of transformed data from the second iteration as:

$$(1 - B)(1 - B^4)Y_t = a_t + 0.137a_{t-2} - 0.878a_{t-4} .$$

Table 3.

		ARIMA parameters									
Outer Iteration	Inner Iteration	$\theta_1$	$\phi_1$	Type	Time	Size	RMS	LB	Q		
I		0.867	-0.226				0.0003	24			
	1	(17.48)	(-2.42)	V	77						
II		0.919	-0.214				0.0001	19			
	1	(23.93)	(-2.28)	IO	77	0.05					
	2			AO	78	0.04					
	3			AO	49	0.03					
	4			TC	46	-0.02					
	5			LC	110	0.02					
III		$\theta_1$	$\theta_4$								
		-0.137	0.878				0.00004	28			
		(-2.85)	(19.69)								

**(iv) Real General Government Transfer Payments.**

Figure 5 indicates the behaviour of this (logged) series over the sample period. It will be recalled that this series is used as a 'control series' in the sense that it represents expenditures that have not been cash limited. The initial ARIMA model selected was :

$$(1 - f_1 B^4)(1 - B^4)(1 - B)Y_t = a_t - \rho_1 a_{t-1} - \rho_2 a_{t-8}$$

No variance change was detected for this model, although a Transient Change was detected at time period 47 ( 1974:3 ), and an Additive Outlier was detected at time period 89 ( 1985:1 ) (see Table 4). Both of these effects seem to be indicated in Figure 5, although the first predates the era of cash limits. The data were adjusted for these two intervention effects and a new model for the next outer iteration was estimated. This was of the form:

There were no significant coefficients in either the autocorrelation or partial auto correlation function of the residuals. For this model, used in the next outer iteration, no interventions of any kind were detected. Our conclusion that cash limits left this series unaffected is expected, but the result does, perhaps, give greater credibility to our linking of intervention effects with cash limits in the other series.

**5. Conclusion.**

Our overall conclusion, with respect to the success of cash limits in significantly reducing the level and/or the rate of growth of public expenditure, must be said to be fairly negative. The intervention effects, where found, seem to be more in the nature of changes in the pattern and timing of expenditure, reflected in variance changes for the series, rather than negative innovational or transient change outliers reflecting reductions in real expenditure. The method of Tsay used in this study does seem, however, to be a potentially useful technique for investigating series which may be subject to structural breaks especially where one is unsure of either the exact break points or the form of the breaks.

Table 4.

## ARIMA parameters

Outer Iteratio	Inner Iteration	$\phi_1$	$\theta_1$	$\theta_2$	Type	Time	Size	RMS	LBQ
n									
I		-0.767	0.195	0.508				0.0016	17
		(-9.21)	(2.34)	(4.53)					
	1				TC	47	0.13		
	2				AO	89	0.08		
II			0.199	0.718				0.0013	17
			(3.33)	(12.08)					

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*Appendix: Cash Limits 1976-7 to 1993-4***1. Central Government Votes**

Year	Cash Limit	Outturn	£ million % Overspread (+) Underspread (-)
1976-7	26,079	25,393	-2.6
1977-8	27,992	27,251	-2.6
1978-9	30,252	29,799	-1.5
1979-80	34,318	34,086	-0.7
1980-1	40,684	40,237	-1.1
1981-2	44,741	43,947	-1.8
1982-3	48,757	47,732	-2.1
1983-4	51,568	50,706	-1.7
1984-5	54,083	53,582	-0.9
1985-6	55,789	55,297	-0.9
1986-7	59,634	58,736	-1.5
1987-8	64,309	63,140	-1.8
1988-9	67,036	66,124	-1.4
1989-90	74,236	73,655	-0.8
1990-1	90,507	90,333	-0.2
1991-2	101,388	100,956	-0.4
1992-3	114,173	113,241	-0.8
1993-4	120,706	119,715*	-0.8

\* a provisional figure

Figures have been rounded to the nearest £million

*Sources: Various white papers on cash limits outturn*

**2. Expenditure not voted in estimates (essentially local government and other bodies' capital expenditure)**

Year	Cash Limit	Outturn	£million % Overspread (+) Underspread (-)
1976-7	2,692	2,229	-17.2
1977-8	4,594	4,080	-11.2
1978-9	5,135	4,599	-10.4
1979-80	5,379	5,264	-2.1
1980-1	6,645	6,382	-4.0
1981-2	7,305	6,347	-13.1
1982-3	7,878	6,685	-15.1
1983-4	6,983	7,349	+5.2
1984-5	6,838	7,777	+13.7
1985-6	7,143	8,043	+12.6
1986-7	7,797	7,981	+2.4
1987-8	8,559	7,913	-7.5
1988-9	5,477	5,313	-3.0
1989-90	6,107	6,050	-0.9
1990-1	10,378	10,251	-1.2
1991-2	11,715	11,598	-1.0
1992-3	13,591	13,436	-1.1
1993-4	12,966	12,802*	-1.3

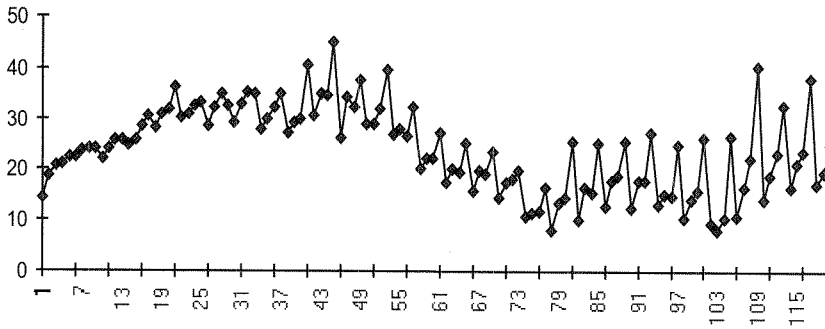
\* a provisional figure

Figures have been rounded to the nearest £million

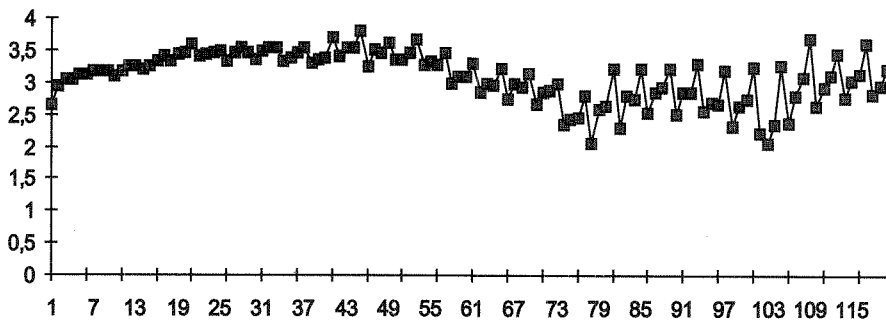
**Sources:** Various white papers on cash limits outturns



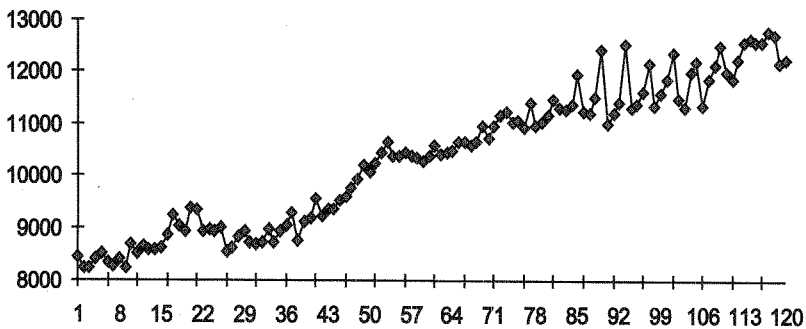
**Figure 1: Real General Government Capital Formation**

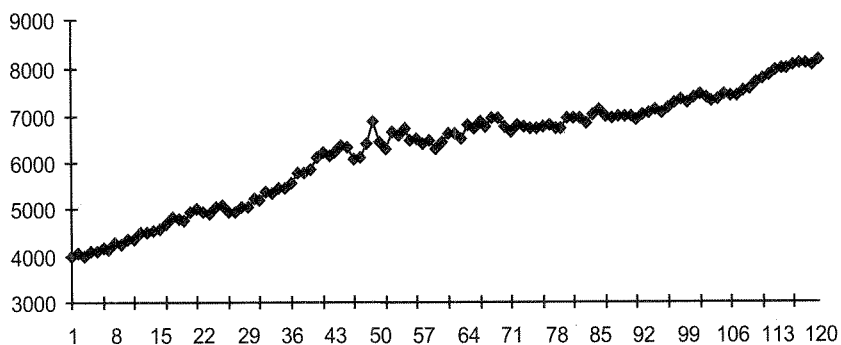


**Figure 2: Real Government Capital Formation ( in natural logs )**



**Figure 3: Real Central Government Consumption**



**Figure 4: Real Local Government Consumption****Figure 5: Real Government Transfer Payments**