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Optimization of Production Processes in the Furniture Industry Using Semi-Markov Models

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Katarzyna Kalisz-Szwedzka[1]

Abstract:

Purpose: *This study aimed to develop a semi-Markov model for assessing the reliability of a machining centre in a furniture industry enterprise. The focus was on minimizing production downtime caused by technical failures, addressing the lack of industry-specific reliability models for medium-volume furniture production. The study seeks to support furniture enterprises in managing production continuity effectively.*
Design/Methodology/Approach: *Empirical data from a furniture enterprise operating continuously in three shifts were analyzed. The semi-Markov model utilized technical documentation and records of 6005 downtimes and failures over two years, focusing on technical failures lasting over 10 minutes. Five key machine elements were modeled, with their times of fitness for use described by exponential distributions. Mathematica software was used to compute the density functions of these distributions, allowing for detailed analysis of the failure intensities of individual elements.*
Findings: *The model predicts the probability of the machining centre transitioning from a state of fitness for use to unfitness, based on failure intensities. It determines the expected failure-free operation time and the frequency of failures, aiding in optimal production and maintenance planning.*
Practical Implications: *The model helps furniture enterprises improve the reliability of complex technical systems by predicting failures and planning maintenance activities. This reduces unexpected downtime and supports production scheduling, particularly for large-scale contracts. Additionally, it can optimize spare parts inventory and maintenance strategies, lowering operational costs. The proactive approach facilitated by the model enhances asset management through condition-based maintenance policies.*
Originality/Value: *The study is an innovative application of semi-Markov process theory to the reliability of technical systems in the furniture industry. Its originality lies in adapting the model to real-world operating conditions and analyzing the reliability of key machine elements using exponential distributions. The findings contribute to predictive reliability models with potential applications in other industries.*

Keywords: *Sustainable development in an enterprise, operational efficiency, stochastic processes, system serial structure, semi-Markov–processes.*

JEL Codes: *C44, L23,C63, Q01.*
Paper Type: *Research article.*

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[1]*Poznan University of Technology, Poland*
ORCID: 0000-0003-3127-7216, katarzyna.kalisz-szwedzka@put.poznan.pl;

1. Introduction

Maintaining high production efficiency in the furniture industry is quite difficult and poses many challenges. In order to implement the production plan in a timely and effective manner, an enterprise must ensure that production is in keeping with a set schedule, a specified time and based on indispensable raw materials. When determining the production plan, the likelihood of unexpected and unplanned technical failures as well as the costs of their removal should also be taken into account.

However, predicting failures is extremely difficult due to the random nature of their occurrence. Thus, the knowledge of stochastic processes, in particular the theory of semi-Markov processes, proves to be helpful here. This paper formulates a research problem that caters for these needs. It presents the development of a semi-Markov model of reliability of a typical machining centre used in furniture industry enterprises. The model was developed on the basis of real data regarding downtime and technical failures contained in the technical documentation and technical maintenance and repair records of the tested machining centre.

The proposed model allows to determine the probability of correct work for any operating time of the analyzed machining centre as a whole, but also its individual elements. Reliability parameters were also determined for the developed model, which include the expected time of fitness for use of the facility, the frequency of failure occurrence and its standard deviation.

2. Literature Review

The furniture industry has a long tradition in Poland and has been an important economy branch for decades. The changes that took place in the 1990s initiated the dynamic development of this type of industry. Large plants were mostly taken over by foreign capital, and smaller enterprises tried to adapt to new conditions. As a result of globalization, product and process flexibility, i.e., the ability to adapt to changing customer expectations and changes directly related to production technology, gained importance (Dyba and Stryjakiewicz, 2014; Kosicka *et al.,* 2006; Migawa, 2013).

Advanced technological processes in enterprises mass-producing furniture currently require the use of specialized machines and devices, adapted to the specifics of the manufactured products, operated by specialized personnel. An assembly of interconnected machines and devices in different configurations is referred to as machining centres. They simultaneously realize complex interconnected working and auxiliary processes. The tasks of a machining centre include making a product from the processed material with the desired shape, size, accuracy, roughness and colour of the surface and with the required mounting holes (through and blind holes)

(Dietrich, 1985; Bajkowski *et al.,* 1988; Karpiński, 2013; Lewandowski *et al.,* 2014; Loska, 2011).

An enterprise, in order to realize the production plan in a timely and effective manner, must assume that production will take place in accordance with the adopted schedule at a specific time and will be based on indispensable raw materials. The production process of several thousand furniture series in a short time, using machining centres, has some limitations in the form of frozen capital and signed contracts for the planned supply of raw materials. In addition, the failure of any component of a machining centre leads to downtimes, and thus financial losses of an enterprise related to its incomplete use (Migawa, 2013; Bobrowski and Jaźwiński, 2001; Szwedzka *et al.,* 2017).

Reliability theory deals with the methods of synthesis and analysis as well as the study of technical systems at the design, construction and operation stages. Reliability issues include questions such as the serial structure of the system and stochastic processes, as well as methods for improving reliability and reliability optimization (Cygan *et al.,* 1994; Rzeźnik *et al.,* 2015; Migawa, 2013; Zółtowski and Tylicki, 2008; Zółtowski and Kołaczyński, 2011). The use of probabilistic models based on semi-Markov processes allows to accurately determine the reliability and operational readiness of technical systems (Grabski, 2001; Limnios and Oprisan, 1999; Linquist, 2009; Macha and Niesłony, 2010; Migawa, 2013; Paska, 2016; Zammori *et al.,* 2011).

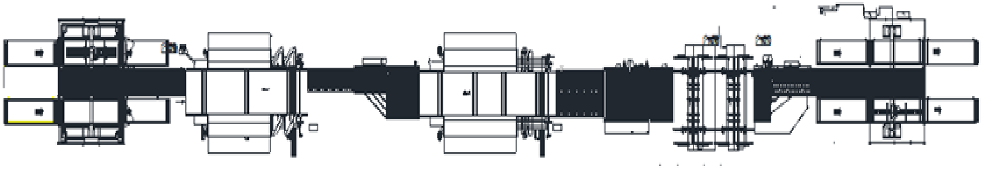
3. Research Methodology

The subject of the research was the development of a reliability model of the analyzed machining centre. The source database consisted of technical documentation and records of technical maintenance and repairs. All downtime periods of the machining centre, including those due to failures, were analyzed over a period of 2 years.

During the analysis, it was assumed that preventive and management downtimes testify to the state of fitness for use of the centre, i.e., that they were not caused by the occurrence of a failure. During the research, downtimes shorter than 10 minutes were omitted, assuming that these were events that did not require the intervention of the Maintenance Services. During the research, it was assumed that the considered machining centre is characterized by a serial reliability structure, consisting of $N = 5$ elements (Figure 1).

It was assumed that the time of fitness for use of each element with number k is a non-negative random variable $\zeta_k : f_k(t),\ t \geq 0$ with an exponential distribution determined by the probability density function (Charzan, 1990; Limnios, 2001; Grabski, 2017):

Figure 1. *Diagram of the Analyzed Machining Centre*



feeder tenoner turning tenoner conveyor drilling feeder

Source: *Own study.*

$$f_k(t) = \lambda e^{-\lambda t} I_{[0,\infty)}(t) \quad (1)$$

The reliability structure of the system shows that it is fit for use if and only if all its elements are in the state of fitness for use. The failure of the system occurs at the moment of damaging any of its components, and the damaged component is renewable. It was assumed that the renewal time (unfitness) of the k-th element of the system is a non-negative random variable $\gamma_k : G_k(t),\ t \geq 0$ with a distribution determined by the distribution function (Grabski, 2017):

$$G_k(t) = P\{\gamma_k \leq t\} \quad (2)$$

Exponential distribution is characterized by the property of lack of memory. This means that the renewal (replacement) of the element is also a renewal of the system. It was also assumed that the successive times of fitness for use ζ_k and unfitness for use γ_k of the element with number *k* are independent copies of random variables $\zeta_1, \zeta_2, ..., \zeta_N$ and $\gamma_1, \gamma_2, ..., \gamma_N$ respectively. It was assumed that random variables $\gamma_1, \gamma_2, ..., \gamma_N$ have positive expected and finite values and positive variances (Grabski, 2017).

During the research, the following states of the analyzed system were assumed:

- *N+1* – operation of a fit system;
- *k*, *k=1,...,N* – renewal (unfitness for use) of the element with number *k*, *k* = 1,....*N*

The reliability model describing the functioning of the analyzed technical system will be the semi-Markov process with a set of states S = {*1,2,...,* *N,N+1*}. The subsets were described as follows (Fleming, 1993):

- $S_N = \{1, 2, ..., N\}$ – a set of states of unfitness for use;

– $S_0 = \{N+1\}$ – a one-element set of states of fitness for use.

The moments when the system state changes $0 = \tau_0 < \tau_1 < ... < \tau_n$ will mean the moments of a system failure or the moments when the use of the system considered fit for use begins. It follows from the above assumption that the state of the system at the moment τ_{n+1} and the duration of the state achieved at the moment τ_n will not depend on the states adopted at the moments $\tau_0, \tau_1, ..., \tau_{n-1}$ and times of their duration. This means that the stochastic process $\{X(t): t \geq 0\}$ is determined by the following relationship (Grabski and Jażwiński, 2009; Grabski, 2017):

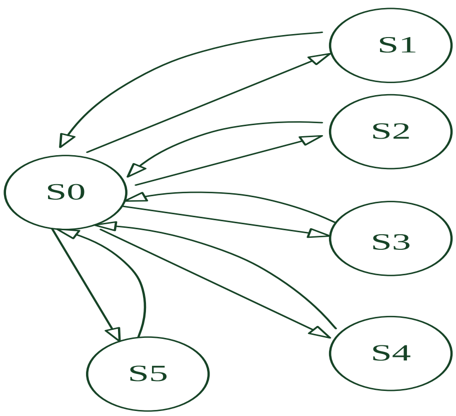
$$X(t) = k \quad dla\ k \in [\tau_n, \tau_{n+1}), \quad k \in S = \{1, \dots, N, N+1\} \tag{3}$$

is a semi-Markov–process.
In the analyzed model, five states of unfitness for use and one state of fitness for use were assumed:

- S_0 – a state of fitness for use (in the model state N+1);
- $S_{1,\dots,}$ S_5 - states of unfitness for use of individual elements of the technical system.

The graph of possible changes in the states of the analyzed technical system is presented in Figure 2.

Figure 2. *Graph of Possible State Changes of the Analyzed Technical System*



Note: $\{X(t): t \geq 0\}$ *for N = 5*
S_0 – a state of fitness for use, S_1 – a state of unfitness for use of the milling-tenoning machine, S_2 – a state of unfitness for use of

the receiver/RBO, S_3 – a state of unfitness for the use of the feeder, S_4 – a state of unfitness for use of the conveyor, S_5 – a state of unfitness for use of the drill
Source: *Author's own development.*

The graph in Figure 2 shows that the technical system can be in only two states: fitness and unfitness for use. System elements cannot be in any intermediate states. It follows that the S_0 state lasts until any functional element of the system has been damaged and passes to one of the unfitness for use states. After the repair, the technical system always returns to the S_0 state.

From the form of the graph describing the functioning of the analyzed system (Fig. 2) results the final form of the kernel of the semi-Markow process *Q(t)* defined by the following matrix (Grabski, 2017):

$$Q(t)=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & Q_{1N+1}(t) \\ 0 & 0 & 0 & 0 & 0 & Q_{2N+1}(t) \\ 0 & 0 & 0 & 0 & 0 & Q_{3N+1}(t) \\ 0 & 0 & 0 & 0 & 0 & Q_{4N+1}(t) \\ 0 & 0 & 0 & 0 & 0 & Q_{5N+1}(t) \\ Q_{1N+1}(t) & Q_{2N+1}(t) & Q_{3N+1}(t) & Q_{4N+1}(t) & Q_{5N+1}(t) & 0 \end{bmatrix}\begin{array}{l} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ N+1 \end{array} \qquad (4)$$

The matrix determined by the relationship (4) is a theoretical model of the reliability of the analyzed system. Therefore, the development of the above model requires the determination of all elements of the kernel of the semi-Markov process *Q(t)*. According to the presented matrix (4), the change in the *N+1* state to *k* (*k*=1,2,3,4,5) at the time not greater than *t* occurs when and only when an event occurs (Jażwiński and Grabski, 2003):

$$\{0 \le \zeta_k \le t, \zeta_i > \zeta_k, i = 1,2,...,k-1,k+1,N\} \qquad (5)$$

Using the independence of the times of being fit for use, it is possible to determine an integral:

$$Q_{k,N+1}(t) = P\{\zeta_k \le t, \zeta_i > \zeta_k,\ i = 1,\dots,k-1,k+1,\dots N\} = \int_D f_1(dt_1) f_2(dt_2) f_3(dt_3) f_4(dt_4) f_5(dt_5) \quad (6) \qquad (6)$$

Matching the above integral equation to the exponential distribution, the following integral is obtained:

$$Q_{k,N+1}(t) = \int_D \lambda_1 e^{-\lambda t_1} \lambda_2 e^{-\lambda t_2} \lambda_3 e^{-\lambda t_3} \lambda_4 e^{-\lambda t_4} \lambda_5 e^{-\lambda t_5} dt_1 dt_2 dt_3 dt_4 dt_5 \tag{7}$$

where $D = \{(\tau_1, \tau_2, \tau_3, \tau_4, \tau_5) : \tau_1 \le t, \tau_2 > \tau_1, ...\}$.

Replacing the integral after the D set with an iterated integral, one obtains (Grabski, 2017):

$$Q_{kN+1}(t) = \frac{\lambda_k}{\Lambda}(1 - e^{-\Lambda t}), \quad \Lambda = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5, \; t \ge 0 \tag{8}$$

The change of the state of unfitness for use k to the state of fitness for use N+1 at the time no greater than t will take place if and only if the following event occurs $\gamma_k \le t$, then (Grabski, 2017):

$$Q_{kN+1}(t) = P\{\gamma_k \le t\} = G_k(t) \tag{9}$$

The final form of the kernel of the semi-Markov process *Q(t)* will assume the form (Grabski, 2017; Grabski and Jażwiński, 2009):

$$Q(t) = \begin{array}{cc} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & G_1(t) \\ 0 & 0 & 0 & 0 & 0 & G_2(t) \\ 0 & 0 & 0 & 0 & 0 & G_3(t) \\ 0 & 0 & 0 & 0 & 0 & G_4(t) \\ 0 & 0 & 0 & 0 & 0 & G_5(t) \\ \frac{\lambda_1}{\Lambda}(1-e^{-\Lambda t}) & \frac{\lambda_2}{\Lambda}(1-e^{-\Lambda t}) & \frac{\lambda_3}{\Lambda}(1-e^{-\Lambda t}) & \frac{\lambda_4}{\Lambda}(1-e^{-\Lambda t}) & \frac{\lambda_5}{\Lambda}(1-e^{-\Lambda t}) & 0 \end{bmatrix} & \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ N+1 \end{array} \end{array} \tag{10}$$

where:
λ1, λ12, λ3, λ4, λ5 – intensity of damage to individual elements of the technical system
$\Lambda = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5$ – cumulative intensity of damage to the technical system

Thus, the above matrix is a model of the reliability of the analyzed technical system. The model assumes that $P\{X(0) = N+1\} = 1$. The results of the statistical analysis of the duration distributions of individual operational states, based on the data contained in the technical documentation of the analyzed system will be used to determine the elements of the kernel of the semi-Markov process *Q(t)*. The input data for the analysis will be: the time of the occurrence of a failure, the time of putting the facility into use, the place of the occurrence of an event and data related to the material produced.

When calculating the semi-Markov kernel boundary (10) when $t \to \infty$, one obtains the probability matrix of transitions of the Markov chain inserted in the semi-Markov process:

$$P = \begin{array}{c} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{\lambda_1}{\Lambda} & \frac{\lambda_2}{\Lambda} & \frac{\lambda_3}{\Lambda} & \frac{\lambda_4}{\Lambda} & \frac{\lambda_5}{\Lambda} & 0 \end{bmatrix} \end{array} \begin{array}{l} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ N+1 \end{array} \tag{11}$$

For the reliability model, reliability parameters can also be determined, which include the expected value of the random variable called the mean time of fitness for use of the facility $E(T)$, the frequency of the occurrence of failures $m(t)$ and its standard deviation $\sigma(t)$.

Random variable T_{N+1} with (Grabski, 2017; Grabski and Jażwiński, 2009):

$$G_{N+1}(t) = \sum_{i=1}^{N} Q_{i,N+1}(t) \tag{12}$$

means the time of fitness for use of the analyzed system. Taking into account the above, the following is obtained:

$$G_{N+1}(t) = 1 - e^{-\Lambda t}, \quad t \geq 0 \tag{13}$$

The above means that the time of fitness for use of the analyzed system has an exponential distribution with the parameter $\Lambda = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5$. Therefore, the expected time of fitness for use of the analyzed system can be determined from the following relationship:

$$E(T_6) = \frac{1}{\Lambda} \tag{14}$$

The determination of the expected time of returning to the state of fitness for use (moment of the first order) is made on the basis of the following relationship (Grabski, 2017; Grabski and Jażwiński, 2009):

$$E(\Theta_{66}) = \frac{1 + \lambda_1 E(\gamma_1) + \lambda_2 E(\gamma_2) + \lambda_3 E(\gamma_3) + \lambda_4 E(\gamma_4) + \lambda_5 E(\gamma_5)}{\Lambda} \tag{15}$$

From the above relationship, the equation for the second moment of the time of returning to the state of fitness for use is obtained (Grabski, 2017; Grabski and Jażwiński, 2009):

$$E(\Theta^2_{66}) = \frac{2}{\Lambda^2}[1 + \lambda_1 E(\gamma_1) + \lambda_2 E(\gamma_2) + \lambda_3 E(\gamma_3) + \lambda_4 E(\gamma_4) + \lambda_5 E(\gamma_5)] + \frac{1}{\Lambda}[\lambda_1 E(\gamma^2{}_1) + \lambda_2 E(\gamma^2{}_2) + \lambda_3 E(\gamma^2{}_3) + \lambda_4 E(\gamma^2{}_4) + \lambda_5 E(\gamma^2{}_5)] \tag{16}$$

After calculating the moment of the first (15) and second order (16), the variance of the time of fitness for use is determined from the relationship (Grabski, 2017):

$$D(\Theta_{66}) = \sqrt{E(\Theta^2{}_{66}) - [E(\Theta_{66})]^2} \tag{17}$$

The knowledge of the variance of the time of fitness for use allows to determine the frequency of failures from the relationship (Grabski, 2017):

$$m(t) = \frac{t}{E(\Theta_{66})} \tag{18}$$

and its standard deviation from the relationship:

$$\sigma(t) = \frac{D(\Theta_{66})}{E(\Theta_{66})}\sqrt{\frac{t}{(\Theta_{66})}} \tag{19}$$

For the developed reliability model of the analyzed system, it is possible to determine its reliability function from the relationship (Niziński, 2002; Macha, 2001; Matuszak, 2012):

$$R(t) = e^{-\Lambda t} \tag{20}$$

$$F(t) = 1 - e^{-\Lambda t} \tag{21}$$

and the function of unreliability from the relationship (Niziński, 2002; Macha, 2001; Matuszak, 2012) (21).

4. Research Results

When developing the reliability model of the analyzed machining centre, at the first stage of the research, a statistical analysis was made to identify all its failures and downtime (including preventive and management downtime), which occurred in the considered period of time. The analysis took into account the fact that production in the furniture industry enterprises takes place in a continuous, three-shift system, skipping only major public holidays. Based on the conducted analysis, a total number of downtimes and failures in the considered period of time was at the level of 6005 events. The total downtime in this period was nearly 200 days.

Following on, only technical failures, i.e., failures lasting more than 10 minutes, were selected from all the failures and downtimes. The number of technical failures in the analyzed period of time amounted to 696 events. For the failures received, the elements of the analyzed system causing these failures were identified. The obtained results are shown in Table 1.

Table 1. *Number of failures of the analyzed system caused by damage to its individual components*

Element of the technical system	**Number of failures [pcs]**
Milling-tenoning machine	192
Turntable	4
Receiver/RBO	80
Feeder	97
Conveyor	20
Drill	303

Source: *Authors' calculations.*

Afterwards, the times of fitness for use of the analyzed system between individual failures were determined. The times of fitness for use of the system were determined between failures caused by damage to its individual components. Then, numerical histograms of failure-free operation times (fitness for use times) of individual elements of the analyzed technical system were made using Mathematica software.

When making histograms, a histogram for the turntable/peavy was not made due to the small number of recorded events (data), which would make it impossible to determine the distribution of the time of fitness for use of this element and the estimation of distribution parameters. It was assumed that the assessment of the reliability of this element will not significantly affect the semi-Markow model of reliability and the results of further analyses using this model.

5. Discussion

On the basis of the developed histograms, for all the elements of the analyzed system, the density function of the distributions of fitness for use times was determined. To determine the distributions, an exponential function was used, which, in accordance with the theory of semi-Markov processes, will allow the development of a reliability model of the tested technical system. The parameters of these distributions were estimated using Mathematica software.

Table 2. *Parameters and form of the density function of the exponential distribution of individual elements of the tested technical system.*

System element	**λ $[\frac{1}{min}]$**	**Form of the distribution density function**
Milling-tenoning machine	0.000212107	$f_{k1}(t) = 0.000212107e^{-0.000212107t}$
Receiver/RBO	0.000106174	$f_{k2}(t) = 0.000106174e^{-0.000106174t}$
Feeder	0.000107465	$f_{k3}(t) = 0.000107465e^{-0.000107465t}$
Conveyor	0.0000275862	$f_{k4}(t) = 0.0000275862e^{-0.0000275862t}$
Drill	0.000164834	$f_{k5}(t) = 0.000164834e^{-0.000164834t}$

Source: *Authors' calculations.*

Having the coefficients λ (damage intensity) of individual elements of the analyzed technical system, it was possible to determine the cumulative damage intensity Λ of the entire system, the value of which is:

$$\Lambda = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 = 0.000618166[\frac{1}{min}] \tag{22}$$

The obtained cumulative damage intensity allowed to determine the equations of the probability function of transition from the state of fitness for use of the analyzed technical system to the state of its unfitness for use caused by the failure of its individual elements. The forms of the above functions are presented in Table 3.

Table 3. *Form of the probability function of transition from the state of fitness for use of the analyzed technical system to the state of its unfitness for use caused by the failure of its individual elements.*

System element	**Function form**
Milling-tenoning machine	$Q_{N+1\,k1}(t) = \frac{0.000212107}{0.000618166}e^{-0.000618166t}$
Receiver/RBO	$Q_{N+1\,k1}(t) = \frac{0.000106174}{0.000618166}e^{-0.000618166t}$

Feeder	$Q_{N+1\,k1}(t) = \frac{0.000107465}{0.000618166} e^{-0.000618166t}$
Conveyor	$Q_{N+1\,k1}(t) = \frac{0.0000275862}{0.000618166} e^{-0.000618166t}$
Drill	$Q_{N+1\,k1}(t) = \frac{0.000164834}{0.000618166} e^{-0.000618166t}$

Source: *Authors' calculations.*

Having determined the equations of the probability function presented in Table 3 and putting these equations in the last row of the matrix (10), the following form of the semi-Markov process–kernel *Q(t)* was obtained:

$$Q(t) = \begin{array}{c} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{0{,}000212107}{0{,}000618166}e^{-0{,}000618166t} & \frac{0{,}000106174}{0{,}000618166}e^{-0{,}000618166t} & \frac{0{,}000107465}{0{,}000618166}e^{-0{,}000618166t} & \frac{0{,}0000275862}{0{,}000618166}e^{-0{,}000618166t} & \frac{0{,}000164834}{0{,}000618166}e^{-0{,}000618166t} & 0 \end{bmatrix} \end{array} \begin{array}{c} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_0 \end{array} \quad (23)$$

The obtained matrix is thus a semi-Markov model of the reliability of the analyzed technical system. On the basis of the above model, it is possible to determine the probability of a change in the state of the analyzed technical system from its state of fitness for use to the state of unfitness for use, caused by the failure of its individual elements, for any time of system operation.

Assuming the time limit $t \to \infty$ for the calculation of the probability values, the probability limits (11) for the change in the state of the analyzed technical system were determined. The matrix of probabilities of changing the state of the analyzed technical system containing the probability limits is presented by the following matrix:

$$P = \begin{array}{c} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0.34 & 0.17 & 0.17 & 0.04 & 0.28 & 0 \end{bmatrix} \end{array} \begin{array}{c} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_0 \end{array}$$

In the above matrix, the sum of probabilities in each row is 1, which confirms the correctness of the calculations and the correctness of the developed semi-Markov reliability model of the tested technical system. The approximate values of the probabilities found in the last row of the above matrix can also be seen in Figure 5.

For the developed reliability model, relationship (14) was used to determine the expected time of fitness for use of the analysed system $E(T)$, which is:

$$E(T_6) = 1617.7[min]$$

Assuming that the analyzed machining centre implements the production process in a continuous, three-shift system, in which the weekly working time is t = 10000 min, using the dependencies (15), (16), (17) and (18), the frequency of failure $m(t)$ was determined for this time, which is:

$$m(10000) = 6.16$$

and the standard deviation $\sigma(t)$ was determined, which is:

$$\sigma(10000) = 0.82$$

Based on the obtained calculation results, it can be concluded that the expected time of fitness for use of the analyzed system is approximately 1618 minutes, and the expected frequency of technical failures (lasting more than 10 minutes) in the time interval of [0, 10000] minutes is approximately equal to 6.16, with a standard deviation of 0.82.

6. Limitation of the Study

Ensuring the high efficiency of the operation of complex technical systems is possible if decisions regarding the management of the operation of such facilities are rational and based on reliable data, recorded reliably by operators and maintenance staff. Based on these records, it is possible to map the actual technical system using a reliability model. The model should make it possible to get to know and analyze the tested system and processes at such a level of detail that it would be possible to develop rational and effective methods of controlling activities carried out by these systems.

The stochastic modelling of the reliability of the analyzed machining centre presented in the paper is an excellent method of supporting decision-making processes and enables a more rational implementation of the production process in the furniture industry enterprises. The developed semi–Markov reliability model of the tested machining centre allows to:

- determine the value of the probability of its transition from the state of fitness for use to the state of unfitness for use, caused by the failure of its individual elements, for any period of operation;
- determine the maximum time for the centre to remain in the state of fitness for use;
- indicate the number of failures in the specified period of operation;
- determine the reliability and unreliability function of the tested centre.

From the perspective of an enterprise, this is valuable information, because by indicating any given time of manufacturing one full batch of a product, the probability of technical failures can be predicted. This information makes it possible to change the production planning strategy from the optimal number of pieces to the optimal time of use of the machining centre.

Thus, this model will enable more effective planning of the production plan and maintenance and repair work, contributing to the reduction of unplanned downtime and more effective management of the operating time of this type of machining centeres, and thus to reducing production costs in in the furniture industry enterprises.

The presented two-state model can be expanded in a situation where an in-depth analysis of selected aspects of the reliability of the analyzed machining centre would be necessary. In addition, it is also possible to assess the place and cause of a given failure. Possessing information about which elements of the system most often fail and the specificity of these failures, it will be possible to better plan parts inventory, as well as determine their design requirements in order to increase their reliability.

The methodology proposed in the paper can be applied to other technical systems with a serial structure, provided that data available on the number of manufactured elements, system load, and information on technical downtimes and failures where they occur are available.

7. Conclusions

The study demonstrates the coherence of the application of semi-Markov models to optimize production processes in the furniture industry, showing their effectiveness in predicting system reliability and production line operational performance. The main conclusions include the ability to accurately predict downtime and plan maintenance activities, thereby reducing production interruptions.

The application of this model addresses key sustainability challenges by minimizing waste and optimizing resource utilization in industrial production. The limitation is the focus on a single type of machining center with a series structure, leaving room to explore more complex configurations such as parallel or hybrid systems.

In future research, the model should include different machine configurations and explore its applicability to other industries. This would increase its adaptability and offer broader insights into sustainable production management. Moreover, incorporating real-time monitoring and data analysis technologies into the model could further improve its accuracy and responsiveness to dynamic operating conditions.

In summary, this work fills a gap in the production of furniture on automated lines with a batch structure. Aligning with sustainability principles, it provides a roadmap for reducing operational inefficiencies and promoting environmentally friendly manufacturing practices.

The findings not only expand academic knowledge, but also provide practical guidance for entrepreneurs to achieve more sustainable and efficient manufacturing processes.

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