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Study of the Nature and Dynamics of Processes in Terms of Fractals on the Example of Selected Joint Stock Companies

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Motto:
How wonderful it is to feel the unity of a complex of phenomena that seem to be torn apart by direct reception.
Albert Einstein

Abstract:

Purpose: *The main purpose of the article is to highlight the role and impact on modern management of natural processes of knowledge about their nature.*
Design/Methodology/Approach: *The authors accepted the thesis that complex processes which contemporary management encounters should be solved with the use of complex formal tools.*
Practical Implications: *Thanks to the new and formal paradigms of modern science we are able to penetrate deeper into the nature of the real processes (economic, social, banking and even into the nature of the stock exchange - which was studied) and their complexity (structure).*
Originality/value: *The originality of the content of the article lies in the combination of theoretical concepts related to the research on the fractal nature of some reality processes with empirical research.*

Keywords: *Fractals, generative nature of fractals, diversity of processes, decision making, management.*

JEL classification: *M11, M12, M21.*

Paper Type: *Research study.*

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1. Introduction

The main goal of this article is to identify and analyze the nature and dynamics of processes in terms of fractals, on the example of components affecting the market value of companies listed on the Polish stock market. Because the mathematical tools (of fractal theory and mathematical statistics) allow, from their behavior, to extract, as it were, those important components that create the nature of the process.

It was shown in the content of empirical data analysis (Grönlund, Yi, and Kim, 2012). Nature encompasses processes that science is able to express using differential calculus, but there also are–at times in the very same processes–some aspects that do not lend themselves to differentiation. These have an additional and different variety compared to the so–called mathematically smooth objects, i.e., the differentiable ones.

Objects other than differentiable are studied in the field of fractal theory (Timoftea, Casian Botezb, Scurtuc, and Agop, 2011). What is a fractal? Generally speaking, a fractal[5], is a certain construct (structure) that can be both a real object and an abstract (man–made) creation. In both cases, the objects share a common property. It consists in the similarity between the whole and its parts (Mitchell and Murray, 2012).

However, one comes across a methodological barrier related to the framing of that similarity within a single conceptual system. One of the inherent features of both fractals and nature is their generativity, that constitutes an opposition, so to speak, to differentiability of objects and processes.

To paraphrase a famous quote from Albert Einstein as our research hypothesis, we are faced with the problem: what defines the structure of a process if both features of the nature of processes, i.e., generativity and differentiability, are combined. We might say that if these two qualities define the entirety of researched objects and processes, it has to mean that one serves as a complement to the other, neither being preeminent (in the sense of domination, as everything depends on the ontology of a given process).

The issue of that duality will be discussed throughout the paper. The duality of occurrence of these notions does not mean, however, that they are symmetrical (as properties) to one another, as is the case for instance in logic. In the preparation phase of the empirical study, the following hypotheses were accepted:

1. The value of a given stock at time t does not really (or to a very small extent) depend on its value taken in previous states.
2. time affects the market value of joint-stock companies listed on the Polish stock market.

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[5] *The term was introduced into scientific discourse by Benoit Mandelbrot (1982).*

The study conducted was aimed at answering the following research questions:

P1: What are the components that affect the market value of public companies listed on the Polish stock market?
P2: Whether and how the value of shares of joint-stock companies is affected by the days of the week and which of them turn out to be more and which less important during the week.
P3: Does generativity combine all the different forms and categories of fractals?
P4: Is there a property capable of expressing the important category of fractals, which is their self-similarity, by a single (identical) formal representation in terms of structure complexity?
P5: Is there and what is the mutual influence of determinism and randomness?

2. Benoit Mandelbrot - Creator of New Geometry

For centuries, science was ruled by the paradigm of geometry that had been introduced by Euclid. It was as late as in the 20th century that Benoit Mandelbrot was able to look at objects and processes in a manner different than they were perceived and conceived until that moment. He developed a new language of their description, and hence of describing the reality. To the three-dimensional space extended by the time parameter, he added an additional dimension, another metrics of looking at space and understanding it.

Then, much like Euclid before him, he compiled into a single whole the observations made by mathematicians dealing with objects difficult to define within the framework of Euclidean geometry and integrate the results of their researches with those of his own. Thus, he was able to develop a geometry of nature. A geometry based on two fundamental qualities, asymmetry and roughness, i.e., non–differentiability of a great number of real objects. For if they are non–differentiable, what is their nature?

The basic, ontologically relevant difference between the ideal objects in Euclidean geometry and those of everyday life, resulting from nature's activity, processes and randomness, in part caused by man, i.e., the difference within the domain of nature lies in the fact that in Euclidean geometry the closer we look at objects, the simpler they become. As an example of that we can look at a projection of objects onto a plane, a process grounded in abstract thought, consisting in larger number of dimensions being reduced in the projected version of the object, one that is simpler, less complex, all the way to zero dimension.

The opposite proves to be the case regarding natural objects. The closer we approach an object, the greater the number of emerging details. The same is the characteristic feature of fractals.

Attempting to typify fractals on the basis of their inherent properties, we are left with those created by nature, represented by freshets, fronts of the space where they are located, while on the other hand artificial fractals, characterized by their statistical self–similarity in relation to time, among which there are fractal time correlations.

That is why at this juncture the most important problem that requires solving seems to be the issue whether there exists a property able to express the essential category of fractals, that is their self–similarity, with a single (identical) formal representation in terms of the complexity of its structure. Detailed knowledge of fractal objects could probably introduce fundamental changes in our perception of reality, and at the same time could trigger the development of new scientific paradigms.

New scientific paradigms tend to lead the way to different perception and account of reality, oftentimes better than the existing ones. In fact, it could all be reduced to the mental aspect of man's nature, for there is truth in the claim that nature does not conceal anything from us, nor does it provide information selectively, as man often does, withholding some data and emphasizing other. Nature does not possess consciousness in the human sense of it, but it is clear and unequivocal (for itself).

Nature's behavior is nature itself, logically speaking nature constitutes a tautology (I am I). It is us who want to get to know it. However, does nature care about being understood by people, who are after all a part of it? It is possible, then, that it is the relation between man and nature that encompasses the meaning of asymmetry, due to the fact that nature existed before us and therefore it is us who have to make attempts at grasping it? That problem was also tackled by Benoit Mandelbrot.

He demonstrated in an analytical (operational) manner that in nature there are objects whose dimension is not an integer, instead being expressed with a fractional value, meaning that mathematically these constitute only a fragment of topological space (part of a whole) in which these objects are situated.

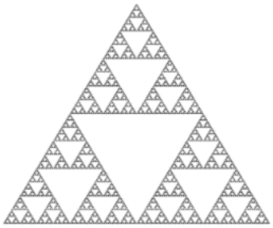
The discovery made by Benoit Mandelbrot might be compared to the concept of Medieval Arab mathematicians acknowledging the existence of zero and to the introduction of negative numbers by the mathematicians of India[6]. As scholars started to become aware of the existence of objects with fractional dimension in nature, the range of application of mathematical description grew to include virtually all fields of knowledge; all that in their research consider such notions as: number, time, spatial order and dimension.

The figures in Euclidean geometry possess definite and deterministic shapes. Fractal shapes can also be created in various ways. The simplest method to construct deterministic fractals is an iterative use of a generative rule;[6] obviously, the rule has

[6] *Mandelbrot came upon the idea of developing "fractal geometry" when studying the works of P. Fatou (1919) and G. Julia (1918).*

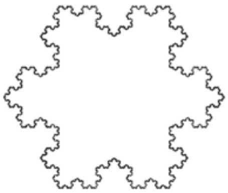
to be known. In academic literature the typical examples of fractals generated using that procedure include the Sierpiński triangle and the von Koch snowflake (Figures 1, 2, and 3).

Figure 1. *Sierpiński triangle.*



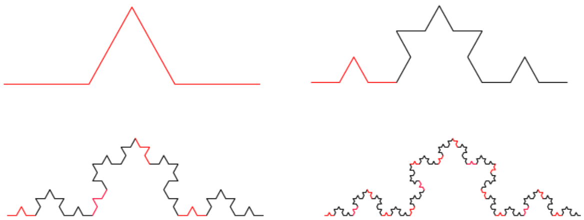
Source: *W. Sierpiński, Sur une courbe dont tout point est un point de ramification, "C. R. Acad. Sci. Paris" 160 (1915), 302-305.*

Figure 2. *Von Koch's snowflake*



Source: *Own work.*

Figure 3*. Von Koch's curve (the basic element of the snowflake) in a generative view.*



Source: *Own work.*

For instance, looking at the geometric shape of the Sierpiński triangle, or at von Koch's snowflake one can clearly observe that both figures are symmetrical in the way they are created. Meanwhile, analyzing other natural and artificial processes, we cannot but notice a striking lack of symmetry in some of them.

The objects mentioned actually exist, they were created (particularly the artificial ones) to attain some strictly defined objectives; therefore, they have to possess a teleological character, certain structure, which displays a stability of construction and hence of behavior over time.

In order to describe an actual object, e.g., in the observation of a time series, one has to infer the principle allowing for expressing the series in the form of a fractal. The process consists in the property of self–similarity of the respective elements of the fractal to the whole loosing the quality of being symmetrical. In the signifying (content) aspect of the process, it means that we are unable to deterministically forecast the value of, say, a given equity in the stock market.

At the end of our text, we are going to quote the empirical results of several joint–stock companies listed in the Polish stock market, i.e., their variability over time. The Hurst exponent will serve as a clear indication of their indeterminacy. It proves difficult to predict (forecast) what value a given equity will assume next.

At the subsequent stage of our consideration of the nature of processes we should ask about the way randomness counteracts the loss of symmetry (the fact it is ruined). An attempt at answering this question has been provided in the text below. At the early stage of development of the science of fractals, mathematicians would work on single objects, classified today as deterministic fractals (Cantor, 1883; von Koch, 1904; Sierpiński, 1915; Julia, 1918; Fatou, 1919).

The figures generated during that period were yet to be identified under their current name. It was only at some point of that discipline's progress when scientists realized that fractals are for the most part created by nature and to a certain extent they are of random character.

In other words, random fractals result from chance development of the generative rules in various scales. Here, we can refer to a conceptual category of randomness in temporal processes. The question is whether it is a universal category of nature, or whether it merely has a local character.

Generativity is universal, because it is an important natural category. It manifests itself in natural language, the foundation of human communication. Due to embryogenesis, generativity is also to be found in the anatomy of man and in many other aspects of reality. The scaling of fractals consists in that the respective elements of a division differ in size or the role they play in the process of generative division. For a specific grammar, in which the rules of discourse have been given certain probabilities, one may define the diversity by calculating the entropy of that system.

However, the diversity generally tends to be accounted for by evolution (Peters, 1997, 10-11). It may therefore be claimed that the natural rule responsible for the diversity of scales is associated with the adaptive abilities of the analyzed system. Natural selection, in itself potentially (locally) random, fosters the emergence of randomly scaled fractals. Such combination of randomness and deterministic generative rules (their mathematical structure), i.e., causality, makes fractals potentially useful in analyzes of many processes of very complex, or still unknown and mathematically non–differentiable structure.

The property of generativity is beneficial for getting to understand as yet unknown objects. Generativity is a new conceptual category which may prove to be somewhat complementary to the notions of continuity and differentiability. Nevertheless, as a conceptual category it has been known to science for quite a long time in the form of combinatorial systems (Ajdukiewicz, 1935; Chomsky, 1959).

However, within fractal theory generativity finds its new, significant place, one that it deserves. It constitutes a perspective on reality different from the one we know from mathematical linguistics. In the operational aspect of fractal analysis, mathematical linguistics did not change its character (which is fundamentally algebraic). Within it, however, the need emerged to analyze fractal objects in space, such as the Cantor's set, the von Koch's curve, as well as fractal time series.

3. Categories of Processes in Temporal Dimension

In our reality, comprising as it does one temporal and three spatial dimensions, three categories of processes operate: the random, the persistent and the least known of the three, the antipersistent ones. All the aforementioned process categories are included in the image of the world as we know it. It is possible that the number of such categories is far greater, that they may be ordered around a certain property, parameter, or in another manner altogether, but currently in terms of the fractal theory of time series only the three above–cited have been indicated.

Willing to identify the fractal dimension of a research object, one has to measure the degree of its density, that is to discover the scale of how the object densifies in space. Many methods of calculating the fractal dimension have been established.

All these metrics consist in discovering the fractal shape (of a volume, surface area or line) and defining how it changes when the unit of the shape's measurement is decreased. The coastal lines of countries or continents are a good example of estimating the fractal dimension. Geometrically, coastal lines exhibit similarity to time series. The observed pattern constitutes a valuable scientific indication, for it enables one to transfer the calculation methods from one domain to another. A costal line and the graph of a time series share a common property — they are both rugged. Now, how would one go about measuring the irregular coastline, say, of Norway?

First, we must select an appropriate unit of length. The lower the value of this unit, the more precise the measurement of the researched coastline will be. However, Benoit Mandelbrot (1982) claimed that it would be impossible to measure a coastline, as the result will depend on the measure adapted. Length proves inappropriate as a metric for calculating the degree of ruggedness of a coastline, or for the variety of a time series.

Instead, the suggestion of the originator of the new geometry, pertaining especially of real–world objects, was to use the fractal dimension in comparisons of the degree of ruggedness of coastlines or of variance in equity.

Benoit Mandelbrot presented the following argument to support his postulate, the coast is rugged, therefore, its dimension (the sum total of all its constituent sections) is greater than the dimension of the section, which includes the measured coastline, which means that it exceeds the length of one. Now we come to the question, what is and how do we calculate the fractal dimension of the coastline? Several algorithms for determining the fractal dimension have been developed; however, none of them has been perfect.

Generally, the fractal dimension tends to be measured using the degree of ruggedness of the edge of a given set. In order to do that, one has to calculate the number of circles of a specified diameter, required to cover the entire run of the maritime coastline. The number of circles is exponentially related to the length of the circle's radius according to the following formula:

$$N \times (2 \times r)^D = 1 \tag{1}$$

where:
N – the number of circles,
r – radius,
D – fractal dimension,
× – multiplication sign.

After taking the log of formula (1), we arrive at the fractal dimension in the form of:

$$D = \frac{\log N}{\log\left(\frac{1}{2 \times r}\right)} \tag{2}$$

4. Significance of Fractal Dimension for Risk Assessment

The theories, metrics and objects created by scholars acquire particular significance only when they can find practical application. Such utilitarian approach is particularly advocated in the case of purely mathematical creations.

Through formal structures developed on the grounds of mathematics we acquire the possibility to transfer certain properties of these objects from one field of science to another, and thus to use them in a functional manner. It has to be acknowledged, quite paradoxically, that such exactly is the power of mathematical abstractions. A condition *sine qua non* for benefitting from the great potential provided by mathematics is the need to possess the mental capability to recognize analogies between the researched objects.

For example, the dimension of the Norwegian coastline is 1.52, that of the Great British coastline is 1.36, while the Polish measures 1.14 (Winnicki, 2010; 176). The data just cited means that the coast of Norway is the most irregular, whereas that of Poland, the least. This scientific finding is treated as a fact, therefore, the very same reasoning can be applied to the comparison of changes in value of equity within a finite period of time. By their nature, shares in the stock market do not differ from coasts.

However, when our considerations come to include the temporal factor, the coastlines of countries are assumed to be invariant. Shares, on the other hand, may be the subject of rapid changes over time. Variability generates risk. The greater the variability of a given share, the higher its risk (Markowitz, 1952).

Thus, classically understood risk is measured using the statistical metric of spread, that is by variance (standard deviation). Generally speaking, it is acknowledged that a large spread involves high probability of major changes in the rates of return. Meanwhile, in the case of the measure of spread (general metric of the process' variability), standard deviation may constitute an appropriate metric only in the case of a random system. If there occurs some correlation in values, the time series displays self–correlation largely limiting the usefulness of standard deviation as the measure of spread.

5. Fractal Time Series

Thanks to the use of fractal geometry, our existing perception of the world has been transformed. Not only are we now able to create, but also to study complex figures using the method of iteration of simple generative rules. The new geometry introduces into the various branches of knowledge and some specific disciplines in particular an, as yet unknown, dimension of looking at objects and processes.

For instance, in economy a very significant role has been given to technical and fundamental analysis. Many analysts expected the newly developed fractal geometry to result in the identification of novel properties of markets and economies.

However, Edgar E. Peters (1997) expressed the opinion that true, practical meaning of fractals is more profound, for natural processes do not constitute a series of regularly iterative formations (sets, structures). Instead, they are locally random and globally ordered. The observed property of randomness and determinism in real–world processes brings major advantages for scientific analyzes, particularly those focusing on the market and the economy.

The fractal dimension can be described using many qualitative and quantitative properties. One of its key characteristics is the manner in which a time series (but also a shape or a geometric figure) permeates its space, as already mentioned. However, an important role is played here by statistical data (randomness) at the micro and macro economic level.

On the basis of such data, investors can draw inferences to support statistical assessment of the value of assets. More often than not, the behavior of investors is not uniform. Most of them tend to react to a change when the trend has already been markedly established.

Meanwhile, the information obtained is of statistical nature. The rate of influx of data varies over time and involves the so-called random walks, in which the preceding value does not have a bearing upon the changes about to occur. Hence, when evaluating the influence that fractal dimension had on probability distribution, one has to consider the possibility of occurrence of long–term correlations in fractals and fractal time series (the property does not have to be reduced to the so–called random walks, whereas fractal time series do not have to follow the normal distribution).

6. Hurst Exponent

The Hurst exponent (*H*) is currently widely applied in the analysis of many naturally ordered processes (Harold Edwin Hurst (1880-1978).

The order pertains to a sequence of events (x_t) occurring in time; thus, for instance to a series of:

$$\ldots, x_{t-1}, x_t, x_{t+1}, \ldots$$

the Hurst exponent involves a small number of assumptions regarding the studied system (process). On the basis of such measure, we are able to classify time series and to distinguish random from nonrandom time series. The task can be carried out also in the case of non–Gaussian random series, i.e., those that do not fall within the normal distribution.

During his research, Hurst discovered that majority of natural systems are not affected by random walks, either of the Gaussian type, or of any other.

Hurst measured the variance of water level in the reservoir in comparison to the mean water level over a given period. The range of the results he obtained varied and depended on the duration of the measurement. If the character of the series was random, the range (*R*) should change proportionally to the square root of time (*T*), that is:

$R \approx \sqrt{T}$

From a mathematical standpoint, it would be beneficial to apply a uniform measure of variance, independent of time. By dividing the range of variance by standard deviation, he was able to obtain a dimensionless indicator. That is why this type of analysis is referred to as the rescaled range analysis.

Now, from scientific perspective it is interesting whether the results obtained by Hurst in his studies of natural phenomena of time series may be generalized to time series within the economy, and to other man–made processes. To do this, we must first define the notion of range, corresponding to the variance of the water level in the reservoir.

If the time series = t, comprises = n observations and:

$$X_{t,N} = \sum_{n=1}^{t} \left(x_n = \overline{X}_N \right), \tag{3}$$

where:
$X_{t,N}$ – cumulated deviation in n periods,

x_n – the value (inflow) in a period *n* (year, quarter, month, …),

$\overline{X}_N$ – mean value of x_n in *N* periods.

With the accepted assumptions the range is equal to the difference between the maximum and the minimum level found in the formula (3):

$$R = \max\left(X_{t,N}\right) - \min\left(X_{t,N}\right), \tag{4}$$

where:
R – the range of *X*,
max(X) – the maximum value of *X*,
min(X) – the minimum value of *X*.

In practice, it is necessary to compare various types of time series, both the natural and the man–made ones. By dividing the range by standard deviation of the original observations, Hurst obtained a rescaled range, in which the level of values is expected to grow with the flow of time. On the basis of this observed property, Hurst formulated the following correlation:

$$R / S = \left(a \times N\right)^H \tag{5}$$

where:
R/S – the rescaled range,

N – the number of observations taken in the given period,
a – a constant,
H – the Hurst exponent.

7. Statistical (Quantum) Mechanics and Hurst Exponent

Time is a significant factor that influences the structure of a series. Therefore, the category of time should explain the shaping and the nature of the real–world process modeled by that series. In order to achieve that, time has to be expressed in a clear–cut, quantitative manner and then interpreted in its content (qualitative) aspect.

Within statistical mechanics, if a time series has the property of random walk, the value of the Hurst exponent is 0.5. This characteristic means that the range of cumulated deviations should grow proportionally to the square root of time, i.e. in the physical dimension to the square root of the number of observations (N) taken into consideration. The subsequent events maintain the memory of their predecessors for quite a long period and, according to Peters, not the Markovian–memory, but some other type thereof (Peters, 1997).

Regarding the nature of the studied processes and their properties of the time series they were represented by, the value of H other than 0.5 is assumed to be an indication of the fact that the observations are not independent. Each observation retains the memory of all previous events and at the same time influences the future ones. Over time, the mutual influence between observations decreases (weakens). The present has an impact on the future, meaning that time is an iterative process (as in the above— discussed chaos game). The correlation at hand can be mathematically phrased with the following formula:

$$C = 2^{(2H-1)} - 1 \tag{6}$$

where:
C – the measure of correlation,
H – the Hurst exponent.

There are three classes of values of the Hurst exponent:

1) H= 0,5,
2) $0 \le H < 0{,}5$
3) $0{,}5 < H \le 1$.

In the case when the Hurst exponent assumes the value of 0.5, we are dealing with a random series and the solution to the equation (6) is equal to zero. Then, the present has no influence over the future, where the probability density function may (though does not have to) resemble the normal curve. That means that the *R/S*analysis may enable one to recognize an independent series, regardless of the shape of its distribution curve.

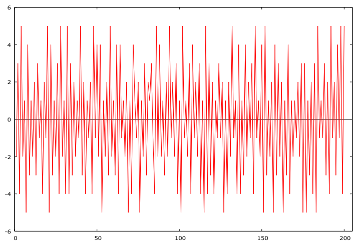
Thanks to Hurt's discovery, we are able to reject the existing conjecture that, although nature does produce various distributions, when the number of elements is high they could be replaced in statistical calculations with the normal distribution, as with regard to the processes commonly occurring in nature the H exponent tends to exceed 0.5. The time series then emerging typically represent fractional Brownian motion and reinforced random walks.

Such series are characterized by a tendency to reinforce the trend; therefore, they are referred to as persistent series. Fractal theory studies the causes of persistence; that is, the stability of trends in a series and the nature of the "memory" effect: how far back such "memory" may reaches, i.e., its range. This notion is only explained here, though for the first time it appeared in the section where we discussed the fractal dimension.

In the situation when H assumes values within the range 0 to 0.5 ($0 \leq H < 0.5$), the series is antipersistent. Another name often encountered with regard thereto is that of an ergodic system, i.e., one tending to reverse to the mean, meaning that if during a given period the system assumed a value above the mean, if it spiked, there is a highly probability that in the subsequent period it is going to take a dip, or the reverse.

The more a system's behavior is ergodic, the closer the value of the Hurst exponent is to zero. At that instant, an ergodic series becomes more variable than a random series. The property is reflected in the geometric representation of such series, which is highly rugged. An example (Figure 4).

Figure 4. *Antipersistent time series with H = 0.41437.*



Source: *Own work based on https://www.money.pl/gielda/spolki-gpw*

7.1 Calculating Hurst Exponent and Fractal Dimension

The rescaled range (5) established by Hurst as a result of dividing the range by standard deviationof the original observations served as the basis for finding the Hurst exponent and the fractal dimension. The formula of the basic correlation (*R* / *S*

= ($(a \times N)^H$) is exponential; therefore, in order to estimate the parameters of the equation we took the log of both sides:

$$\log(R/S) = H \times \log(N) + \log(a) \tag{7}$$

Then, we substituted the following in place of the logged values:
$Y = \log(R/S)$
$X = \log(N)$
$b = \log(a) = X$

As a result, we obtained a markedly analytical form of a linear function:

$$Y = H \cdot X + b \tag{8}$$

where:
Y – dependent variable,
X – independent variable,
H – the slope,
b – the intercept.

Empirical data provides the *X* and *Y* variables. In order to determine the analytical form of function (8), one has to estimate the structural parameters of *H* and *b*. By finding out the inclination of the graph of the R/S logs over the time logs axis (the number of observations), we determine the value of *H*. In this calculation, no statistical assumptions regarding the shape of a given distribution are made, and hence also regarding the regression model.

However, for a high number of N, resulting from the fact that the “memory” effect weakens over time, one should expect the series to tend to H = 0.5. This means that the regression equation (7) pertains only to the data prior to the tendency of H towards 0.5. Further important observations regarding the calculation of the Hurst exponent (H) concern the measure of correlation included in equation (6) and the quantity of data. The metric of correlation used does not apply to all increments in value of the analyzed series, for it is not related to the auto correlation function (ACF) of Gaussian random variables.

The autocorrelation function is used with good results in determining short–term correlations and generally does not appropriately reflect the long–term correlation of non–Gaussian series. In the case of small number of empirical *n* (data), the regression equation proves unsuitable for estimating the H exponent. By assuming that *a* in equation (7) is equal to 0.5, Hurst designed a formula on the basis of which we are able to determine the value of H with a single R/S value[7].

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[7]*This form is reflected in equation* $H = \log(R/S)/\log(n/2)$ *(Peters 1997).*

In the course of the paper, we have been trying to find the answer to the question of the mutual influence between determinism and randomness. Various aspects of the phenomenon have been investigated, leading to the conclusion that despite the effort made, it is still difficult to provide an unequivocal solution. The one certain thing is that these two ontological variabilities may exist alongside one another. In all likelihood, they provide benefits to one another resulting from such vicinity.

Thus, nature would be playing with itself. What is observed is the game of nature vs nature in the real–world dimension of (3×3) . The distribution of payoffs is to be determined. It may be hypothesized that the stronger process turns out victorious, but it is equally possible that the stronger process could lose its dominance in favor of the weaker one.

For instance, when a random process is adjacent to a *persistent* process, the random process acquires an outline of a trend, whereas the *persistent* process records a slightly lower value of the Hurst exponent as compared to the previous state, i.e., it looses the stability of the trend. This proves sensible in relation to any other game, in which the final value does not have to equal zero. As nature plays with itself, it does not win nor loose anything in the end, even when playing a game with another player of different qualitative characteristic (Volgin, 1970).

8. Fractal and Statistical Aspect of Properties of Time Series Based on the Analysis of Empirical Data

In fractal analysis, like in any other theory, there are some required conditions that have to be met in order for the scientific research on these processes or objects to be performed. Compared to statistical analysis, in the domain of fractals no statistical assumption are used, or at least they are limited to the minimum.

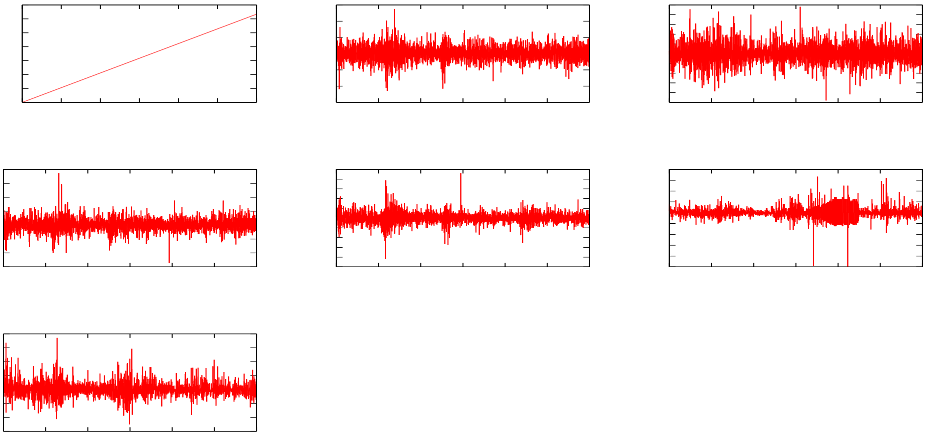
Willing to obtain the real value of the Hurst exponent in the research of the degree of variability of time series in the form of processes, objects and other ordered elements of the surrounding reality, one has to take into consideration the properties that comprise the dimension of a series. Many of the properties stem from the knowledge amassed by science to date.

Other results are obtained by experiments, or in further (in–depth) studies pertaining to the analyzed problems, such as the issue of coherence of the market, or, most broadly speaking, the relations occurring between the environment and the non–linear statistical time processes (Vaga, 1991; Peters, 1997). Therefore, we will now present some selected results obtained from the analysis of data recording the behavior of a number of joint–stock companies in the Polish economy (https://www.money.pl/gielda/spolki-gpw).

We will start from a graphic presentation, which will visually indicate the shape of behavior taken by the respective companies in the stock market over time, i.e., dynamically (Figure 5).

When looking at the graphs above, one may and should ask about the qualitative and essential (through numbers) content conveyed by the (diagrams), as well as the methodological tools that should be applied here in order to obtain the data on the quantitative and qualitative nature of their behavior. The majority of time series contains their basic measurements within, which can be expressed through the magnitude of their statistical and fractal parameters (the knowledge of which is the fruit of the scientific methodology to date).

Figure 5. *Behavior of joint–stock companies over time.*



Source: *Own work based on https://www.money.pl/gielda/spolki-gpw*

8.1 Interpretation of Results Obtained from Figure 4 and Table 1, Table 2

The graphs presented above are of very indeterminate character. Such properties are indicated in the statistics and coefficients grouped in Table 1 above. Looking at the values drawn from the calculated statistics of the respective businesses, one may notice two categories of values concerning the nature of the studied objects (joint–stock companies).

These are, the statistical ones, expressed by the basic statistical data (the mean, the median and others), and those associated with fractal parameters (correlation and the Hurst exponent). The former, particularly the median, indicate the symmetry of value distribution of the researched variables over time (in terms of the number of negative and positive stock quotes).

Table 1. *A selection of values for observation from the sample 1-3169 (Average, Median, Standard deviation, Hurst exponent)*

Object (Variable)	Average	Median	Standard deviation	Hurst exponent
PKN_Orlen_SA_PKN	0,0524172	0	2,21802	0,52489
PGNiG_SA	0,0523509	0	1,95456	0,46324
Grupa_Lotos_SA	0,0466235	0	2,2416	0,6103
KGHM_Polska_Miedź_SA	0,088479	0	2,721	0,5881
Polimex_Mostostal_SA_PXM	-0,0119407	0	4,61515	0,55769
Kruszwica_SA_KSW	0,0561534	0	2,32399	0,54798

Note: *Coefficient of linear correlation for observation from the sample 1-3169. Critical value (with a 5% of critical areas on both sides) = 0.0348 for n = 3169.*
Source: *Own work based on https://www.money.pl/gielda/spolki-gpw*

Table 2. *A selection of values for observation from the sample 1–3169 (Time Correlation Coefficient, Coefficient of variation, Skewness, Kurtosis).*

Object (Variable)	Time Correlation Coefficient	Coefficient of variation	Skewness	Kurtosis
PKN_Orlen_SA_PKN	0,0185	42,3148	-0,0314611	1,91141
PGNiG_SA	-0,0005	37,3358	0,12967	1,56269
Grupa_Lotos_SA	0,0305	48,0787	0,224171	3,84436
KGHM_Polska_Miedź_SA	-0,024	30,7531	0,187086	5,64972
Polimex_Mostostal_SA_PXM	-0,0051	386,506	0,107418	13,076
Kruszwica_SA_KSW	-0,0285	41,3865	0,709454	5,72498

Note: *Coefficient of linear correlation for observation from the sample 1–3169. Critical value (with a 5% of critical areas on both sides) = 0.0348 for n = 3169.*
Source: *Own work based on https://www.money.pl/gielda/spolki-gpw*

However, the values of the calculated coefficients of correlation and of the Hurst exponent point to a rather significant variability of these equities over time (see the value of the variability coefficient, standard deviation, as well as the values of skewness and *kurtosis*). Above all, however, we are able to determine that the behavior of the researched objects is not correlated with the parameter of time. The PGNiG corporation exhibits the variability of the *antipersistent* type.

The behavior of LOTOS Group displays the most marked trend, as its Hurst exponent assumes the value of H = 0.61 and is relatively the highest (with KGHM coming close second). It seems rather bizarre that over such a long period a company as dynamic as the ORLEN Group noted the Hurst exponent almost at the level of randomness, that is H = 0.500 (the actual result was in fact H = 0.525).

There are two likely explanations of the fact. Either the company is badly managed (which indeed does find support in evidence, as the management board changed very frequently), or it was influenced by the market and by the behavior of investors.

At this juncture in the article, after all the above–presented deliberations and results of calculations, the most fitting seems to be: what are the factors influencing the behavior over time of the system we have researched. It has been generally stated that the theory of complexity deals with processes in which a great number of seemingly independent factors operate in a coherent manner. Such complexity may constitute a dynamic process or an object.

Therefore, in order to discover the causes behind the behavior of a given object, one has to consider the meaningful factors to represent it with. If there is such an abundance of the latter, which of them prove to be the most significant? The first factor is time. This parameter can be broken down into days, weeks, etc., for the values of the studied objects do posses the dimension of a day, week… Hence, we proceed to perform an analysis more detailed than the one before.

A Polish joint–stock company known as KGHM Polska Miedź S.A. is one of the leading and best known worldwide conglomerates in the field of copper production.

The data recording its stock–market behavior in the period between 2005 and 2018 are presented in Table 1, while its Hurst exponent has the value H = 0.58810. We are going to see, then, how the behavior of the company (i.e., the value of its shares) is affected by days of the week, and which of these turn out to be more and less important within the week.

8.1.1 Principal Components (Role of Weekdays in Variability of KGHM Shares Behavior) (Table 3, Table 4)

Table 3. *Covariance matrix of individual values.*

Factor	Own value	Share (%)	Cumulative share of variance (%)
1	8,6657	0,2343	0,2343
2	8,1984	0,2216	0,4559
3	7,4028	0,2001	0,6560
4	6,8273	0,1846	0,8406
5	5,8967	0,1594	1,0000

Source: *Own work based on https://www.money.pl/gielda/spolki-gpw*

Table 4. *Eigenvalue vectors (component charges).*

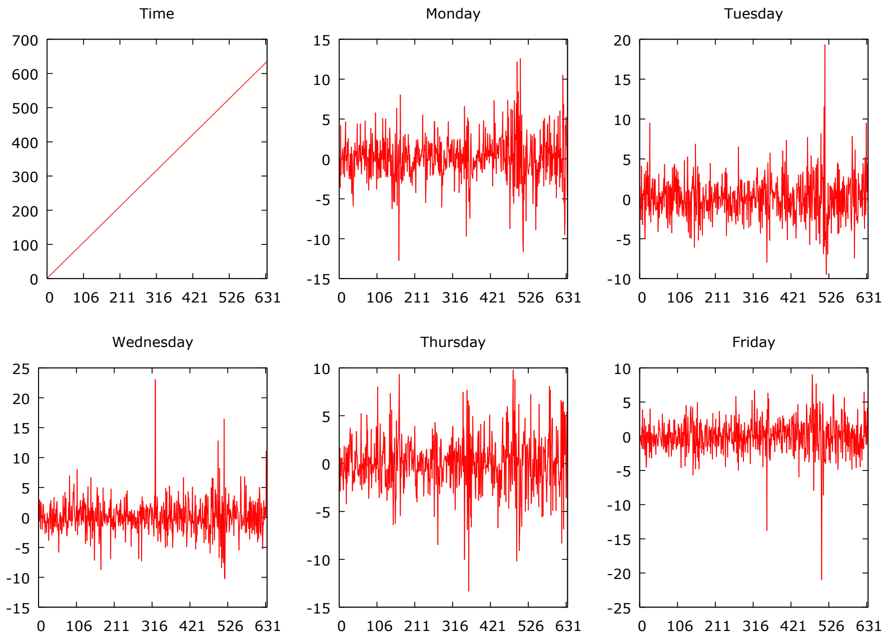
	PC1	**PC2**	**PC3**	**PC4**	**PC5**
Monday	-0,762	0,127	0,444	0,38	-0,247
Tuesday	0,231	-0,051	0,438	-0,853	0,159
Wednesday	-0,128	-0,954	-0,089	-0,001	-0,255
Thursday	0,570	-0,131	0,773	0,237	-0,070
Friday	-0,157	-0,232	0,078	0,268	0,918

Source: *Own work based on https://www.money.pl/gielda/spolki-gpw*

Each day of the week has an important contribution to the weekly behavior, and consequently to the general stock–market behavior of KGHM (Figure 6). Colored fields mark the highest correlation between the variable (the day of the week) and the factor (the principal component).

For the most part, such factors (principal components) do not significantly differ throughout the week, as every day has its own marked contribution to the entire stock–market behavior of a company. This conclusion may seem self–explanatory. One must remember, however, that the intuition is not always congruent with the real–life behavior of a studied process in terms of statistics. The analysis of data confirmed such tentative assumption.

Figure 6. *Graphs for KGHM with a division according to the day of the week.*



Source: *Own work based on https://www.money.pl/gielda/spolki-gpw*

8.1.2 Week Day Variability (of Behavior) for KGHM - Hurst Exponent and Statistical Parameters (Table 5, Table 6)

Looking at the values of the Hurst exponent, one clearly notices that the days of the week constitute here two separate classes of behavior in terms of variability (though standard deviations do not differ much). The values of H at the beginning and at the end of the week are similar. There is an outline of a trend (persistence). However, in the middle of the week investors (through the environment) generally behave in a different manner. A process of this kind is of an antipersistent character (displaying no trend and high probability that the subsequent value of the shares will be opposite to what it previously was).

Table 5. *Descriptive statistics for the observations from the sample 1–635 (Average, Median, Standard deviation, Hurst exponent).*

Object (Variable)	Average	Median	Standard deviation	Hurst exponent
Monday-a	0,117102	0,0900	2,82386	0,64669
Tuesday-b	0,117465	0,0800	2,64874	0,49393
Wednesday-c	0,151827	0,0000	2,83717	0,48428
Thursday-d	0,077953	0,0000	2,79041	0,48489
Friday-e	0,001197	0,0000	2,48294	0,55778

Note: *Critical value of the correlation coefficient (with a 5% of critical areas on both sides)* $r^{*}= 0.0778$ *for* $n = 636$.
Source: *Own work based on https://www.money.pl/gielda/spolki-gpw*

Table 6. *Descriptive statistics for the observations from the sample 1–635 (Time Correlation Coefficient, Coefficient of variation, Skewness, Kurtosis).*

Object (Variable)	Time Correlation Coefficient.	Coefficient of variation	Skewness	Kurtosis
Monday-a	0,0395	24,1144	-0,0895808	2,88767
Tuesday-b	0,0280	22,5492	0,778418	5,18341
Wednesday-c	0,0190	18,6869	1,14843	9,36183
Thursday-d	0,0134	35,7962	-0,139323	1,99700
Friday-e	0,0367	2074,56	-1,08710	9,39384

Note: *Critical value of the correlation coefficient (with a 5% of critical areas on both sides)* $r^{*}= 0.0778$ *for* $n = 636$.
Source: *Own work based on https://www.money.pl/gielda/spolki-gpw*

Still, why do these classes of days differ so greatly? It is exactly the differences of that type that can be sought for in the complexity (nature) of such time processes. In the values of the Hurst exponent for the respective days of the week one may discover and interpret the widely discussed here notion of generativity (Blikle 1971; Chomsky 1959). Below, we will present the generative grammar of days of the week for KGHM Polska Miedź S.A. and a graphic representation of the parse tree of the word x = aebcd.

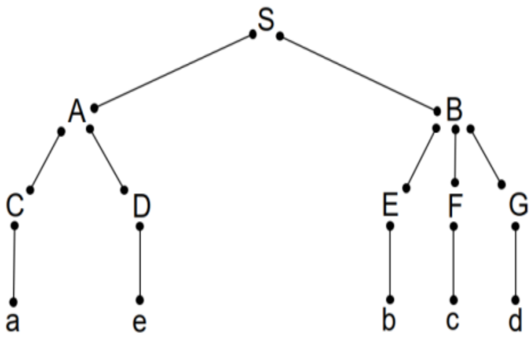
Definition. By generative (formal) grammar there is mean 4–touple $G = \langle V_A, V_T . P, S \rangle$, where V_A and V_T are respectively the auxiliary and terminal alphabets $(V_A \cap V_T = \mathrm{O})$, P is the set of productions in the alphabet $V_A \cup V_T$ and S is the axiom (initial symbol of generation).

Then KGHM days of week can be represented by the following grammar: $V_A = \{A, B, C, D, E, F, G, S\}$, $V_T = \{a, b, c, d, e\}$, S - axiom and set of productions:

$$P: \left| \begin{array}{lll} S & \to & AB \\ A & \to & CD \\ B & \to & EFG \\ C & \to & a \\ D & \to & e \\ E & \to & b \\ F & \to & c \\ G & \to & d \end{array} \right|$$

This grammar can be represented by its tree generation of word x= aebcd presented belove and generated by list of productions P:

Figure 7. *Generative and geometrical representations of KGHM week days as fractal.*



Source: *Own work based on https://www.money.pl/gielda/spolki-gpw*

8.1.3 Generative Grammar of the Von Koch Curve

The grammar presented above concerned in fact a process to a large extent random. However, it also represented a process displaying fractal behaviour. We now want to present an instance of a fractal of the deterministic type, in the form of the von Koch curve, in order to demonstrate that both these generative representations do not introduce a distinction between fractals; that is, they do not separate them into the random and deterministic ones.

We have above posed the issue of a language able to combine the various categories of fractals. The answer seems to be that it is generativity that brings together all the various forms and categories of fractals.

8.1.4 The formal grammar for von Koche's curve

We have done formal grammar

$$G = \langle \Sigma, V_T P, S \rangle,$$

where:

$\Sigma = \{S, A, B\}, \quad V_T = \{a, b\}$, S-axiom (initial symbol)
and set of productions:

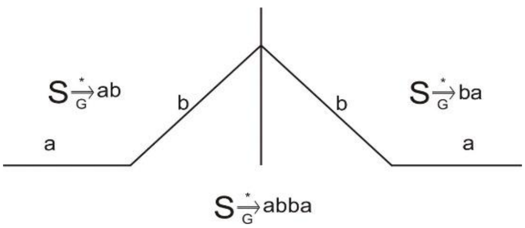
$$P: \left| \begin{array}{llll} (1) & S \to aB & (5) & A \to a \\ (2) & S \to bA & (6) & B \to bS \\ (3) & A \to aS & (7) & B \to aBB \\ (4) & A \to bAA & (8) & B \to b \end{array} \right|$$

This context-free grammar generate language *L(G)* that contains set of all words from V_T^* alphabet about the same numbers of symbols a and b. Typical generations in grammar G are the following

$$\begin{array}{ll} (a) & S \overset{(1)}{\Rightarrow} aB \overset{(6)}{\Rightarrow} abS \overset{(2)}{\Rightarrow} abbA \overset{(5)}{\Rightarrow} abba, \\ (b) & S \overset{(2)}{\Rightarrow} bA \overset{(5)}{\Rightarrow} ba, \\ (c) & S \overset{(1)}{\Rightarrow} aB \overset{(8)}{\Rightarrow} ab. \end{array}$$

Above sentences (a), (b) and (c) are representing the fractal structure of von Koche curve in Figure 8. Axis of symmetry:

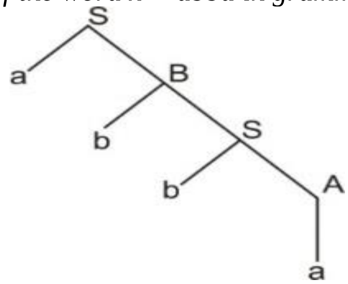
Figure 8. *Generative and geometrical representations of von Koche curve as fractal.*



Source: *Own work.*

This fractal (von Choch's curve) can be representing also by belove generative tree (Figure 9):

Figure 9. *Generative tree of the word x = abba in grammar G.*



Source: *Own work.*

8.2 Further Calculations

The graphs presented in Figure 7 indicate the behavior of various companies over time; hence, they possess the property of compactness (continuity). However, the respective observations are in fact discrete. Therefore, also the graphs expressing the dependence of a discrete variable of the time x on the value of the variable y seem interesting. Let us then present the distribution of points for days of the week in the case of KGHM (Figure 10).

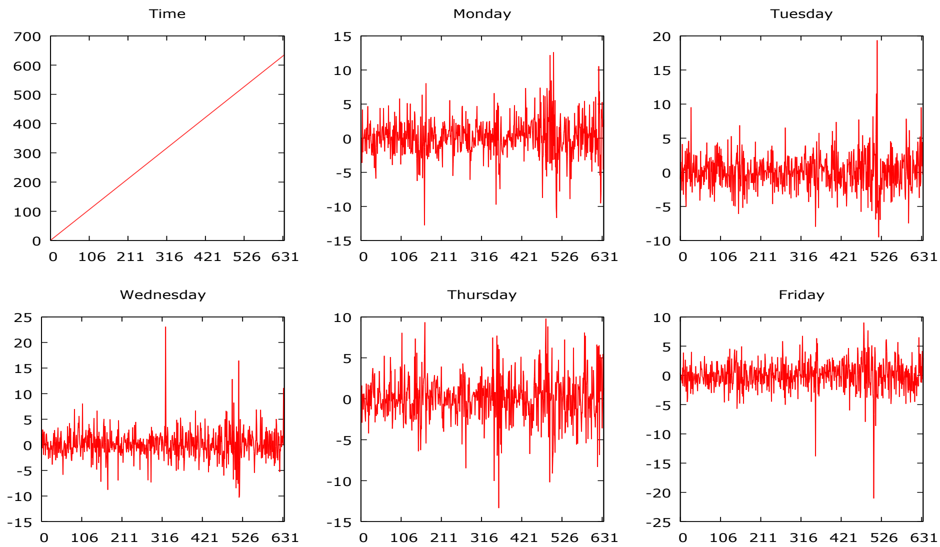
For other companies these graphs look similarly. First of all, as demonstrated in the calculations, there is no significant correlation with time (see the results presented in the tables). This piece of information implies (suggests and indicates) an approach of sorts to analyzing such a complex—due to its diversity—research problem.

The value of a given share at the moment t does not in fact (or to a very little extent) depend on its value assumed in previous states. Nevertheless, each of the analyzed companies has been in the stock market for a long period of time, and hence has to retain some form of its own structure.

That is why our objective is to study their nature by indicating (distinguishing) the components influencing the market value of each company, as the behavior of every dynamic process depends partly on its environment. In our analysis, we treat TIME as the factor that influences the values of shares of the studied entities listed in the stock market. It represents the environment, i.e., the market.

It does involve a far–reaching simplification, but we are researching the nature of companies from the perspective of the relations between their internal factors (as it is these that operate over time). If a system possesses a strong, well–organized structure, able to adjust to the ever–changing conditions of the surroundings, its value (the price) should assume high numerical values (and in a worst–case scenario stable ones — positive mean), whereas for Mostostal the parameter even dipped into negative values.

Figure 10. *KGHM - days of the week*



Source: *Own work based on https://www.money.pl/gielda/spolki-gpw*

8.2.1 Nature of Notions (Concepts) in Coherence Theory (the Theory of Coherents)

Some time ago, the scientific world saw the emergence of a concept of coherence, that is of consistent participation of notions, be they qualitatively different, involved in a given studied process. Initially, it only concerned researches on the behavior of natural systems with many degrees of freedom, subject to numerous sources of influence. It allowed for these instances to be grouped and treated as parameters of order (Callan and Shapiro, 1974).

Such a parameter of order summarizes the external influences, thus diminishing their number (degrees of freedom). With regard to linear space, principal components that we have applied here constitute such a model.

Generally, however, many formal and natural models are of non–linear and statistical character. Such concept (patterned after the coherence of natural processes) was developed by Vaga (1991). His coherent market hypothesis is a non–linear statistical model. The fundamental assumption behind that hypothesis entails the probability distribution in the market changes over time depending on:

- the fundamental (i.e., economic) environment of the market,
- the degree (amount) of collective thinking.

When these combined factors change, so does the market (the environment of the processes). The functions representing given processes change their density, shape, i.e. their existing nature. The presented hypothesis lists four phases (stages) of the market:

1. *Random walk:* Investors act independently of one another, whereas the information is quickly reflected in the prices.
2. *Transition phase:* When the level of "collective thinking" grows, the degree of (psychological) moods pervading the market may cause the impact of information on the state of shares to become long–term. Investors wait for the change in the market situation (environment). There is an analogy to be drawn here to the change in the weather, when we await sunny days to come.
3. *Chaotic phase:* Individual moods of investors are significantly susceptible to the influence of collective (group) thinking, but the fundamental factors are neutral (objective) and uncertain (unknown). Such situation may result in volatility of the market.
4. *Coherent market phase:* As a result of strong positive (or negative) fundamental news (information), associated with strong emotions of investors, the market may gain coherence. On such an occasion, the trend is markedly incremental (or decreasing), and the risk is minor (major).

In our considerations, we referred to the theory of coherence in order to (consistently) frame within a single theory the notions we have used in our paper. These included among others: continuity, discreteness, generativity, fractal dimension as well as many terms from the field of statistics. Many of them we used in our calculations. Some of the terms oppose one other (they display a contrariness of sort).

However, the coherence theory cited enables one to treat them all synoptically, as they manifest to be advantageous some over others when the environment passes through a given phase over a certain period. Moreover, some sectors of the economy may experience different phases at the same time.

Then, in data analysis we may find variance in indicators and parameters for stock–market companies. This is what encapsulates the complexity and coherence of processes through the nature of their non–linearity and multidimensional statistics (Aczel, 2010). While all processes are situated in a certain topological volume (of space).

9. Summary and Conclusions

The problems discussed in the paper concern largely the analysis of fractal theory, it being one of the more current and crucial scientific paradigms. In nature, the processes— including fractals— are situated in space and time. Concurrently, time and space constitute the carriers of their structure and behavior.

Additionally, it is assumed that the internal structure of processes is chaotic, but their individual elements generally tend not to be so.

Therefore, of key importance is the question, what is the perspective to analyze this apparent chaos from. Who is the observer and what is her or his position? The choice of methods to discriminate the processes and to develop them also proves problematic. The article poses all the above questions and attempts to answer them.

The novelty in looking at behavior of processes, and of single objects, lies in the combination of the two categories of nature's variability: differentiability (smoothness) of objects and generativity, a category of a discrete rather than continuous character. Other notions significant for and discussed in the text are somehow related to these two categories. Thus, we have assumed the philosophy of reductionism.

According to that theory, there are primary notions and then there are those that are associated with the former and dependent upon them; hence, though important, they are of secondary importance and may often be generated from the primary ones by way of logical analysis.

Two categories of values concerning the nature of the objects under study (stock companies) can be noted. These are statistical values, expressed by basic statistics (mean, median, principal components, correlation and others) and values related to fractal parameters (Hurst exponent, randomness, persistence, antipersistence). The former, especially the median, indicate the degree of symmetry of the distribution of the values of the studied variables over time (in terms of the number of negative and positive quotations).

On the other hand, values of calculated correlation coefficients together with principal components and Hurst exponent indicate the level of variability of those values in time (cf. the value of variability coefficient, standard deviation and values of skewness and kurtosis and the category of this variability) and the category forming the group of variables - their new dimension.

Above all, however, we are also able to conclude that the behavior of the studied objects is not correlated with the time parameter. It turned out that all companies have a stable structure of their functioning in the stock market over time - ergodic probabilities.

In the course of the research we tried to find an answer to the question about the mutual influence of determinism and randomness. Various aspects of this phenomenon have been analyzed, which leads to the conclusion that, despite the effort made, it is difficult to find an unambiguous solution, because the relationship lies in the organization of nature, its structure, but locally it can be studied on the basis of empirical data. The only certainty is that these two ontological variations occur side by side.

After all, when a globally given system is disrupted by randomness we can only assess its probability of occurrence and this on the basis of scientific analysis of methodologically correct empirical data collected. And as reality shows, the two categories do not destroy each other, and perhaps must exist for the development of dynamic systems.

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