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Dynamic Time Warping in Financial Data – Modification of Algorithm in Context of Stock Market Similarity Analysis

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Abstract:

Purpose: *The purpose of this paper is to use Dynamic Time Warping algorithm along with two statistical tests, Fourier transform and Random permutation, to analyze the similarity of company charts based on stock market data.*
Design/methodology/approach: *The analyzed data from the Warsaw Stock Exchange will detect the similarity between different companies at different and the same time, and between different times for the same company. The research has been carried out based on data from the Warsaw Stock Exchange for selected companies on which DTW analysis has been performed, using AAiFT (Fourier transformations) and RP (Random permutations).*
Findings: *Obtained results indicate that it is possible to detect high similarity between charts by DTW and verify this similarity using statistical tests. Boundary conditions were indicated for the analyses performed and various analysis cases were investigated.*
Practical Implications: *Conducted research reveals the relevance of the use of DTW in a financial context and the possibility of further development as a tool to support investing.*
Originality/Value: *Previous analyses using the DTW method have not included the use of statistical tests in the analysis of the resulting similarity. The use of such a solution is of particular importance in stock market data, allowing the analysis and detection of similarity, along with the determination of the probability of its occurrence.*

Keywords: *Dynamic Time Warping, Warsaw Stock Exchange, Stock Similarity, Random Permutation, Fourier Transformation.*

JEL classification: *D12, D47, D53.*

Paper type: *A research study.*

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1. Introduction

Similarity measures are an important part of the analysis of stock market indicators. The ability to correctly read information, from data contained in charts or tables, is a key characteristic of a successful investor. In order to be able to assume that the analysis of data has a chance to lead to conclusions that may arise in the future, it should be pointed out that financial stock exchange indicators assume the existence of certain repetitive patterns. The assumption of repeatability and interrelationships between the cause and effect of the market is often an unspeakable, but also an indispensable basis for conducting analyses relating to the study of the similarity of certain patterns or the prediction of patterns based on the data available. The ability to effectively determine, even in general terms, the development of share prices is a knowledge that gives investors the opportunity to invest effectively and, consequently, profitably.

The first step towards an assumption about the possibility of predicating certain patterns is an assumption about the similarity of certain actions. The possibility of determining the similarity is therefore a source of valuable information about whether a given pattern happened in the past and what consequences it had. The purpose of this article is to apply the Dynamic Time Warping method to the analysis of the data of the Warsaw Stock Exchange in order to examine the effectiveness of this algorithm on the data held and to compare the results obtained with the classically applied methods.

Such a solution has a chance to allow in the future for free comparison of two companies in different time intervals in order to detect and analyse patterns that have already occurred in the past, as well as the subsequent consequences that have occurred for a given company. This algorithm is able to determine not only whether the two company benchmarks are similar, but also the degree of similarity. This measure is likely to be an important factor in making subsequent investment decisions. In this article we present the classic application of DTW, a second method to check compliance by producing a similar series of analyses using random permutations or Fourier transformation.

2. Literature Review

Dynamic Time Warping (DTW) is an algorithm of matching two time series that can be shifted in time. It is widely used to analyse data in time series. Data that can only be processed after taking the time aspect into account are an important issue in many scientific fields. It is used to process ECG signals, meteorological or financial data

(Tsinaslanidis, 2018). DTW was first introduced in 1994 by Berndt and Clifford. Since then, DTW and its variations have been an important aspect of the exploration of time series, allowing the analysis of sequences of different lengths to find the optimal alignment that minimises the distances between the sequences (Thongmee *et al*., 2014). Calculating correlation of series after DTW results in very high values, thus it raises the common concern about the usability of this method (Kim *et al*., 2001). To answer the question on how to interpret the values of correlation after DTW the method of statistical proof was proposed. DTW allows to calculate the degree of similarity between two series of data, even if they do not have the same length (Puspita *et al.,* 2020). The appropriate application of statistical significance tests for the resulting cross-correlations reduces the risk of overmatching the two data series and finding similarity where it does not exist.

The purpose of this Article is to use the DTW algorithm to find similarities between two series of data. A classical calculation of the correlation measure will be presented, and it will be analysed by a statistical test on randomly generated copies of the series. This approach will then be used to analyse financial data from the Polish stock exchange (Yi *et al*., 1998). It will allow searching for similarities between two companies in terms of the same or different time frames and time windows, as well as for one company in terms of earlier quotations (Agarwal *et al*., 2015). This solution will be compared with classical similarity testing methods, which are widely used to determine certain patterns that may occur within a listing.

2.1 Contribution

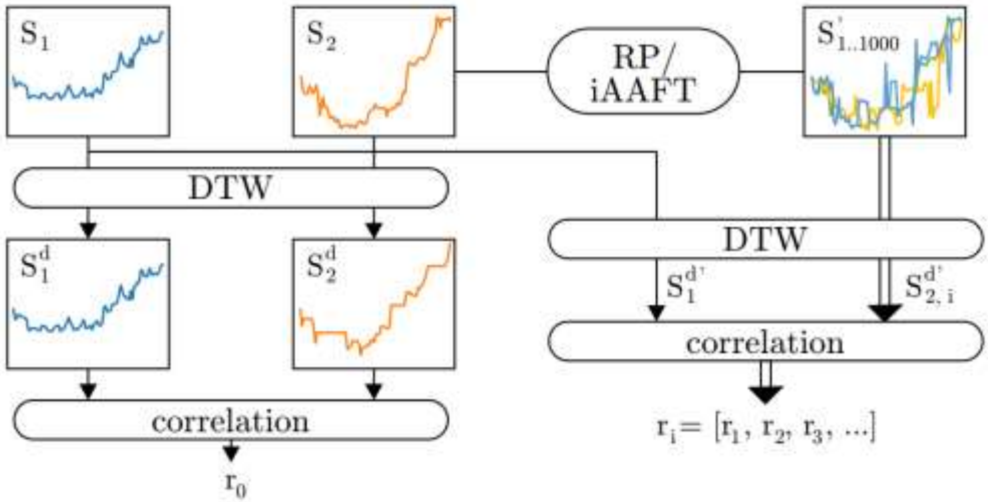
The choice of the DTW algorithm was dictated by its wide application in other and related fields and the possibility of its extension with subsequent modifications, which allows for a wide application of this tool to financial data. However, it should be remembered that DTW is also widely used in other areas, such as the analysis of data from sensors used for early detection of Alzheimer's (Varatharajan *et al*., 2018), recognition of mixed actions of equipment in specific cycle measurements (Kim *et al*., 2018), recognition of gestures based on the analysis of motion trajectory in the image (Tang *et al*., 2018) or functional analysis of fMRI communication at rest (Meszlényi *et al*., 2017).

A wide range of applications makes it clear that there are currently many modifications and improvements to DTW that can provide better parameters for specific data. Network DTWs are being developed (Cai *et al*., 2019), calculation time acceleration models (Silva and Batista, 2016) and solutions linking DTWs to neural recurring networks are being used (Yu *et al*., 2019).

3. Materials and Methods

The need for the research conduction is to develop a simple and fast method of finding similar stock. To find the similarity between two time series (S_1, S_2), one can calculate the cross-correlation $r(S_1, S_2)$. However, in stock prices, two companies can share the same trend, but one can be delayed. To omit the delay effect, the DTW can be performed. Unfortunately, research shows (Pandria and Kugiumtzis, 2014) that calculating correlation after DTW can be unreliable. The nature of DTW makes the outcome series artificially significantly more similar, with $r(S_1^d, S_2^d) > r(S_1, S_2)$. Authors suggested the set of statistical tests to additionally check the result of correlation after DTW. The concept of statistical test is presented in Figure 1 and described in section 3.4.

Figure 1. *Flowchart of the methodology*



Source: *Own elaboration based on research.*

The aim of the statistical test is to perform the significance test after randomization and resampling, stating the hypothesis

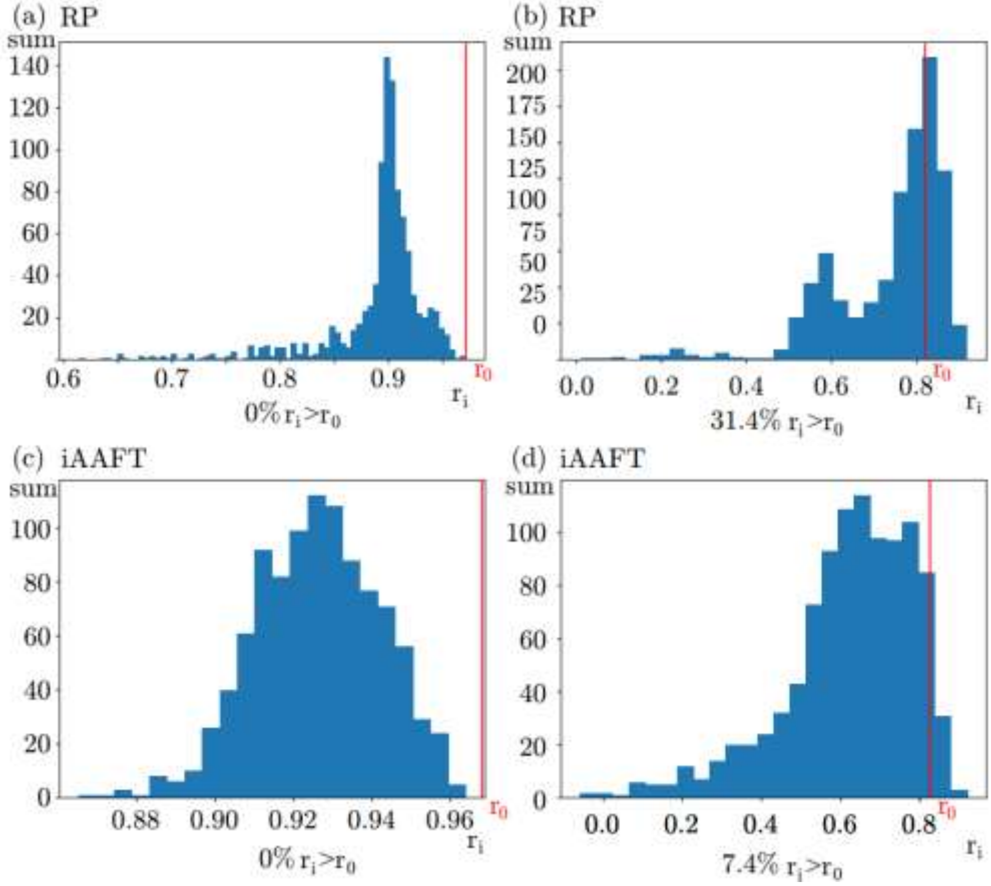
$$H_0 : \rho(S_1^d, S_2^d) = 0 \tag{1}$$

that the pair of series are not similar. Thus to prove the similarity, the hypothesis should be rejected. To test the hypothesis, there is a need of generation of randomized copies $(S_1^{d'}, S_2^{d'})$ of (S_1^d, S_2^d) that are consistent to H_0. In this approach, we generated 1000 copies of input series with use of 2 algorithms:

- RP - Random Permutation, that randomly shuffles the samples preserving same marginal distribution as original series.
- iAAFT - Iterated Amplitude Adjusted Fourier Transform, that preserves the original power spectrum. The surrogates of original series are created with use of pyunicorn library.

The aim of the test is to compare the original cross-correlation $r_0(S_1^d, S_2^d)$ with the one calculated from artificially generated series $r_i(S_1^d, S_2^d), i = 1, 2, ..., 1000$. Hypothesis H_0 is rejected if r_0 lies at the tails of the rand ordering sample distribution. Examples of statistical test results of series from Figure 4 are presented in Figure 2. Early research shows that both algorithms gives similar results with significant differences in less than 5% cases. Those differences has no impact on the thesis tested in this research.

Figure 2. *Histograms of correlation after DTW of artificially generated samples r_i (blue) created by RP (a-b) and iAAFT (c-d) with comparison to original correlation r_0 (red)*



Source: *Own elaboration based on research.*

3.1 Data and Sampling

The comparison is performed between two same length time series. In order to normalize the data, the values of the stock in time are divided by the first value, making the series the plot of stock return value. Depending on the user preferences the length of the series and sampling is variant.

In the performed test, the sampling is set to read the stock price every 1 hour. Each day, there are 9 hours of stock market operation. The set of experiments were performed for 1 week (5 work days, 5 × 9 = 45 samples), 2 weeks (10 work days, 10 × 9 = 90 samples), 1 month (22 work days in tested month = 198) and a quarter (549 samples). The exemplary two stock return plots are presented in Figure 3.

3.2 DTW

The Dynamic Time Wraping (DTW) is an algorithm frequently used to measure similarity between two time series, which may vary in speed. This algorithm is well known and frequently used in many applications. In stock market the DTW is used to find similar trends neglecting small delays.

It is important to introduce the constrain. We used the Sakoe-Chiba constrain in form of a distance *d* that is the distance to diagonal of the matrix. In other words, in 1 hour sampling, the Sakoe-Chiba constrain $d = 15$ means that the pattern from S_1 can be connected with same pattern from S_2 that occurred up to 15 hours earlier or later.

The output of the DTW is a path, that is a series of pairs. Figure 4 presents the example of DTW of a 2 week series presented in Figure 3.

$$(S_1^d, S_2^d) = (S_{1,*}, S_{2,*}), (S_{1,*}, S_{2,*}), (S_{1,*}, S_{2,*}), \dots \quad (2)$$

where $*$ is the element of series S_1 or S_2 that ensures the smallest distance between series.

3.3 Cross Correlation

The cross-correlation is the measure of similarity. Having two input series S_1 and S_2 of the same length, the cross-correlation is calculated as

$$r(S_1, S_2) = \frac{(S_1 - \overline{S_1}) \cdot (S_2 - \overline{S_2})}{||(S_1 - \overline{S_1})||_2 ||(S_2 - \overline{S_2})||_2}. \quad (3)$$

The same formula is used to calculate the similarity measure of two series before DTW, after DTW and in statistical tests with use of artificially generated series.

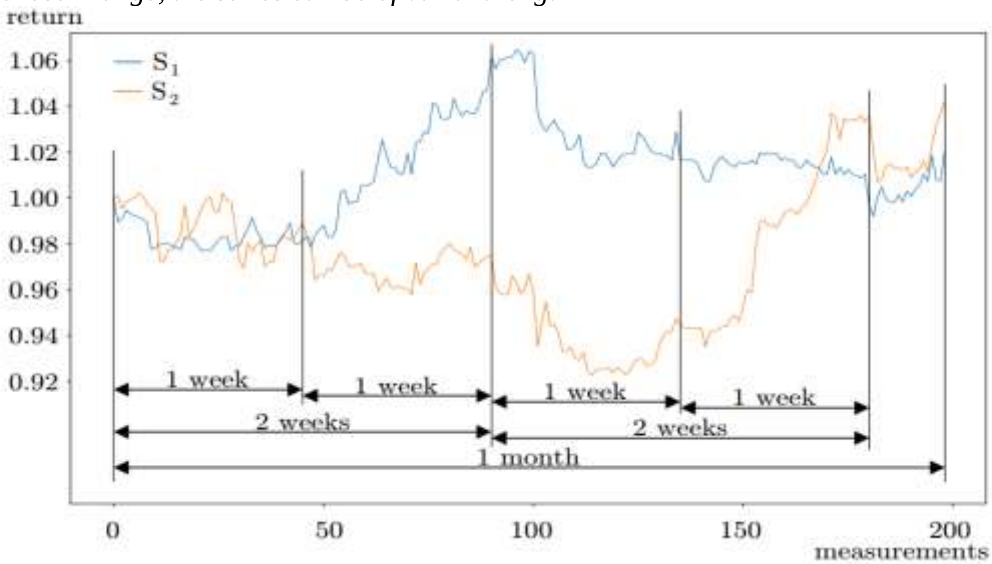
3.4 Tests

According to stated theorem, the aim of the research is to find a threshold value of the cross-correlation after DTW that ensures the H_0 hypothesis rejection. It is expected that the value can be different depending on the length of series and the Sakoe-Chiba constraint. The testing procedure is performed according to Figure 3 and the following algorithm:

1. Load all stock prices from a database from a desired date range and sampling. In the test, the 1 week, 2 weeks, 1 month and a quarter date range are used with 1 hour sampling.
2. For each pair from loaded data, calculate the cross-correlation similarity measure value $r_0(S_1^d, S_2^d)$.
3. Create 1000 artificial series similar to the selected pair for both methods of generating $S_2^{d'}$.
4. Test H_0. The result is a similarity measure value of the series pair $r_0(S_1^d, S_2^d)$ and the percentage of $r_i > r_0$. The hypothesis is rejected when $P(r_i > r_0) > 25$ that is > 2.5% of all artificial series.
5. Plot the scatter plot and histogram of test result. Find the cross-correlation threshold value θ that ensures that all the tests above this correlation rejects the H_0.
6. Repeat all steps for different series length and DTW constraint.

The model thus presented allowed for a series of tests depending on the length of the data and the statistical test chosen.

Figure 3. *The example of return values of two different stocks. Depending on the chosen range, the series can be of variant length*

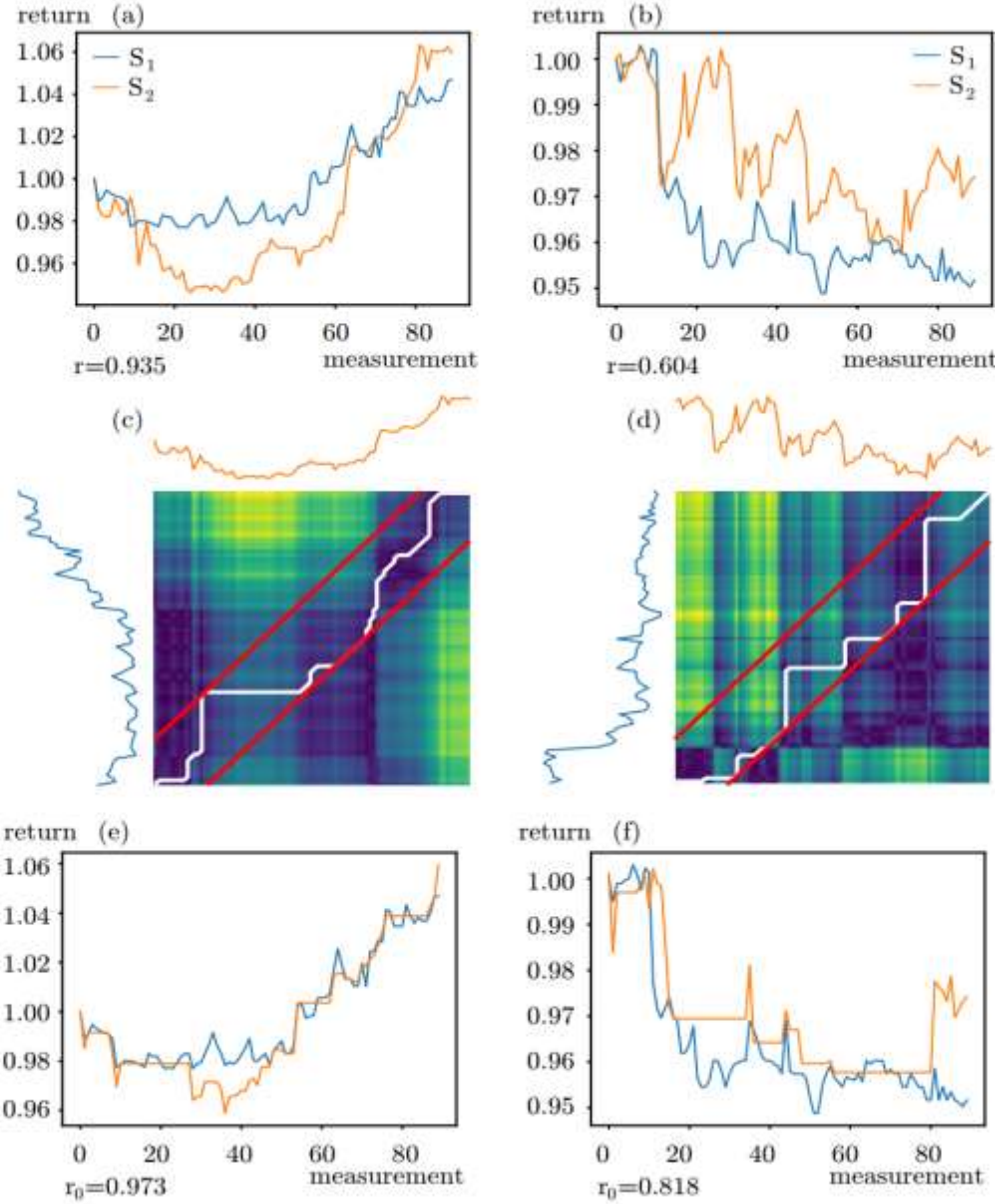


Source: *Own elaboration based on research.*

4. Results

The aim of the test is to find the cross-correlation threshold value θ that ensures that all the tests above this correlation rejects the H_0, meaning that the tested pair is in fact similar. This demand comes from a practical application, where user would like to obtain a list of similar stock values. Depending on the user preferences, there are some parameters that can be tuned, and those parameters influences the time of calculation and the results.

Figure 4. *Example of DTW algorithm used to wrap two series. (a-b) Input series are subject to DTW algorithm. (c-d) The matrix presents the distance between each point of two series and the lowest cost path (white) that connects two diagonals, that does not exceed the Sakoe-Chiba constrain (marked in red). (e-f) The S_2 series wrapped to match S_1 according to lowest cost path.*



Source: *Own elaboration based on research.*

4.1 Parameters

In accordance with sections 3.1 and 3.2, there are two main parameters: Number of samples and DTW Sakoe-Chiba constrain. It is also important to notice that applying tests with long series (high number of samples) with low constrain parameter makes no sense, because no sufficient number of similar pairs are found.

4.1.1 Samples

Data length and sampling. Depending on the user preferences, one can be interested in long term daily returns with one day sampling, other can be interested in short term intraday trading with frequent sampling. However, from an algorithm point of view, all of those parameters in fact influences only one last variable, that is the number of samples. According to section 3.1, the tests were performed for series consisting of 45, 90, 198 and 549 samples.

4.1.2 DTW Sakoe-Chiba constrain

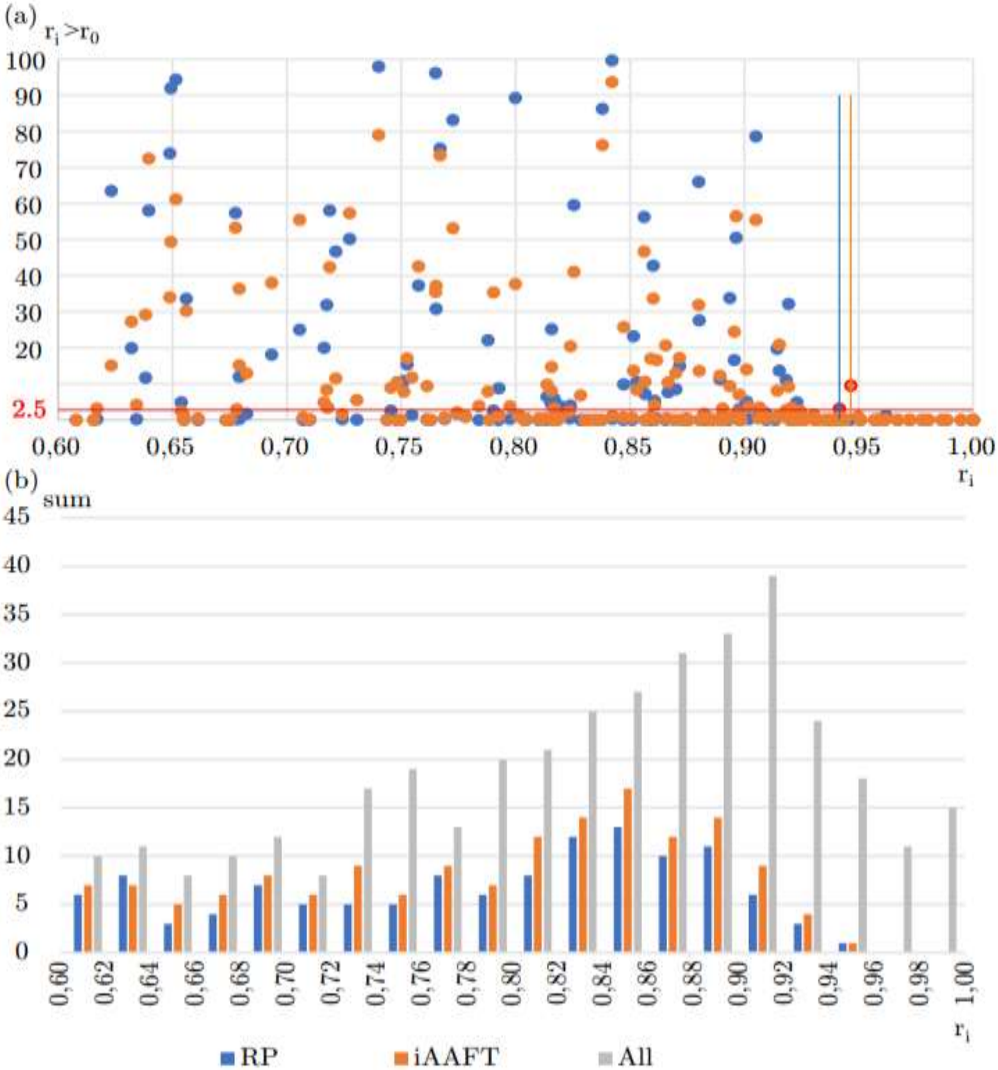
The second parameter, less visible for the user is the Sakoe-Chiba constrain in DTW algorithm. This parameter is tuned depending on the length of the series and willingness of pattern matching distance. According to section 3.2, the tests were performed for constrain 5, 15 and 30.

4.2 Cross-Correlation Threshold Value

According to test procedure described in section 3.4, the result of the test for a specific set of parameters is the list of potentially similar series with their cross-correlation similarity measure value $r_0(S_1^d, S_2^d)$ and the percentage of $r_i > r_0$, that is the basis for H_0 rejection. The hypothesis is rejected when $P(r_i > r_0) > 25$ that is > 2.5% of all artificial series. The aim of the test is to find the cross-correlation threshold value θ that ensures that all the tests above this correlation rejects the H_0. To visualize the data one can draw the test results in form of a histogram and a scatter plot. The results of test for 90 samples and Sakoe-Chiba constrain equal 15 is presented in Figure 5.

In this test, 268 pairs with correlation higher than 0.6 were detected. Each pair is subject to statistical test with comparison to 1000 artificially generated series with use of RP and iAAFT algorithm. The result of the statistical test is presented on the scatter plot as a percentage value of $r_i > r_0$. If the dot lies above the red line, the test passes, proving that the input series S_1,S_2 are not similar and this pair is a part of histogram bar. Reading the histogram, the dominant correlation is 0.9-0.92, however among 39 pairs with such correlation, 9 of iAAFT test and 6 of RP test proven to be not similar. Looking at the trend of H_0 acceptance depending on the correlation, it can be noticed, that the value is dropping to 0. Thus, it can be observed that there is a value above which the H_0 is alvays accepted. From this test, the threshold values θ are equal to 0.94 for RP and 0.95 for iAAFT. This values are presented in Table 1. The test is repeated for other parameters.

***Figure 5** The result of statistical tests. (a) The percentage placement of r_0 in r_i distribution depending on the cross-correlation r_0. (b) Histogram of test results, where gray is the total number of tests in the correlation range and the coloured bars correspond to accepted H_0.*



***Source:** Own elaboration based on research.*

4.3 Test Results

The summary of the testing procedure is the set of cross-correlation threshold value θ that guarantees the H_0 rejection, depending on the testing method and input parameters that are the length of series depending on the number of samples and Shakoe-Chiba constrain. The results are presented in Table 1 and discussed in section 5.

Table 1. *The cross-correlation threshold value θ that guarantees the H_0 rejection*

Samples	Sakoe-Chiba constraint					
	iAAFT			RP		
	5	15	30	5	15	30
45	0.96	0.96	0.96	0.96	0.97	0.97
90		0.95	0.96		0.94	0.96
198		0.93	0.94		0.93	0.94
549			0.92			0.92

Source: *Own elaboration based on research.*

One can read the results of Table 1 as: For a given samples and constrain, two series are proven to be similar if their cross-correlation is greater than calculated threshold $r_0 > \theta$. In that case the time consuming statistical test proving the similarity is not necessary to be performed, because it will be always passed, with H_0 rejection. Smaller values of θ points that the condition is easier to satisfy, thus the value of θ in table is conditionally gradient coloured.

5. Discussion

As it was expected, the value of the threshold of the cross-correlation after DTW that ensures the H_0 hypothesis rejection is dependent on the length of the series and DTW constrain. Analyzing the results shown in Table 1, the naive conclusions are that the number of samples should be kept high and Sakoe-Chiba constrain low.

For Sakoe-Chiba constrain equal to 1, the DTW does not work, because the samples are not wrapped. On the other hand, for low restricted constrain, the samples can be wrapped very far from each other. In case of stock prices, high constrain value can result in comparing stock prices from few days apart, which makes no practical sense. This causes the high cross correlation for plots that are in fact not similar. Those highly correlated plots fails the tests and in result are shifting the threshold value.

For higher samples count, it is more difficult to find the similar pair, thus by the nature of stock prices, the cross-correlation value drops. Moreover, for higher samples count and low Sakoe-Chiba parameter, the number of found pairs were not enough to perform the reliable test.

There is also another crucial conclusion from Table 1. Both algorithms of generating random similar series for statistical tests, that are iAAFT and RP, are providing similar threshold results, thus it is enough to perform only one of those tests.

In conclusion, this test proves that such threshold can be found. We provide a test method to calculate this value during the preparation and development of the application for a set of input parameters and constraints, so that the value can be used to find the set of similar pairs just by fulfilling the cross-correlation threshold parameter.

6. Conclusions

The purpose of this analysis was to indicate the applicability of Dynamic Time Warping based on data from the Warsaw Stock Exchange. The obtained results were analyzed using two statistical tests allowing for an additional measure of significance to the demonstrated similarity between the two stock charts. The purpose of testing for similarity between the two charts was to find companies as similar as possible to each other in terms of stock price changes. Such similarity is one of the first steps to further analysis, which can be used in portfolio construction or in attempts to predict changes in companies.

It should be noted that the similarity search is possible on the chart of the same company at different times, different companies at different times or different companies at the same time. It is indicated that the analyzed data came from the Warsaw Stock Exchange and it would be highly recommended to perform similar analyses for data from other stock exchanges. The use of the Dynamic Time Warping algorithm has the potential to be a new trend in the analysis of stock market trends, which shifts the focus away from fundamental analysis towards advanced algorithms.

Complexity of financial data requires having some economic background and experience in analyzing certain recurring patterns, use of Dynamic Time Warping enables detection of similarities of certain behavior of shares on other companies and can be a valuable tool in the hands of an investor. In further work, it is planned to perform the resulting analyses on cryptocurrency exchanges as well and take steps to optimize and speed up the algorithm.

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