*European Research Studies,*
*Volume XIII, Issue (2), 2010*

# The Distribution of London Metal Exchange Prices: A Test of the Fractal Market Hypothesis

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***Abstract***

*The purpose of the present work is to study the fractal properties of the London Metal Exchange (LME) returns time series. Special emphasis is given to the fundamental issue of detection, identification, and measurement of scaling behaviour of LME returns time series. A fractal approach through ARFIMA models is used to analyze the LME time series. The stable distribution has also been used in order to test the Fractal Market Hypothesis (FMH) in the case of LME market. It is demonstrated that LME returns data possess to some extent fractal properties. The findings are in line with the FMH.*
***Keywords****: ARFIMA model, stable distribution, Fractal Market Hypothesis*

## 1. Introduction

The study of the distribution of stock and commodity price changes has been examined in several empirical studies (Houthakker, 1961), (Cornew, Town and Crowson, 1984), (Blattberg and Gonedes, 1974), (Hall, Brorsen and Irwin, 1989), (Poitras, 1990).

The main conclusion of these empirical results is that the distribution of price changes is not Gaussian or normal but leptokurtic. In addition, it is well known that the crucial assumption of the Capital Market Theory is that returns are normally distributed, which is based on the Efficient Market Hypothesis (EMH). This implies that the future would be unrelated to the past with no possibility of identifying trends or cycles. If the returns are not Gaussian then the Capital Market Theory does not hold.

The concept of fractals has been introduced in financial time series by (Mandelbrot, 1963). Mandelbrot found a similarity between various charts of cotton price changes with different time resolution. He concluded that such scale invariance could help to characterize many complex phenomena seen in physical sciences. Fractal concepts have been applied to a wide range of economic subjects, providing

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a useful tool to investigate the complex behaviour of financial time series. Since the first work by (Mandelbrot, 1963), many efforts have been made in order to examine Mandelbrot's hypothesis according to which returns follow a family of stable Paretian distributions.

The development of fractal mathematics - (Feder, 1989) and (Falconer, 1990) - has shown that highly irregular observations such as stock returns can be quantified in a new way (Mandelbrot, 1982). This irregularity is statistically "self-similar", i.e. it is the same at any scale on which the object is viewed. In fact, scaling is the dominant property of fractals.

Closely related to fractals are stable distributions. Stable distributions are frequently associated with fractional Brownian motion and therefore are related to processes with memory effects.

This study uses the Hurst exponent and ARFIMA models to detect both a possible fractal structure and long run dependence in the returns. However, the Hurst exponent gives no direct information about the underlying distribution of returns. Thus, this study also examines the hypothesis that the distribution of metal price returns is stable. Empirical studies that have rejected the EMH have applied alternative statistical models to account for the rejection of the EMH model. Such a well known model is the Fractal Market Hypothesis (FMH) which is related to the stable distribution.

The investigation of the FMH for metal commodities is of special importance. To our knowledge, there are few studies that have applied the long memory prices in the London Metal Exchange (Panas, 2001b). However, these studies did not examine whether the commodity returns follow Gaussian or stable distribution: an important question from both empirical and theoretical points of view.

The outline of the paper is as follows. In Section 2 a brief review of FMH is presented. Section 3 develops the R/S and ARFIMA statistical models. Section 4 includes a description of stable law and the method of estimation of its parameters. Section 5 summarizes the results.

## 2. Fractal Market Hypothesis

Bachelier's hypothesis (Bachelier, 1900) that stock price changes are normally distributed was left unchallenged until (Mandelbrot, 1963). Bachelier developed and tested, on commodity price data, a mathematical model based on the assumption that prices should have independent additive increments.

The first investigation of the probability distribution of stock price changes can be found in Osborne's study (Osborne, 1959). Assuming that (i) price changes across transactions are identically, independently distributed (iid) random variables with finite variance and (ii) transactions are evenly distributed over time and occur in a large number over a period of days, weeks or months, it follows that daily, weekly and monthly price changes will be the sum of iid random variables, and according to the central limit theorem, they must converge to a normal distribution.

Since the stock market has a large number of investors, there is an underlying assumption that today's change in stock price is caused only by today's unexpected new information. In other words, this implies that there are no "memory" effects: today's returns are independent of the behaviour of returns yesterday. According to earlier capital market efficiency theories (Fama, 1970), this means that stock price returns follow normal distribution and the information arrives at an investor's linearity.

Stock returns are normally distributed and follow a random walk model. The so called random walk hypothesis of Osborne has evolved into Fama's efficient market hypothesis (EMH) (Fama, 1970). The random walk ensures that past events have no effect and that the best guess of future stock prices is the current stock price plus a random variable.

Samuelson (Samuelson, 1965) developed the EMH to rationalize the random walk behaviour. He argued that the current stock price $p_t$ fully reflects all relevant information. However, in almost all cases the stock returns show a higher peak around the mean and fatter tails. This indicates that returns are not normally distributed.

Mandelbrot, using the assumption of independence, concluded that a stable distribution exists, where as k increases the rescaled log-k returns, $k^{-1/\alpha}(\log p_{t+k} - \log p_t)$ would tend towards a stable random variable of characteristic exponent α. For cotton prices the estimated α equals 1.7, which corresponds to a Hurst exponent $H = \frac{1}{\alpha} = .59$. This scaling behaviour deviates considerably from the expected EMH hypothesis of the Gaussian distribution where $H = \frac{1}{2}$. Mandelbrot (Mandelbrot, 1963) showed that the independence of price changes and the theoretically desirable property of stability of the distributions of price returns could be reconciled with the leptokurtosis (fat tails) found in the empirical distributions.

Non-linearity of stock returns has been investigated extensively in the literature, the most recent studies being those of (Akgiray and Booth, 1988), (Jansen and de Vries, 1991), (Buckle, 1995), (Mantegna and Stanley, 1995), (McCulloch, 1997), (Panas, 2001a) and (Kanellopoulou, Panas, 2008). Since the stock returns showing non-normality are independent and show characteristics of nonlinearity, this is evidence of market inefficiency. This reflects the EMH hypothesis, which is not justified by real data.

In place of the EMH, the Fractal Market Hypothesis (FMH) is a new model proposed by Peters (Peters, 1990), (Peters, 1994). The Fractal Market Hypothesis emphasizes the impact of information and investment horizons on the behaviour of investors. In other words, rather than emphasizing market efficiency, the FMH focuses on liquidity as the cornerstone holding markets together (Peters, 1994).

Peters (Peters, 1994) proposed the following assumptions for the FMH:

the market consists of many individuals with a large number of different investment horizons;

information has a different impact on different investment horizons;

the stability of the market is largely a matter of liquidity (balance of supply and demand). Liquidity is present when the market is composed of many investors with many different investment horizons;

prices reflect a combination of short-term technical trading and long-term fundamental valuation;

if a security has no tie to the economic cycle, then there will be no long term trend. Trading, liquidity and short-term information will dominate.

Furthermore, Weron and Weron (Weron and Weron, 2000) have developed a new model to justify asset returns, adopting the Fractal Market Hypothesis, while Blackledge (Blackledge, 2008) has shown that market volatility can be predicted by a signal directly related to the fractal dimension and has used as an example the FTSE close-of-day data for the time period 1980-2007.

A model which has been employed by researchers to examine the FMH is the family of stable distributions. The stable distributions are fractals with a self-similar (power-law scale invariant) behaviour with respect to time (Walter, 1990). They have thick tails, and hence increase the likelihood of the occurrence of large shocks (crashes, booms, discontinuous price jumps in the stock market). The efficient market hypothesis (EMH) has $\alpha=2$, while the fractal market hypothesis (FMH) has $\alpha$ in the range $1<\alpha<2$.

## 3. Time series characteristics of metal prices

The London Metal Exchange (LME) market is the largest active metal market in the world. The metal futures prices are derived from the LME. In particular, the near one–month contracts comprising daily closed metal prices are provided by the LME and cover the period from January 1989 to December 2000 (2987 observations) (the tin series begins in August 1989). Thus, in this study we analyse the daily pricing for non-ferrous metals: aluminium, copper, lead, tin, nickel and zinc.

Descriptive statistics of metal prices

The price series obtained from the LME are used to calculate series of returns for each of the individual industrial metals using the relationship:

$$Z_t = \log\left(\frac{p_t}{p_{t-1}}\right) \qquad t = 1,...,2987$$

The descriptive statistics for these daily returns are shown in Table 1.

**Table 1:**
***Descriptive statistics of metals price returns***

| Statistics | Aluminium | Copper | Lead | Zinc | Nickel | Tin |
|---|---|---|---|---|---|---|
| Skewness | -.07994 | -.00793 | -.37802 | -.82493 | -.13957 | -.46867 |
| Excess Kurtosis | 7.41081 | 9.2245 | 12.167 | 11.308 | 6.8835 | 9.4250 |
| Bera-Jarque | 2424 | 4820.6 | 10527.5 | 8928.1 | 1886.1 | 5027.6 |
| Q(30) | 68.14 | 116.5 | 121.8 | 75.7 | 58.2 | 62.1 |
| QS(30) | 1399.9 | 1118.7 | 1387.8 | 351.4 | 709.8 | 516.1 |

From Table 1 it follows that the return series are all negatively skewed, i.e. the distribution of these series is skewed to the left. The kurtosis coefficients are in all cases significantly leptokurtic, in which case the tails of their distribution taper off to zero more gradually than the tails of a normal distribution of the same variance do. It is interesting to note that the lead and zinc returns appear to be more leptokurtic than the other metal returns. The combination of a significant asymmetry and leptokurtosis indicates that the metal returns are non-normally distributed. In addition, the Bera-Jarque test rejects the null hypothesis of a normal distribution for all price returns. These results imply that returns on non-ferrous metals are not normally distributed, and according to (Fang, Lai and Lai, 1994), "the significant deviations from normality can be a symptom of non-linear dynamics".

## Stationarity

If we have a stationary time series, then their statistical properties do not change over time. Thus, stationarity is an important property. To test the stationarity of a time series $\{Z_t = \log(P_t / P_{t-1})\}$, the conventional Dickey-Fuller (Dickey and Fuller, 1979), (Dickey and Fuller, 1981) and Phillips-Perron (Phillips and Perron, 1988) $\tau$ and z tests are used, the null hypothesis being that the series contain a unit root. Table 2 presents the $\tau$ and z tests for returns in the metal market.

**Table 2:** ***The Augmented Dickey-Fuller (τ), Phillips-Perron (z) and KPSS tests***

| Statistics | Aluminium | Copper | Lead | Zinc | Nickel | Tin |
|---|---|---|---|---|---|---|
| $\tau$ | -23.5 | -24.6 | -25.4 | -24.3 | -23.8 | -24.6 |
| z | -54.5 | -61.7 | -62.9 | -55.1 | -53.9 | -55.6 |
| $n_\mu$ | .106 | .061 | .057 | .038 | .073 | .092 |
| $n_r$ | .168 | .255 | .122 | .047 | .106 | .111 |

Note: The critical values at the 5% level are .463 and .146 for the $n_\mu$ and $n_\tau$ statistics respectively.

Using the augmented Dickey-Fuller and Phillips-Perron (PP) tests, the root hypotheses for all the return series of metals commodities are rejected – see Table 2. Thus, the results strongly reject the presence of a unit root, implying that the first differences $(\log p_t - \log p_{t-1})$ or the returns are stationary.

In contrast to the Dickey-Fuller and Phillips-Perron tests, the Kwiatkowski-Phillips-Schmidt-Shin test (KPSS) (Kwiatkowski, Phillips, Schmidt and Shin, 1992) breaks a time series variable into two components, a stationary component and a random walk component. The KPSS test provides an alternative way of testing the null hypothesis of stationarity against the alternative of a unit root. Table 2 also presents the results of the KPSS tests $\eta_\mu$ (for the null of level stationarity) and $\eta_\tau$ (for the null of trend stationarity).

The KPSS tests show that for aluminium and copper the null hypothesis is rejected, while for the remaining commodities the null hypothesis is accepted. In the case of these two commodities – aluminium and copper – the PP and KPSS tests provide conflicting results. The contradictory inference obtained from the PP and KPSS tests may be evidence that these commodities have low-frequency behaviour, i.e. a fractional differencing process may provide a more appropriate representation. In addition, with regards to the $\eta_\tau$ KPSS statistic, since the aluminium and copper returns reject the null hypothesis of trend stationarity and given that efficient market theory predicts $E(r_{t+1}) = 0$, the presence of a trend in the returns is unlikely.

**Structure of autocorrelations**

The patterns of autocorrelations offer valuable information for the purpose of modeling linear or nonlinear dynamics. Table 1 reports the Ljung-Box Q statistics for up to 30 lags for returns – Q(30) – and squared returns – QS(30). The Q(30) statistic for testing the hypothesis that all autocorrelations up to lag 30 are jointly equal to zero in the LME market is greater than the value of $x^2$ distribution with 30 degrees of freedom at the 5% level, suggesting that the null hypothesis of the independence of returns should be rejected. Thus, linear serial dependencies seem to play a significant role in the dynamics of London Metal Exchange returns. The next question, and the most important one for the study of the behaviour of nonlinear dependencies in metals returns, is: do these returns also exhibit nonlinear serial dependencies? The easiest way to answer this question is by examining the autocorrelation behaviour of squared daily returns. The values of QS(30) - see Table 1 - provide strong evidence of non-linear dependence.

The results up to this point suggest that there is very strong evidence of non-linear structure in daily LME return series. These results are consistent with the hypothesis that these returns are being generated by some sort of non-linear stochastic process, such as long memory process, or a deterministic process such as chaos. Consequently, non-linearity can be analysed on the basis of long memory and chaos theory. In perspective, these approaches could provide new elements useful in the analysis of non-linearity of daily LME returns.

**4. Long memory process**

Einstein originally proved that the distance R covered by a particle undergoing random collisions is proportional to the square root of time T to measure it:

$$R = \kappa \cdot T^{\frac{1}{2}}$$

Hurst (Hurst, 1951) generalized the above equation of Einstein, which is valid for the Brownian motion, in order to investigate the dynamic properties of a time series. In this framework, the generalization proposed by Hurst was:

$$\frac{R}{S} = \kappa \cdot T^{H}$$

where (R/S) is the range of the cumulative deviations from the mean divided by the standard deviation. H denotes the Hurst exponent. The statistic was used to quantify the persistence or antipersistence of feature details. We note that H exponent values range between 0 and 1. If H=.5, the behaviour of the time series is similar to random walk, i.e. the market has a 50% chance of going up (or down) the next day; if H>.5 a persistent trend is characterized by repetitive behaviour. For example, if a high price value of aluminium occurs at time t=κ then at time t=κ+1 one would expect the probability of another high price value of aluminium to be greater by .5. In this case (H>.5) the time series is a black noise process and indicates that the process has persistence, or memory. If H<.5 the behaviour of the time series is antipersistent, that is, if a price value of aluminium occurs at time t=κ then at time t=κ+1 one would be more likely to see a low price value and vice versa. In this case (H<.5) the system is termed a pink noise antipersistent process. Thus, the H statistic summarizes the persistence or antipersistence of a time series. For a time series X(t) embedded in the space (X(t), t) the fractal dimension D, is given by

$$D = 2 - H \qquad (1)$$

For a time series with total observations T and an integer n, n≤T, the R/S statistic is defined as:

$$Q(n) = \frac{R(n)}{S(n)}$$

where R(n) is the range given by

$$R(n) = \max\left(\sum_{j=1}^{n}\left(X_j - \bar{X}\right)\right) - \min\left(\sum_{j=1}^{n}\left(X_j - \bar{X}\right)\right), \quad 1 \le j \le n$$

and S(n) is the sample standard deviation of $X_t$ over the period of n. As n increases, the following holds:

$$\log\left(\frac{R(n)}{S(n)}\right) = \text{constant} + H \cdot \log(n) \qquad (2)$$

where H is the Hurst exponent.

Thus, the Hurst exponent can be obtained by regressing $\log(R(n)/S(n))$ on $\log(n)$ for different values of n.

The main advantage of the above R/S analysis is that the procedure of H estimation is independent of the distribution assumption for a given time series. However, the R/S statistic was reported to have bias when (i) the series contains the short-term memory (ii) the series is characterized with heterogeneities and (iii) the series is non-stationary.

Lo (Lo, 1991) proposed a modified version of the R/S statistic, which is robust even in the presence of a short memory process and heterogeneity. Lo's modified R/S statistic can be defined as follows:

$$Z(n) = \frac{R(n)}{[S(n)]_q} \qquad (3)$$

where S(n) is replaced by $[S(n)]_q$:

$$[S(n)]_q^2 = c_0 + 2\sum_{j=1}^{q}\left(\omega_j(q) \cdot c_j\right)$$

and $c_j$ is the $j^{th}$ order autocovariance of $X_t$ and $\omega_j(q)$ is the Bartlett window weight:

$$\omega_j(q) = 1 - \frac{j}{q+1}, \quad q < n$$

and q is the optimal lag of autocovariances.

The Hurst exponent H is used to measure the intensity of so-called long-range dependence. According to Mandelbrot (Mandelbrot, 1982), long-term correlations and self-similar patterns in time series can be evaluated by techniques based on fractal concepts. The Hurst exponent is also known as a self-similarity parameter since the processes $\{X(\lambda t), t \in T\}$ and $\{\lambda^H \cdot X(t), t \in T\}$ have identical finite-dimensional distributions for all $\lambda > 0$. Thus, the Hurst exponent H has proved a meaningful way of characterizing long memory phenomena or self-similar correlations in physical, biomedical and economic systems.

The economics literature recognized its usefulness in economic time series when Granger (Granger, 1966) first reported the empirical finding that the typical shape of an economic time series followed the pattern of a long memory. Diebold and Rudebusch (Diebold and Rudebusch, 1989) found significant long memory estimates for various measures of US GNP series, while Sowell (Sowell, 1992) found long memory estimates for quarterly postwar GNP series. Baillie, Chung and Tieslau (Baillie, Chung and Tieslau, 1996) and Hassler and Wolters (Hassler and Wolters, 1995) considered the inflation rates of ten and five industrialized countries respectively.

Long memory processes have been found using financial market time series. Greene and Fielitz (Greene and Fielitz, 1977) and Aydogan and Booth (Aydogan and Booth, 1988) reported long memory for stock returns. Ding, Granger and Engle (Ding, Granger and Engle, 1993) considered the power transformation of stock returns and found long memory results. In addition, major applications were made to exchange rates and real interest rates. For real exchange rates under the gold standard, Diebold, Husted and Rush (Diebold, Husted and Rush, 1991) found long memory results, while Shea (Shea, 1991), Backus and Zin (Backus and Zin, 1993) and Crato and Rothman (Crato and Rothman, 1994) found long memory results for various bond yields and real interest rates. Baillie (Baillie, 1996) has presented an empirical review of the long memory findings in the economics literature.

While the existence of a long memory effect was questioned in the economic literature, the metal commodities literature has very little empirical evidence on this issue. When, Richard Baillie made his literature review on long memory processes in 1996, studies of applications on metal prices did not exist. We consider this a serious omission.

Recently, Labys, Lesourd and Badillo (Labys, Lesourd and Badillo, 1998), in their attempt to study metal prices and the business cycles, examined the time series characteristics of metal prices and concluded: "The R/S analysis, the exponent of Hurst and the ARFIMA test results suggest the presence of an anti-persistence phenomenon. This means that some phases of a price increase have a tendency to be followed by some phases of price decrease and that the included series display some type of non-periodic short cycles. Finally, the tests for chaos also reject that independence is caused by non-linearity of a stochastic nature." The small number of observations - a factor of primary importance in this type of tests - these authors used to test the non-linear dynamics, casts doubt on the reliability of the above conclusion. The purpose of the present study is to examine whether metal commodities time series really possess long memory or whether these series can be better modeled by other models.

In this study, equation (2) was estimated for daily LME returns. Table 3 shows the results for Hurst exponent which in the economic literature varies between H=.45 and H=.6 (Mandelbrot, 1963), (Evertsz and Berkner, 1995) and (Müller, Dacorogna, Olsen, Pictet, Schwarz and Morgenegg, 1990). The Hurst exponent for the series is higher than the expected .5 (random walk series). The main result of our analysis is that, for all metal commodities, the Hurst exponent is well above .5. This result is an indication of the existence of long-range dependencies in the fluctuation dynamics.

**Table 3:** ***Hurst exponent (H) and Lo's Z(n) statistics***

| Statistics | Aluminium | Copper | Lead | Zinc | Nickel | Tin |
|---|---|---|---|---|---|---|
| H | .58604 | .59368 | .58294 | .67235 | .58682 | .51420 |
| Z(n) | 2.10* | 1.485 | 1.575 | 1.556 | 1.390 | 1.323 |

Notes: (i) The null hypothesis is that $H_0$: $Z_t$ carries no long memory.
The alternative hypothesis is $H_1$: $Z_t$ as long memory.
(ii) Critical values – see Lo (1991): 10%:1.62 and 5%:1.747. Significant at the 5% level.

In the case of zinc, we have H= .67235; if the market was up, it would have a 67% chance of staying up the next day. Our results are consistent with estimates of the Hurst exponent for the various stocks and exchange rates reported. Since metal commodities - see Table 3 - have H values greater that .5 they are considered as fractal. An additional observation is that zinc has a higher value of H than other commodities. A high Hurst exponent value shows less noise, more persistence and clearer trends, than lower values do. In other words, the higher the value of H is, the lower the corresponding risk is.

However, the above results for the Hurst exponent remain to be clarified further since the R/S analysis is sensitive to heterogeneities. For this case we used Lo's Z(n) statistics. Table 3 reports the estimates of Z(n) statistics. As suggested by Z(n) estimates, aluminium, copper and lead contain long memory structure while zinc, nickel and tin are not long memory processes.

An additional check to determine whether the series $\{r_t\}$ follows a long memory process or not is based on the estimation of an autoregressive fractionally integrated moving average model of order (p, d, q), denoted by ARFIMA (p, d, q). A long memory process is identified if $\{r_t\}$ satisfies the equation:

$$Q(L)(1-L)^d \cdot r_t = \Theta(L) \cdot u_t, \quad u_t = iid(0, \sigma_u{}^2)$$

where d is allowed to be any real number, and $(1-L)^d$ is the fractional differencing operator defined by:

$$(1-L)^d = \sum_{k=0}^{\infty} \frac{r(k-d)r^k}{r(k+1)r(-d)}$$

Both polynomials, $Q(L)$ and $\Theta(L)$, are assumed to have no common roots, and these roots of the AR polynomial $Q(z)$ and of the MA polynomial $\Theta(z), z \in C$ are assumed to lie outside the unit circle. For $-.5 < d < .5$ the process is stationary and invertible, while for $|d| > .5$ the variance of $r_t$ is infinite and $r_t$ is non-stationary. The ARFIMA (p,d,q) process exhibits long memory for d>0 while d=0 corresponds to the presence of short memory. The value of d may be estimated using several techniques, such as semiparametric estimation (Geweke and Porter-Hudak, 1983), (Robinson, 1995), approximate maximum likelihood estimation in the frequency domain (Fox and Taqqu, 1986), exact maximum likelihood estimation in the time domain (Sowell, 1992), Bayesian techniques (Koop, Ley, Osiewalski and Steel, 1997) and bootstrapping (Andersson and Gredenhoff, 1998). Here, the d value for each of the commodities is estimated using Sowell's maximum likelihood approach. Table 4 reports the exact maximum likelihood estimates for d for various ARFIMA models.

**Table 4:** ***Maximum Likelihood Estimates of Long Memory Parameter***

| Metal Commodities | (0, d, 0) | (0, d, 1) | (1, d, 1) |
|---|---|---|---|
| Aluminium | .05881<br>(1.69) | .08984<br>(1.80) | .09595<br>(1.88) |
| Copper | .0692<br>(1.85) | .04764<br>(2.5) | .05243<br>(2.3) |
| Lead | .02820<br>(1.5) | .03406<br>(1.38) | .2930<br>(.66) |
| Zinc | -.02315<br>(1.6) | -.04568<br>(1.1) | -.08468<br>(1.3) |
| Nickel | .01621<br>(.92) | .03799<br>(1.1) | .07825<br>(1.03) |
| Tin | -.03886<br>(1.71) | -.04001<br>(1.2) | -.03841<br>(.67) |

Note: Asymptotic absolute t-values displayed in parentheses.

Schmidt and Tschering (Schmidt and Tschering, 1993) discuss the identification of ARFIMA models using information criteria. Using the Akaike (AIC) and Swartz Bayesian information (BIC) criteria for all markets, an ARFIMA (1,d,1) is chosen. The estimates of d from the ARFIMA (1,d,1) do not equal zero for aluminium and copper. The estimates of the ARFIMA models provide evidence for long memory in the aluminium and copper returns. The conclusion that the returns of the aluminium and copper have long memory behaviour means that correlations between price changes die out very slowly so that the actual movements in the market are stochastically influenced by the recent to the furthest past.

In the case of lead and nickel returns, the null hypothesis Ho:d=0 was accepted, suggesting that the behaviour of these time series exhibits short memory process. From Table 4 it follows that d is less than zero for zinc and tin. At a first glance, it appears that these returns follow an anti-persistent process in which these values in the returns are negatively correlated over short time scales.

## 5. Estimation of stable distributions for LME

In this section we investigate the use of stable distributions for closed metal prices. The justification for this is the evidence in section 2 (time series characteristics in metal prices) which proves that closed metal prices are almost always heavy tailed. We first make a short introduction to stable distributions and then, we go on to estimate stable parameters for LME data.

The α-stable distribution is a generalization of the Gaussian distribution which satisfies two properties. First, it satisfies the stability property, which states that if X, $X_1$ and $X_2$ are α-stable independent random variables of the same distribution, then there exists a positive number *v* and a real number d such that:

$$vX + d \overset{d}{=} \mu_1 X_1 + \mu_2 X_2 \tag{4}$$

where $\mu_1$ and $\mu_2$ are constants and d denotes equality in distributions. Note that

$$v^\alpha = \mu_1^\alpha + \mu_2^\alpha$$

for some $\alpha \in (0,2]$, which is called the index of stability or characteristic exponent. X is called an α-stable random variable. When d=0, X is called a symmetric α-stable random variable. The implication of (4) is that the tail index is unchanged when independent stable random variables are summed. The economic implication is that if individual stock returns are stably distributed, the stability property of stable distributions implies that portfolio returns are also stably distributed as well. Second, an α-stable distribution satisfies the generalized central limit theorem (Samorodnisky and Taqqu, 1994).

Due to the lack of a closed form formula for densities, the stable distribution can be conveniently described by its characteristic function $\varphi(t)$. The characteristic function $\varphi(t)$ is the Fourier transformation of a probability density function and for stable distributions is given by

$$\varphi(t) = \exp\{j\lambda t - \gamma|t|^\alpha\}[1 + j\beta \operatorname{sign}(t)\omega(t,\alpha)]$$

where

$$\omega(t,\alpha) = \begin{cases} \tan\frac{\alpha\pi}{2}, \text{if } \alpha \neq 1 \\ \frac{2}{\pi}\log|t|, \text{if } \alpha = 1 \end{cases}$$

and

$$\operatorname{sign}(t) = \begin{cases} 1, & \text{if } t > 0 \\ 0, & \text{if } t = 0 \\ -1, & \text{if } t < 0 \end{cases}$$

The parameters λ, α, β and γ describe completely a stable distribution. The characteristic exponent or "degree of fractality" α controls the heaviness of the tails of the stable distribution. The characteristic exponent α, can take values in (0,2]: a smaller value implies heavier tails and α=2 is the Gaussian case. All stable distributions with α<2 possess infinite variance. The index of skew β, controls the symmetry of the stable distributions and takes values in [-1,+1]. When β=0, the distribution is symmetric, β>0 implies that the distribution is skewed to the right and β<0 implies skewness to the left. The dispersion parameter γ, which determines the spread of the density around its location parameter, takes values in $(0,+\infty)$. The location parameter, λ, corresponds to the mean for $1 < \alpha \le 2$ and the median for $0 < \alpha \le 1$. It takes values in $(-\infty, +\infty)$.

The most popular method of parameter estimation is that of (Fama and Roll, 1968) and (Fama and Roll, 1971), which was later extended by (McCulloch, 1986). This is the method used in this study to estimate the parameters and a short description is given below.

The four stable parameters are estimated as follows: The $p^{th}$ quantile of a set of data is defined as the value $x_p$ that satisfies $f(x_p)=p$. Thus for a data set of size N, $\hat{x}_f$ refers to the $p(N+1)^{st}$ order statistic of the set, which we can use as an estimate of $x_p$. The four stable parameters are estimated as follows:

$$\hat{k}_\alpha = \frac{\hat{x}_{.95} - \hat{x}_{.05}}{\hat{x}_{.75} - \hat{x}_{.25}} = \varphi_1(\alpha, \beta)$$

$$\hat{k}_\beta = \frac{\hat{x}_{.95} + \hat{x}_{.05} - 2\hat{x}_{.5}}{\hat{x}_{.95} - \hat{x}_{.05}} = \varphi_2(\alpha, \beta)$$

The $\varphi$ functions can be inverted to yield $y_1$ and $y_2$:

$$\hat{\alpha} = y_1(\hat{\kappa}_\alpha, \hat{\kappa}_\beta)$$

$$\hat{\beta} = y_2(\hat{\kappa}_\alpha, \hat{\kappa}_\beta)$$

McCulloch supplies tables of the values of these functions for determining α and β. The scale parameter can be estimated by:

$$\hat{\gamma} = \frac{\hat{x}_{.75} + \hat{x}_{.25}}{\varphi_3(\hat{\alpha}, \hat{\beta})}$$

Finally, the location parameter λ can be estimated by:

$$\hat{\lambda} = \hat{x}_{.50} + \hat{\gamma}\varphi_5(\hat{\alpha}, \hat{\beta})$$

The McCulloch type estimates of the four stable parameters for daily data are reported in Table 5. Tables 5, 6 and 7 show the estimates of the four parameters of stable distribution for daily, weekly and monthly returns respectively. For daily data the estimates of $\hat{\alpha}$ range from 1.48 to 1.62; the estimates of $\hat{\beta}$ are between -.009 and .12. For weekly data, the estimates of $\hat{\alpha}$ are between 1.4 and 1.51; two of the estimates of $\hat{\beta}$ are negative. For monthly returns the estimates of $\hat{\alpha}$ are between 1.36 and 1.58.

The stability index α of all six metal commodities does not equal 2. The most important feature of the estimates of the index of stability α is that it determines how non-Gaussian a particular density becomes. Our empirical evidence – see Section 2 – tells us that the distribution of metals returns deviate significantly from Gaussian distribution, exhibiting excess kurtosis and fat tails. We can therefore conclude that the metal returns are not normally distributed.

If the underlying distributions of commodities are stable then the estimates of the stability index α and the index of skew β for daily, weekly and monthly data should be statistically indistinguishable invariant of stable distributions i.e. two stable distributions can be added or subtracted from each other without changing the shape of the distributions. This follows from the stability property since the weekly and monthly data are linear combinations of daily series.

From Tables 5, 6 and 7 it follows that the stability index α is not increasing with the sum size. Tables 5, 6 and 7 show that the stability index and skewness do not differ that much in daily, weekly and monthly scales. This is a clear validation of the stability property. This finding means that the stable law hypothesis could be accepted in metal commodities returns of LME and therefore, we could say that the metal returns are characterized by a fractal structure.

**Table 5: Stable Law Parameter Estimates for LME (Daily data)**

| Metal Commodities | $\hat{\lambda}$ | $\hat{\beta}$ | $\hat{\alpha}$ | $\hat{\gamma}$ |
|---|---|---|---|---|
| Aluminium | -.00054 | .021 | 1.62 | .0074 |
| Copper | -.0005 | -.01 | 1.57 | .0087 |
| Lead | -.0006 | .11 | 1.59 | .0093 |
| Nickel | -.00011 | .042 | 1.51 | .010 |
| Tin | .00002 | -.009 | 1.47 | .0056 |
| Zinc | -.0006 | .091 | 1.48 | .0085 |

**Table 6: Stable Law Parameter Estimates for LME (Weekly data)**

| Metal Commodities | $\hat{\lambda}$ | $\hat{\beta}$ | $\hat{\alpha}$ | $\hat{\gamma}$ |
|---|---|---|---|---|
| Aluminium | -.00008 | .038 | 1.45 | .0079 |
| Copper | -.00014 | -.01 | 1.49 | .009 |
| Lead | -.0008 | .14 | 1.51 | .0093 |
| Nickel | -.0003 | .056 | 1.45 | .0096 |
| Tin | -.00002 | -.01 | 1.40 | .006 |
| Zinc | -.0005 | .036 | 1.47 | .0081 |

**Table 7: Stable Law Parameter Estimates for LME (Monthly data)**

| Metal Commodities | $\hat{\lambda}$ | $\hat{\beta}$ | $\hat{\alpha}$ | $\hat{\gamma}$ |
|---|---|---|---|---|
| Aluminium | -.00404 | .099 | 1.36 | .008 |
| Copper | .00004 | -.018 | 1.48 | .009 |
| Lead | -.0007 | .118 | 1.52 | .010 |
| Nickel | -.0007 | .039 | 1.42 | .0095 |
| Tin | .00012 | -.009 | 1.41 | .05 |
| Zinc | -.0004 | .018 | 1.58 | .0100 |

## 6. Conclusions

This study has examined the presence of the long memory property and the Fractal Model Hypothesis in daily returns of metal commodities traded on the London Metal Exchange (LME). The scaling form may reveal important information about the fundamental interactions that take place in a financial time series. We have demonstrated the scaling properties of metal commodities which are similar to those observed in stock returns. Probability density function estimates indicate that the returns of metal commodities are stably distributed; hence, the results are compatible with the Fractal Market Hypothesis (FMH). The presence of fatter tails indicates "memory" effects which arise due to non-linear stochastic processes; as a consequence, the information flow to an investor is irregular rather than smooth.

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