
Grey Method of Selecting a Contractor in the UN Public Procurement

Submitted 29/09/21, 1st revision 16/10/21, 2nd revision 08/11/21, accepted 13/12/21

Arkadiusz Borowiec¹, Marcin Nowak²

Abstract:

Purpose: The article proposes a new method for selecting a contractor in United Nations public procurement. Approach: This method is based on the use of the grey number decision model.

Findings: The presented model, which belongs to the grey systems theory, enables to combine quantitative and qualitative criteria in the assessment of the contractors.

Practical Implications: The method is particularly applicable in case of the presence of subjective features among the evaluation criteria which are difficult to express on a quantitative scale. An example of its use may be the so-called green public procurement, in which, apart from the quantitative criteria, such as price, there are qualitative criteria related, for example, to the impact of the offer on the social and ecological environment. The first section of the article presents the basics issues related to public procurement. In the second section, the essence of the grey systems theory is described. The third part of the article presents the structure and procedure in the grey method of selecting a contractor in public procurement on the example of the United Nations.

Value: Finally, the developed method as presented on the selected case study can be a guide for future cases.

Keywords: Public procurement, grey system theory, grey number decision model, grey decision-making.

JEL Classification: M2, H57.

Paper Type: Research article.

¹ Poznan University of Technology Faculty of Engineering Management, Poznań, Poland;
arkadiusz.borowiec@put.poznan.pl;

² Poznan University of Technology Faculty of Engineering Management, Poznań, Poland;
marcin.nowak@put.poznan.pl;

1. Introduction

1.1 The Essence of the Issue of Public Procurement

The public procurement system operates in almost all countries around the world. Public procurement is related to this sphere of the state's activity which concerns the acquisition of goods and services necessary for the proper performance of its functions. They are based on economic, legal, and social elements. The public procurement market is a very peculiar market since it deals with the purchase monopoly. The factor that shapes demand is not a price mechanism but budget plans. In the market, there also doesn't exist the classic division of economy into three spheres, i.e., production, distribution, and consumption. Its specificity also excludes (or at least should) the area of the grey economy and black market.

The state, as the holder of public funds, is obliged to use them rationally according to economic laws. The guarantee of ensuring this rationality is a system of legal regulations which should give a relative balance between the entities involved in the public procurement process. In each country, this system should foster rational, honest, and effective management of public funds and constitute an important instrument for reviving the economy, and thus, improving the economic existence of the state. The essence of public procurement has not changed since ancient times.

For centuries it has been seen as an extremely effective instrument of the state's influence on the economy. History shows that public procurement has often led the economies of many countries out of the economic crisis and reduced unemployment. With the public procurement system, recipients and administrators of public funds receive ordered and regulated by law mode of conduct, which not only facilitates business trading but also favors the protection of their mutual interests.

1.2 Selecting a Contractor in the UN Public Procurement

The value of the United Nations' public procurement market is estimated to be over USD 17 billion a year - at the same time the market is steadily growing. The market is very diverse and dynamic, covering all countries of the world. Over 30 United Nations system agencies purchase goods and services from 194 countries. The United Nations public procurement market increased by USD 11 billion only from 2003 to 2013. 43.21% of this sum was assigned to ten countries most involved in the provision of goods and services.

One of the key mechanisms affecting the correctness of public procurement in this market is honest and non-discriminatory choice of the contractor offering the best terms of the contract from the social and economic point of view. When selecting a contractor, the contracting authority is obliged to follow the principle of equal treatment of participants and the principle of fair competition, which ensure proper disposal of public funds and give access to public procurement to all entities capable of performing contracts.

The principle of equal treatment ensures that all candidates in public procurement process are treated in the same way, at all stages of the proceedings. This means establishing identical requirements for all potential contractors as well as an obligation to assess compliance with the same requirements and to provide identical information to all participants of the proceedings. To maintain fair competition, the contracting authority is obliged to eliminate the offer which constitutes unfair competition. To decide regarding the selection of the most advantageous offer, it is necessary to comprehensively examine all offers in terms of formal, technical and financial aspects.

The formal assessment used by the United Nations consists in examining, inter alia, whether the offer is signed by a person authorized to represent the contractor; whether the offer contains all the required documents; whether the contractor secured the bid bond in accordance with the requirements set out in the tender documentation; whether the contractor is registered in the UNGM database³. Failure to meet any of the above requirements may result in a request for missing information or a rejection of the offer.

The technical assessment refers to checking whether, or to what extent, the offer corresponds to the object of the contract. The consequence of failure to comply with this requirement is generally the rejection of the offer - this means that the contractor has not offered what the contracting authority intends to buy. The technical criteria may refer not only to the parameters of the ordered goods and services but also to the contractor's properties. Previous experience of contractors gained in a similar field, available technical resources, qualifications, and experience of the staff that the contractor will have at the time of the contract are, inter alia, evaluated. Finally, the financial assessment.

This assessment is based on criteria strictly related to the costs to be borne by the contracting authority due to the contract. Apart from the price, life cycle costs are also taken into account, along with the cost of acquisition, operation, repair, modernization, utilization etc. The selection of appropriate criteria for the evaluation of offers depends on many factors, such as the object of the contract, the procedure, and special circumstances of the proceedings.

2. The Grey System Theory

The beginnings of the Grey System Theory go back to the 80s of the twentieth century (Deng, 1982). In 1982, in the article "Grey Control System" J. Deng presented the concept of the grey system for the first time. In 1989, the same author, in the publication "Introduction to the Grey System Theory", presented the foundations of the contemporary concept of the Grey System Theory (Deng, 1989). The Grey System Theory deals with modeling information uncertainty. In this context, it constitutes an alternative to the Probability Theory, the Fuzzy Set Theory (fuzzy logic), the Rough Set

³The UNGM is used by 26 UN system organizations, including UN/PD, UNOPS, UNICEF, UNESCO, WHO, PAHO, WFP, UNDP, UNIDO, ILO, UNRWA, WMO, ADB. The registration is free of charge.

Theory (Mierziak, Xie, Nowak, 2018). Data uncertainty is a distinctive feature of many systems, especially organizations whose immanent element is people. One of the key aspects of uncertainty is related to the amount of available information about the system. Information uncertainty may result either from an excess of information generated by a given system or a shortage of information (Karr-Wisniewski and Lu, 2010).

In the case of an excess of information, the Big Data concept is intensively developed in supporting management (LaValle, Lesser, Shockley, Hopkins, and Kruschwitz, 2011). The area of applicability of the Grey Systems Theory includes the situation of the lack of information - in this context the Grey System Theory is the opposite of the methods belonging to the Big Data family. Because in the case of management problems, there is information uncertainty regarding the information shortage, the popularity of application of the Grey System Theory grows (Liu, Forrest, and Yang, 2012).

The Grey System Theory finds several applications in economic as well as social and technical sciences (Delcea, 2015). In recent years, methods belonging to the Grey System Theory have been used for example for:

- evaluation of enterprise integrity (Ma and Zhu, 2016),
- product development (Hsiao, Lin, and Ko, 2017),
- selection of suppliers (Rao, Goh, and Zheng, 2017),
- evaluation of the health system financing (Pourmohammadi, Shojaei, Rahimi and Bastani, 2018),
- prediction of the tourist income (Ma, Liu, and Wang, 2019),
- diagnosis of crises in family businesses (Wiecek-Janka, Nowak, and Borowiec, 2019),
- prediction of the performance of franchise industry (Day, Wang, and Dang, 2017),
- airplane boarding strategies (Delcea, Cotfas, and Paun, 2018).

Among the most popular models developed in the Grey Systems Theory are Grey Prediction Model, Grey Decision-Making Model, Grey Relational Space, Grey Generating Space, Grey Control Model (Liu, Yang, Xie, and Forrest, 2016).

The basic concept of the grey systems theory is a grey number. The grey number is usually marked with the symbol $\otimes G$. The grey number is a numerical value which is only known to be in a certain range, but its exact value is not known. This range is marked as follows: $:\otimes \rightarrow [\underline{G}, \overline{G}]$. \underline{G} indicates the lower limit of the range, and \overline{G} is the upper limit of the range. A special case of the grey number is a white number. The grey number is called the white number when the lower and upper limits have the same value. A black number is a number whose lower limit is $-\infty$ and the upper limit is $+\infty$. In formal terms, the grey number can be defined in the following way.

Definition 1. The grey number $\otimes G$ is called the real number d^* , which fulfils the following condition:

$$\{d^* \in [\underline{G}, \overline{G}]\} \wedge \{\underline{G} \neq \overline{G}\} \wedge \{(\underline{G} \vee \overline{G}) \neq \pm\infty\} \quad (1)$$

As the logical consequence of the definition 1, the grey number may be in the following ranges:

$$1) \quad \otimes G \rightarrow [-\infty, \bar{G}], \text{ where } \bar{G} \neq +\infty \quad (2)$$

$$2) \quad \otimes G \rightarrow [-G, +\infty], \text{ where } -G \neq -\infty \quad (3)$$

$$3) \quad \otimes G \rightarrow [-G, \bar{G}], \text{ where } (-G \neq -\infty) \wedge (\bar{G} \neq +\infty) \wedge (-G \neq \bar{G}) \quad (4)$$

If the grey number is described on the interval 3), it is called the Gray Interval Number. The basic arithmetic operations on the Grey Interval Number are as follows (Fang *et al.*, 2017):

$$1) \quad \otimes G_1 + \otimes G_2 = [\underline{G}_1 + \underline{G}_2, \bar{G}_1 + \bar{G}_2] \quad (5)$$

$$2) \quad \otimes G_1 - \otimes G_2 = [\underline{G}_1 - \bar{G}_2, \bar{G}_1 - \underline{G}_2] \quad (6)$$

$$3) \quad \otimes G_1 \times \otimes G_2 = [\min(\underline{G}_1 \underline{G}_2, \underline{G}_1 \bar{G}_2, \bar{G}_1 \underline{G}_2, \bar{G}_1 \bar{G}_2), \max(\underline{G}_1 \underline{G}_2, \underline{G}_1 \bar{G}_2, \bar{G}_1 \underline{G}_2, \bar{G}_1 \bar{G}_2)] \quad (7)$$

$$4) \quad \otimes G_1 \div \otimes G_2 = [\underline{G}_1, \bar{G}_1] \times \left[\frac{1}{\underline{G}_2}, \frac{1}{\bar{G}_2} \right] \quad (8)$$

The presented principles of arithmetic are the basis for the process of ranking of grey numbers. In the process of comparing grey numbers, the degree of grey possibility is used. In accordance with this indicator, the probability that one grey number ($\otimes G_2$) is greater than the second grey number ($\otimes G_1$) is determined using the following formula (Eshtaiwi *et al.*, 2017):

$$P\{\otimes G_1 \leq \otimes G_2\} = \frac{\max[0, L^* - \max(0, \bar{G}_1 - \underline{G}_2)]}{L^*} \quad (9)$$

where $L^* = L(\otimes G_1) + L(\otimes G_2)$

Degree of grey possibility is a theoretical basis for the Grey Number Decision Model.

3. The Structure of the Grey Method of Selection a Contractor in the UN Public Procurement

The structure of operations in the proposed method consists of 8 steps (Li *et al.*, 2007).

Step 1. Determining the weights of the decision-making criteria

The weights of individual decision-making criteria in the proposed method are determined using a panel of experts. It is possible that the scales are determined by one expert as well as many experts. In this respect, the method has no limitations.

Experts assigned to each criterion value expressed in the following scale linguistic: very low importance (VL), low importance (L), medium importance (M), high importance (H) and very high importance (VH). Subsequently, each evaluation was presented in the form of a grey number in accordance with the scheme presented in the Table 1.

Table 1. Scale of the values of decision-making criteria expressed in the form of grey numbers

Linguistic evaluation	Criterion weight $\otimes w$
Very low importance (VL)	[0.0, 0.2]
Low importance (L)	[0.2, 0.4]
Medium importance (M)	[0.4, 0.6]
High importance (H)	[0.6, 0.8]
Very high importance (VH)	[0.8, 1.0]

Source: Own study.

The weights of subsequent decision-making criteria will be expressed using the following formula (10).

$$\otimes w_j = \frac{1}{K} (\otimes w_j^1 + \otimes w_j^2 + \dots + \otimes w_j^K) \tag{10}$$

where $\otimes w_j^K$ is the weight j th of the decision-making criterion indicated by the expert K th.

Step 2. Determining the value of subsequent decision-making attributes

The values of subsequent decision-making attributes are determined using the formula (11).

$$\otimes G_{ij} = \frac{1}{K} (\otimes G_{ij}^1 + \otimes G_{ij}^2 + \dots + \otimes G_{ij}^K) \tag{11}$$

where $\otimes G_{ij}^K$ is the value of the i th attribute for the j th decision-making criterion indicated by the expert K th.

Step 3. Elaboration of the grey decision matrix

$$D = \begin{bmatrix} \otimes G_{11} & \otimes G_{12} & \dots & \otimes G_{1n} \\ \otimes G_{21} & \otimes G_{22} & \dots & \otimes G_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes G_{m1} & \otimes G_{m2} & \dots & \otimes G_{mn} \end{bmatrix} \tag{12}$$

where $\otimes G_{ij}^K$ is the value of a variable expressed in the form of grey numbers.

Step 4. Normalization of the grey decision matrix

$$D^* = \begin{bmatrix} \otimes G_{11}^* & \otimes G_{12}^* & \dots & \otimes G_{1n}^* \\ \otimes G_{21}^* & \otimes G_{22}^* & \dots & \otimes G_{2n}^* \\ \vdots & \vdots & \ddots & \vdots \\ \otimes G_{m1}^* & \otimes G_{m2}^* & \dots & \otimes G_{mn}^* \end{bmatrix} \tag{13}$$

where $\otimes G_{ij}^*$ indicates:

1. For a benefit attribute:

$$\otimes G_{ij}^* = \left[\frac{G_{ij}}{G_j^{\max}}, \frac{\overline{G_{ij}}}{G_j^{\max}} \right] \tag{14}$$

2. For a cost attribute:

$$\otimes G_{ij}^* = \left[\frac{G_j^{\min}}{G_{ij}}, \frac{G_j^{\min}}{\overline{G_{ij}}} \right] \tag{15}$$

Step 5. Developing the weighted normalized grey decision matrix

$$D^* = \begin{bmatrix} \otimes V_{11} & \otimes V_{12} & \dots & \otimes V_{1n} \\ \otimes V_{21} & \otimes V_{22} & \dots & \otimes V_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes V_{m1} & \otimes V_{m2} & \dots & \otimes V_{mn} \end{bmatrix} \tag{16}$$

where $\otimes V_{ij} = \otimes G_{ij}^* \times \otimes w_j$

The development of the matrix (16) enables taking into account weights for each decision-making attribute.

Step 6. Determining the referential value vector in individual decision-making criteria

For m possible bidders $S = \{S_1, S_2, \dots, S_m\}$ to be analyzed, the referential value vector in individual decision-making criteria is as follows: $S^{\max} = \{\otimes G_1^{\max}, \otimes G_2^{\max}, \dots, \otimes G_n^{\max}\}$. The vector is determined using the following formula:

$$S^{\max} = \left\{ \left(\max_{1 \leq i \leq m} V_{i1}, \max_{1 \leq i \leq m} \overline{V}_{i1} \right), \left(\max_{1 \leq i \leq m} V_{i2}, \max_{1 \leq i \leq m} \overline{V}_{i2} \right), \dots, \left(\max_{1 \leq i \leq m} V_{in}, \max_{1 \leq i \leq m} \overline{V}_{in} \right) \right\} \tag{17}$$

Step 7. Determining the grey possibility degree between compared S and referential vector S^{\max}

$$P\{S_i \leq S^{\max}\} = \frac{1}{n} \sum_{j=1}^n P\{ \otimes V_{ij} \leq \otimes G_j^{\max} \} \tag{18}$$

The result of step 8 is the assignment of the value of the grey possibility degree between compared S referential vector S^{\max} to each bidder. The lower the value of the determined probability coefficient for an individual bidder, the better.

4. Case Study

The method of selecting a contractor in the UN public procurement with the use of the Grey System Theory will be presented on the example of a public institution making

a public contract for the supply of 10 city buses. The contracting authority bases the award of a public contract on the most economically advantageous tender. The initial stage of the implementation of the method is the determination of a set of decision-making criteria by the management of the contracting authority in the selection of the most advantageous offer. A group of experts of the contracting authority selected a set of criteria for selection of a contractor in three categories presented in Table 2.

Table 2. A set of decision-making criteria (C_j)

Economic criteria (E_i)	Qualitative criteria (Q_i)	Socio-ecological criteria (SE_i)
Price (E_1)	Technical properties (Q_1)	Social value added (SE_1)
Operating costs (E_2)	Functional properties (Q_2)	Ecological value added (SE_2)
Service costs (E_3)	Aesthetic properties (Q_3)	–

Source: Own study.

Step 1. Determining the weights of the decision-making criteria

Four experts (Ex) - people working in managerial positions in the contracting authority - took part in determining the weightings of subsequent decision-making criteria. Table 3 presents the results of the weight determination process for individual decision-making criteria.

Table 3. Determining the weights of decision-making criteria

C_j	Ex_1	Ex_2	Ex_3	Ex_4	$\otimes w_j$
E_1	VH	VH	VH	VH	[0.800; 1.000]
E_2	VH	H	VH	VH	[0.750; 0.950]
E_3	VH	VH	VH	VH	[0.800; 1.000]
Q_1	H	VH	VH	VH	[0.750; 0.950]
Q_2	H	H	VH	H	[0.650; 0.850]
Q_3	M	L	L	H	[0.400; 0.600]
SE_1	M	L	L	H	[0.350; 0.550]
SE_2	L	M	L	M	[0.300; 0.500]

Source: Own study.

Experts regarded the price (0.800, 1.000) and service costs (0.800, 1.000) as the most important criteria for selecting a contractor in public procurement. Among the least important criteria were ecological value added (0.300, 0.500) and social value added (0.350, 0.550).

Step 2. Elaboration of the grey decision matrix

The values of economic criteria are expressed in the form of white numbers (specific values expressed most often in monetary units). The possibility of using the white numbers in the grey model results from the fact that the white number is a special case of grey number, namely the case in which the lower and upper limits of the grey number have the same value ($\underline{G} = \overline{G}$). It was assumed that in each of the economic criteria, the grey number [100; 100] will be assigned to the offer with the highest price. For each of the other bidders, the value of individual economic criteria will be expressed in a grey

number (with the same lower and upper limits), reflecting the ratio of the offered price to the maximum price. Table 4. shows the scale of attribute values for qualitative criteria (technical properties, functional properties and aesthetic properties) and for socio-ecological criteria (social value added and ecological value added).

Table 4. The scale of attribute values of qualitative and socio-ecological criteria expressed in the form of grey numbers

Linguistic evaluation	Criterion weight $\otimes G$
Very low (VL)	[0, 2]
Low (L)	[2, 4]
Medium (M)	[4, 6]
High (H)	[6, 8]
Very high (VH)	[8, 10]

Source: Own study.

As part of a sample public contract, there were 5 bidders (S_i). Table 5. presents the attribute values in economic criteria in offers of individual bidders expressed in grey numbers.

Table 5. Attribute values in economic criteria in offers of individual bidders expressed in grey numbers

$\otimes G_j$					
E_i/S_i	S_1	S_2	S_3	S_4	S_5
E_1	[95.00; 95.00]	[100.00; 100.00]	[82.00; 82.00]	[76.00; 76.00]	[91.00; 91.00]
E_2	[100.00; 100.00]	[93.00; 93.00]	[81.00; 81.00]	[84.00; 84.00]	[79.00; 79.00]
E_3	[88.00; 88.00]	[63.00; 63.00]	[75.00; 75.00]	[64.00; 64.00]	[100.00; 100.00]

Source: Own study.

Table 6 presents the attribute values in the qualitative and socio-ecological criteria in the offers of individual bidders expressed in grey numbers.

Table 6. Attribute values in the qualitative and socio-ecological criteria in the offers of individual bidders expressed in grey numbers.

S_j	Ex_1	Ex_2	Ex_3	Ex_4	$\otimes G_j$
Q₁					
S_1	VH	H	VH	VH	[7.500; 9.500]
S_2	VH	VH	VH	VH	[8.000; 10.000]
S_3	H	H	H	VH	[6.500; 8.500]
S_4	H	VH	H	M	[6.000; 8.000]
S_5	M	H	M	H	[5.000; 7.000]
Q₂					
S_1	L	M	H	M	[4.000; 6.000]
S_2	H	VH	VH	H	[7.000; 9.000]
S_3	VL	M	L	L	[2.000; 4.000]
S_4	VH	H	VH	H	[7.000; 9.000]

S ₅	M	M	L	M	[3.500; 5.500]
Q₃					
S ₁	M	L	VH	H	[5.000; 7.000]
S ₂	VL	H	L	H	[3.500; 5.500]
S ₃	VH	VH	H	VH	[7.500; 9.500]
S ₄	M	L	H	H	[4.500; 6.500]
S ₅	VH	H	VH	M	[6.500; 8.500]
SE₁					
S ₁	M	L	VH	H	[5.000; 7.000]
S ₂	L	L	M	M	[3.000; 5.000]
S ₃	H	VH	VH	H	[7.000; 9.000]
S ₄	L	M	L	M	[3.000; 5.000]
S ₅	H	H	VH	VH	[7.000; 9.000]
SE₂					
S ₁	VL	L	VL	L	[1.000; 3.000]
S ₂	M	M	M	M	[4.000; 6.000]
S ₃	H	M	H	M	[5.000; 7.000]
S ₄	VH	VH	H	VH	[7.500; 9.500]
S ₅	L	M	H	M	[4.000; 6.000]

Source: Own study.

Step 3. Normalization of the grey decision matrix

The next stage of the proposed method is the development of the normalized grey decision matrix (Table 7).

Table 7. Normalized grey decision matrix

S _i /E _i	E ₁	E ₂	E ₃	Q ₁	Q ₂	Q ₃	SE ₁	SE ₂
S ₁	[0.800; 0.800]	[0.790; 0.790]	[0.716; 0.716]	[0.750; 0.950]	[0.444; 0.667]	[0.526; 0.737]	[0.556; 0.778]	[0.105; 0.316]
S ₂	[0.760; 0.760]	[0.849; 0.849]	[1.000; 1.000]	[0.800; 1.000]	[0.778; 1.000]	[0.368; 0.579]	[0.333; 0.556]	[0.421; 0.632]
S ₃	[0.927; 0.927]	[0.975; 0.975]	[0.840; 0.840]	[0.650; 0.850]	[0.222; 0.444]	[0.789; 1.000]	[0.778; 1.000]	[0.526; 0.737]
S ₄	[1.000; 1.000]	[0.940; 0.940]	[0.984; 0.984]	[0.600; 0.800]	[0.778; 1.000]	[0.474; 0.684]	[0.333; 0.556]	[0.789; 1.000]
S ₅	[0.835; 0.835]	[1.000; 1.000]	[0.630; 0.630]	[0.500; 0.700]	[0.389; 0.611]	[0.684; 0.895]	[0.778; 1.000]	[0.421; 0.632]

Source: Own study.

Step 4. Determining the weighted normalized grey decision matrix

In the next step, the weight of individual decision-making criteria should be taken into consideration in the developed grey decision matrix (Table 8.)

Table 8. Weighted normalized grey decision matrix

S_i/E_i	E_1	E_2	E_3	Q_1	Q_2	Q_3	SE_1	SE_2
S_1	[0.640; 0.800]	[0.593; 0.751]	[0.573; 0.716]	[0.563; 0.903]	[0.289; 0.567]	[0.211; 0.442]	[0.194; 0.428]	[0.032; 0.158]
S_2	[0.608; 0.760]	[0.637; 0.807]	[0.800; 1.000]	[0.600; 0.950]	[0.506; 0.850]	[0.147; 0.347]	[0.117; 0.306]	[0.126; 0.316]
S_3	[0.741; 0.927]	[0.731; 0.927]	[0.672; 0.840]	[0.488; 0.808]	[0.144; 0.378]	[0.316; 0.600]	[0.272; 0.550]	[0.158; 0.368]
S_4	[0.800; 1.000]	[0.705; 0.893]	[0.788; 0.984]	[0.450; 0.760]	[0.506; 0.850]	[0.189; 0.411]	[0.117; 0.306]	[0.237; 0.500]
S_5	[0.668; 0.835]	[0.750; 0.950]	[0.504; 0.630]	[0.375; 0.665]	[0.253; 0.519]	[0.274; 0.537]	[0.272; 0.550]	[0.126; 0.316]

Source: Own study.

Step 5. Determining the referential value vector in individual decision-making criteria

In step 5 of the developed method, the referential value vector was determined in individual decision-making criteria:

$$S^{\max} = \{[0.800, 1.000], [0.750, 0.950], [0.800, 1.000], [0.600, 0.950], [0.506, 0.850], [0.316, 0.600], [0.272, 0.550], [0.237, 0.500]\}$$

Step 6. Determining the grey possibility degree between compared S and referential vector S^{\max}

The final step of this method is to determine the grey possibility degree between compared suppliers and referential vector S^{\max} (Table 9).

Table 9. The grey possibility degree between compared S and referential vector S^{\max}

S_i	$P(S_i \leq S^{\max})$
S_1	0.8641
S_2	0.7544
S_3	0.6910
S_4	0.6456

Source: Own study.

According to the conducted research, the best potential contractor for the UN procurement was the entity S_4 . This was followed successively by S_3 , S_2 and S_5 . The subject S_1 turned out to be the worst potential contractor.

5. Conclusion

The article presents a new method of selecting a contractor in United Nations public procurement. The developed method, based on the grey systems theory, enables to evaluate contractors both in the context of quantitative and qualitative criteria. The article is, therefore, part of the trend of developing expert systems which support decision-makers in public procurement, especially when there is a number of subjective criteria.

The article presents a concept in which white numbers are a special case of grey numbers (with the same lower and upper limits). As a result, it was possible to use the Grey Number Decision Model, which uses both values expressed in the form of white numbers (referring to price criteria) as well as values expressed in the form of grey numbers (referring to qualitative and socio-ecological criteria).

The scope of applicability of the developed method covers all public procurement of the United Nations, understood as a specific decision problem. In particular, it can be used in green public procurement. Public procurement is characterized by taking into account at the evaluation stage a number of subjective factors, difficult to express with the use of a quantitative scale. In this context, the method allows to objectify the process of selecting the contractors.

Limitations of applicability of the method result from the legal restrictions in some countries. In the case of public procurement, in which the sole criterion is the price, the contractor selection process is based solely on a comparison of the prices and choosing the most advantageous offer. In this case, the use of the proposed method is unnecessary. The developed method has the potential for further improvement - especially in the area of the application of decision-making models, differentiating the process of weighting decision criteria, and its adaptation to legal regulations governing public procurement in different countries.

References:

- Day, J.D., Wang, C.N., Dang, H.S. 2017. Predicting the performance of franchise industry using Grey models-case study in United States. In 2017 International Conference on System Science and Engineering (ICSSE), 617-620. IEEE.
- Delcea, C. 2015. Grey systems theory in economics—a historical applications review. *Grey Systems: Theory and Application*, 5(2), 263-276.
- Delcea, C., Cotfas, L.A., Paun, R. 2018. Airplane Boarding Strategies Using Agent-Based Modeling and Grey Analysis. In International Conference on Computational Collective Intelligence, 329-339. Springer, Cham.
- Eshtaiwi, M.I., Badi, I.A., Abdulshahed, A.M., Erkan, T.E. 2017. Assessment of airport performance using the grey theory method: A case study in Libya. *Grey Systems: Theory and Application*, 7(3), 426-436.
- Fang, Z., Zhang, Q., Cai, J., Hu, Q., Liu, S. 2017. On general standard grey number representation and operations for multi-type uncertain data. In *Grey Systems and Intelligent Services (GSIS), International Conference*, 19-26. IEEE.
- Hsiao, S.W., Lin, H.H., Ko, Y.C. 2017. Application of Grey Relational Analysis to Decision-Making during Product Development. *EURASIA J. Math. Sci. Technol. Educ.*, 13, 2581-2600.
- Julong, D. 1982. Grey control system. *Journal of Huazhong University of Science and Technology*, 3(9), 18.
- Julong, D. 1989. Introduction to grey system theory. *The Journal of grey system*, 1(1), 1-24.
- Karr-Wisniewski, P., Lu, Y. 2010. When more is too much: Operationalizing technology overload and exploring its impact on knowledge worker productivity. *Computers in Human Behavior*, 26(5), 1061-1072.

- LaValle, S., Lesser, E., Shockley, R., Hopkins, M.S., Kruschwitz, N. 2011. Big data, analytics and the path from insights to value. *MIT Sloan management review*, 52(2), 21.
- Li, G.D., Yamaguchi, D., Nagai, M. 2007. A grey-based decision-making approach to the supplier selection problem. *Mathematical and computer modelling*, 46(3-4), 573-581.
- Liu, S., Forrest, J., Yang, Y. 2012. A brief introduction to grey systems theory. *Grey Systems: Theory and Application*, 2(2), 89-104.
- Liu, S., Yang, Y., Forrest, J. 2017. *Grey data analysis*. Springer: Berlin, Germany
- Liu, S., Yang, Y., Xie, N., Forrest, J. 2016. New progress of grey system theory in the new millennium. *Grey Systems: Theory and Application*, 6(1), 2-31.
- Ma, X., Liu, Z., Wang, Y. 2019. Application of a novel nonlinear multivariate grey Bernoulli model to predict the tourist income of China. *Journal of Computational and Applied Mathematics*, 347, 84-94.
- Ma, Z.Z., Zhu, J. 2016. An uncertain pure linguistic approach on evaluation of enterprise integrity based on grey information. *Grey Systems: Theory and Application*, 6(3), 353-364.
- Mierzwia, R., Xie, N., Nowak, M. 2018. New axiomatic approach to the concept of grey information. *Grey Systems: Theory and Application*, 8(2).
- Pourmohammadi, K., Shojaei, P., Rahimi, H., Bastani, P. 2018. Evaluating the health system financing of the Eastern Mediterranean Region (EMR) countries using Grey Relation Analysis and Shannon Entropy. *Cost Effectiveness and Resource Allocation*, 16(1), 31.
- Rao, C., Goh, M., Zheng, J. 2017. Decision mechanism for supplier selection under sustainability. *International Journal of Information Technology & Decision Making*, 16(01), 87-115.
- Wiecek-Janka, E., Nowak, M., Borowiec, A. 2019. Application of the GDM model in the diagnosis of crises in family businesses. *Grey Systems: Theory and Application*, 9(1), 114-127.