Full Sea Trial Based Estimation of Hydrodynamic Derivatives of a Research Vessel

Eric Kulbiej[1]

Abstract:

Purpose: The study aims to generate a set of hydrodynamic coefficients allowing for a manoeuvring simulation of an existing vessel.

Design/Methodology/Approach: Several full-scale trials of a research vessel mv Nawigator XXI were carried out. A set of hydrodynamic derivatives based on pre-existing semi-empirical formulae was formed and used as a starting point in the tuning phase. The set was tuned so that the summarised error of manoeuvring indices between the simulation and the experimental data would be minimised.

Findings: A set of hydrodynamic derivatives named N7 was obtained. It allows for a simulation of manoeuvring capabilities of an existing research vessel with satisfactory accuracy.

Practical implications: The pre-existing mathematical model MMG can be supplied with an N7 set generated and tuned in the following study for further simulations of manoeuvres or in derivative studies i.e. concerning planning realistic collision avoidance manoeuvres or track optimisation.

Originality value: The obtained set of hydrodynamic coefficients N7 can be successfully used for simulation of mv Nawigator XXI manoeuvrability as it is a new vessel, never previously simulated using an existing and popular MMG model..

Keywords: Ship mathematical model, ship hydrodynamics, hydrodynamic derivatives, manoeuvring performance, sea trials.

JEL classification: K49, C51, Z00.

Paper type: Research article.

---

[1]*PhD Candidate and Research Grant Beneficiary, Maritime University of Szczecin,*
ekulbiej@gmail.com

1. Introduction

Among many tools enabling researchers to model ship manoeuvring characteristics and simulate movements mathematical motion still play a crucial role. Their use is potentially unlimited, provided that the ship is modelled carefully and resemble the original manoeuvring performance. Since conducting sea trials is a somewhat expensive venture, captive tests are preferred, namely circular motion (CMT) or planar motion mechanism (PMM). The physical model, no matter if in scale or full sea trial, is employed to produce data that can be used as a validation for the mathematical model.

One of the most common algorithms that can be used for ship movement prediction was introduced by the Mathematical Modelling Group (MMG) (Ogawa *et al.,* 1977; Yasukawa and Yoshimura, 2014). The model is sometimes labelled as modular, since it divides the forces acting upon a ship, in its ship-fixed reference frame and coordinate system, into components regarding hull, rudder and propeller. The interaction between those modules is modelled via the incorporation of several parameters or functions, approximating the influence between one another.

For the MMG model to be useful, a set of parameters must be known. Those are hydrodynamic derivatives, used to simulate hydrodynamic forces of the hull; coefficients used to model rudder interactions and propeller-induced thrust forces. Depending on the type of parameter, they can be either calculated using semi-empirical means (Yoshimura and Masumoto, 2012), generated using numerical solutions (Kołodziej and Hoffman, 2021) or obtained from the free-running model or full-scale tests (Hajizadeh *et al.,* 2016). Semi-empirical algorithms are the most convenient to use especially when no real-world data is available. The premise of the following article is to show a method of obtaining a set of hydrodynamic coefficients of a hull for a real existing ship, by the means of reverse-engineering the results of a sea trial.

The following procedure can be successfully used for fining a set of coefficients for any ship, provided that the characteristics of the rudder and propeller system are known. Moreover, the full set of parameters of a said research ship is provided and can be conveniently used for the simulation of manoeuvres.

2. Premise of the Mathematical Model

The MMG employs a dual reference frame and coordinate system. The first one, XY, is fixed to the ship and that is where all the hydrodynamic calculations are done. Then the obtained accelerations and velocities are transformed into an Earth-fixed inertial $X_0 Y_0$ frame. The basic notation, including the sign convention, is displayed in figure 1 below. The details can be found in the references (Ogawa *et al.,* 1977; Yasukawa and Yoshimura, 2014).

Figure 1. *Basic notation and sign convention used in MMG model.*
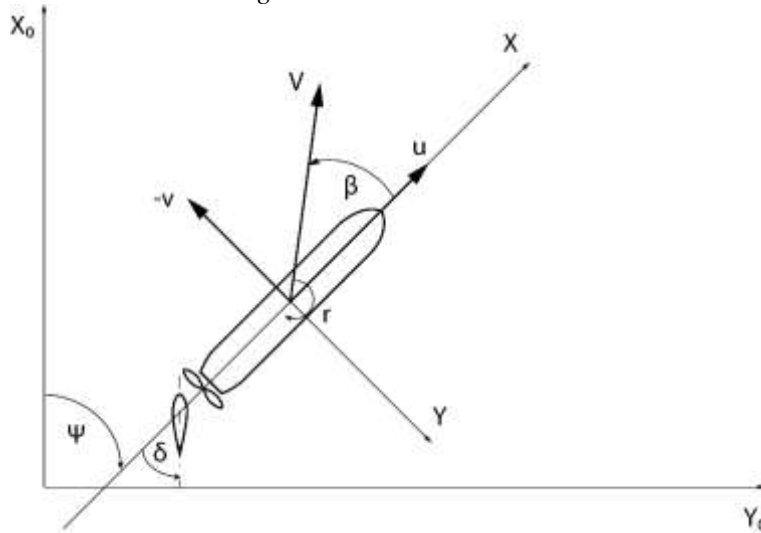
The ship has an effective velocity V which is a sum of longitudinal u and transversal v velocity components. The difference between the direction of ships movement in Earth-fixed coordinate system and ships heading ψ is the drift angle β, r denotes yaw rate and δ means current rudder angle. Should the centre of gravity be different from the beginning of the ship-fixed coordinate system, its position would be described as $G = (x_G, y_G, z_G)$.

The model consists of four integral parts, namely general equations of motions, derived from Newtonian rigid body dynamics, hull, rudder, and propeller hydrodynamic interaction components, all of which are described in the following paragraphs.

2.1 Equations of Motions

The motion of a ship manoeuvring in calm water can be described in up to six degrees of freedom (DOF), out of which the MMG model employs three: surge (sailing in forward/astern direction), sway (sailing in the transverse direction) and yaw (rotation in the XY plane). The equations of said motions, involving terms regarding inertial forces are as follows:

$$m(\dot{u} - vr) = F_X$$
$$m(\dot{v} + ur) = F_Y$$
$$I_{ZZ_G}\dot{r} = M_Z$$

where the dot notation means derivative, in other words $\dot{v} = \frac{dv}{dt}$. Mass of the ship is m and I_{ZZ_G} denotes the ship's moment of inertia around the vertical axis in the centre of

gravity. Both forces and moments are from hereon described simply as forces. Those forces can be then described as:

$$F_X = -m_X\dot{u} + m_Y vr + X$$
$$F_Y = -m_Y\dot{v} - m_X ur + Y$$
$$M_Z = -J_{ZZ}\dot{r} + N - x_G F_Y$$

Where m_X, m_Y, J_{ZZ} denote added mass and moment of inertia when surging, swaying and yawing respectively. Combining those two sets yields the general form as follows:

$$(m + m_X)\dot{u} - (m + m_Y)vr - x_G mr^2 = X$$
$$(m + m_Y)\dot{v} + (m + m_X)ur + x_G m\dot{r} = Y$$
$$\left(I_{ZZ_G} + x_G^2 m + J_{ZZ}\right)\dot{r} + x_G m(\dot{v} + ur) = N$$

where X, Y, N are the sum of hydrodynamical forces acting upon the hull:

$$X = X_H + X_R + X_P$$
$$Y = Y_H + Y_R$$
$$N = N_H + N_R$$

where the subscripts H, R, and P denote respectively forces due to the hull, rudder, and propeller.

2.2 Hull Forces

The hull forces model can be generally described as follows:

$$X_H = 0.5\rho LTV^2 X_H'(v', r')$$
$$Y_H = 0.5\rho LTV^2 Y_H'(v', r')$$
$$N_H = 0.5\rho L^2 TV^2 N_H'(v', r')$$

Here ρ means the density of water, L is the ship's length between perpendiculars, T is the draught and V is the ship's speed. The hull forces coefficients are commonly expressed in a non-dimensional notation (SNAME, 1950) as a partial Taylor series expansion (Abkowitz, 1964). The hydrodynamic forces coefficients effectively take the form of polynomial expressions of ship's resistance in longitudinal motion in calm water R_0', non-dimensional transverse velocity $v' = v/V$, and non-dimensional yaw rate $r' = rL/V$. The coefficients can be thus described as:

$$X_H'(v', r') = -R_0' + X_{vv}'v'^2 + X_{vr}'v'r' + X_{rr}'r'^2 + X_{vvvv}'v'^4$$
$$Y_H'(v', r') = Y_v'v' + Y_r'r' + Y_{vvv}'v'^3 + Y_{vvr}'v'^2 r + Y_{vrr}'v'r'^2 + Y_{rrr}'r'^2$$
$$N_H'(v', r') = N_v'v' + N_r'r' + N_{vvv}'v'^3 + N_{vvr}'v'^2 r + N_{vrr}'v'r'^2 + N_{rrr}'r'^2$$

It is worth noting that since $\beta = \text{atan}(v/u)$, then $v' \approx \beta$ for small angles of drift (effectively less than 20°). There are instances where different orders of hydrodynamic derivatives are used, such as 2nd order. Here, following (Yasukawa and Yoshimura, 2014) 1st and 3rd order is employed for lateral hull force and yaw moment. Values of particular parameters $X'_{vv}, X'_{vr}, X'_{rr}, X'_{vvvv}, Y'_v, Y'_r, Y'_{vvv}, Y'_{vvr}, Y'_{vrr}, Y'_{rrr},$ $N'_v, N'_r, N'_{vvv}, N'_{vvr}, N'_{vrr}, N'_{rrr}$ can be obtained using methods mentioned earlier or according to the procedure shown in the article.

2.3 Rudder Forces

The generic version of the MMG model employs a rudder model assuming a typical case of a conventional stern rudder. The hydrodynamic forces induced by said ruder can be then calculated in the function of rudder angle and rudder normal force F_N:

$$X_R = -(1 - t_R)F_N \sin \delta$$
$$Y_R = -(1 + a_H)F_N \cos \delta$$
$$N_R = -(x_R + a_H x_H)F_N \cos \delta$$

The normal rudder force can be calculated according to the following equation:

$$F_N = 0.5\rho A_R U_R^2 f_\alpha \sin \alpha_R$$

where the velocity of water inflow to the rudder area U_R and effective angle of attack α_R are defined as:

$$U_R = \sqrt{u_R^2 + v_R^2}$$

$$\alpha_R = \delta - \text{atan}\left(\frac{v_R}{u_R}\right)$$

The components of water inflow speed can be calculated accordingly:

$$u_R = \varepsilon u(1 - w_P)\sqrt{\eta\left\{1 + \kappa\left(\sqrt{1 + \frac{8K_T}{\pi J_P^2}} - 1\right)\right\}^2 + (1 - \eta)}$$
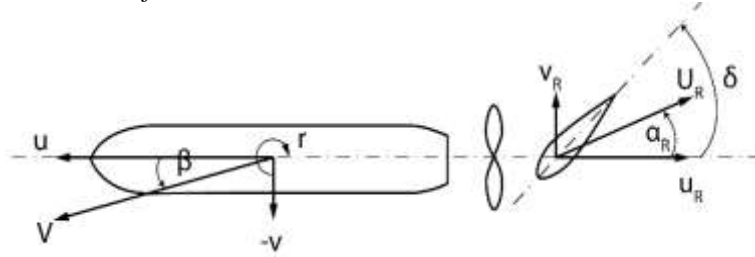
$$v_R = V\gamma_R\beta_R$$

$$\beta_R = \beta - \ell'_R r'$$

$$\varepsilon = \frac{(1 - w_R)}{(1 - w_P)}$$

The velocity of water inflow to the rudder area is shown in Figure 2. Coefficients and parameters $f_\alpha, t_R, a_H, x_H, x_R, \varepsilon, \kappa, \eta, \gamma_R, \ell'_R$ describe, inter alia, geometrical parameters of the rudder, and impact and interactions of propeller and hull on the rudder forces. They can be either obtained empirically or calculated numerically.

Figure 2. *Rudder water inflow.*

Source: *Own study.*

2.4 Propeller Forces

The force due to the propeller is modelled as a function of thrust. It is to be noted that lateral force and yaw moment due to propeller thrust is minor and is thus neglected. The longitudinal force is described as follows:

$$X_P = (1 - t_P)T$$

there the thrust deduction factor t_P is assumed constant for a particular propeller's load. The thrust is then modelled as:

$$T = \rho n_P^2 D_P^4 K_T(J_P)$$

where n_P are the propeller's revolutions, D_P is the diameter of the propeller, J_P is the advance coefficient, and K_T is the propeller hydrodynamical thrust coefficient. K_T describes the characteristics of the propeller in the function of water inflow via advance coefficient. It is commonly approximated as a polynomial of 2nd order:

$$K_T(J_P) = k_2 J_P^2 + k_1 J_P + k_0$$

Values k_2, k_1, k_0 are obtained from open-water tests of the propeller in the design stage. The advance coefficient is described as:

$$J_P = \frac{u(1 - w_P)}{n_P D_P}$$

The wake fraction coefficient w_P can be with decent approximation be treated as a constant value, however, it is advised to model it according to the following algorithm (2):

$$\frac{(1 - w_P)}{(1 - w_{P0})} = 1 + \{1 - \exp(-C_1|\beta_P|)\}(C_2 - 1)$$

where

$$\beta_P = \beta - x'_P r'$$

Here, C_1, C_2, x'_P are constants, w_{P0} is the initial wake fraction coefficient, and β_P is the coefficient of geometrical water inflow to the propeller. It is worth noting that:

$$\frac{(1 - w_P)}{(1 - w_{P0})} \rightarrow C_2, \text{for } |\beta_P| \rightarrow \infty$$

3. Sea Trials

3.1 Analysed Ship

The study is conducted on the example of a research vessel mv Nawigator XXI. This 60-metre long ship's hull geometry is shown in figure 3 and its particulars and properties are displayed in table 1. MV Nawigator XXI is equipped with a four-blade controllable pitch propeller and a Becker's rudder.

Figure 3. *Hull geometry of MV Nawigator XXI.*
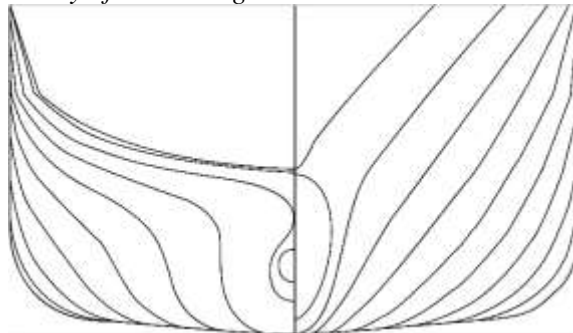
Source: Own study.

Table 1. *Properties of analysed research vessel Nawigator XXI.*

Property	Symbol	Value
Name	-	Nawigator XXI
Length overall [m]	L_{OA}	60.20
Length between perpendiculars [m]	L	56.85
Breadth [m]	B	10.50
Draught [m]	T	3.14
Displacement [m³]	∇	1126
The radius of the gyration around the centre of gravity [m]	k_{ZZ}	13.75

Surface wetted area [m²]	S_W	672
Block coefficient [-]	C_B	0.623
Service speed [m/s]	V_0	5.91
Propeller diameter [m]	D_P	2.26
Propeller rotation speed [1/s]	n	3.7
Propeller expanded area ratio [-]	A_E/A_O	0.673
Propeller pitch ratio [-]	P/D	0.924
Rudder projected lateral area [m²]	A_R	5
Rudder turn rate [°/s]	v_δ	2.5

Source: *Own study.*

3.2 Experimental Setup

The data required for analysis was connected using a laptop with Navdec installed (Figure 4). Navdec is a navigational decision supporting system (NDSS) and was invented and designed by a research team of professor Pietrzykowski from the Maritime University of Szczecin for both ocean-going vessels and pleasure crafts (Pietrzykowski *et al.,* 2012). The system itself is to designed supplement the shipborne navigational equipment, whilst in the future, it may be included in Integrated Bridge System (IBS).

The positive operation of the system requires co-operation with other navigational devices and systems onboard the ship as well as the external ones to acquire and evaluate navigational information automatically and correctly. The main goal of Navdec is to qualify encounter situations with other vessels according to COLREGS and to propose a law-abiding solution to the navigator.

Figure 4. *Laptop connected to the pilot plug with Navdec launched.*

Source: *Own study.*

In the experiment, however, the anticollision module was merely tested, and auxiliary elements of Navdec were mainly used. Namely, the computer was connected to the pilot plug and thus received all relevant navigational data directly in a form of NMEA strings. The strings were captured and recorded by Navdec. An example of a said NMEA string is presented as follows:

```
$OWN,0,261187000,0,0.0,11.4,15.944375,54.969566,356.6,2.0,0.0,
0.0,,5,9161247,SNHA,NAWIGATOR
XXI,99,17,43,4,6,2,5,21,12,0,3.4,SZCZECIN*77
```

From it the relevant information can be obtained, namely:

- current position of the ship (latitude and longitude),
- ship's heading and course over ground,
- ship's speed through water and over ground.

3.3 Turning Circle

The full-sea trials were conducted on the 19th of May 2021 in the Baltic Sea, close to Swinemünde. The water depth was above 30 metres, which for the draft of 3 metres can be treated as deep water. The wind was on average 10 knots, direction NW and sea state was smooth. The difference between a log through water and overground reading for straight sailing implied a lack of current. The hydrometeorological and other environmental influences were thus neglected in the scope of calculation. The manoeuvre was postponed until an absence of other vessels and navigational risks was ensured so that the sea trials would not impede the safe operations of other ships. The test was then executed using hand steering, employing a student of the Maritime University of Szczecin as a helmsman.

Several full-sea trial manoeuvres were executed, among which the most interesting and useful is the turning circle manoeuvre, conducted whilst at full ahead pitch setting and maximum rudder angle. The data obtained during the experiments were transformed so that the positions could be displayed in a Cartesian coordinate system with the axes normalised by the ship's length between perpendiculars. The partial velocities u, v were also smoothed using a moving average filter in a form of:

$$g(x) = 0.2 \cdot (f(x-2) + f(x-1) + f(x) + f(x+1) + f(x+2))$$

so that the major part of the noise could be removed.

The full-scale sea trial manoeuvre that is analysed in the article is the turning circle to starboard with rudder angle $\delta = 35°$, and initial ship's speed $V = u = 5.91 m/s$ and heading $\psi = 000°$. The longitudinal and lateral ships velocity both raw and smoothed are shown in figure 5, whilst the positions of the vessel during turning are shown in figure 6. Trajectory refers to originally registered one and smoothed is the one
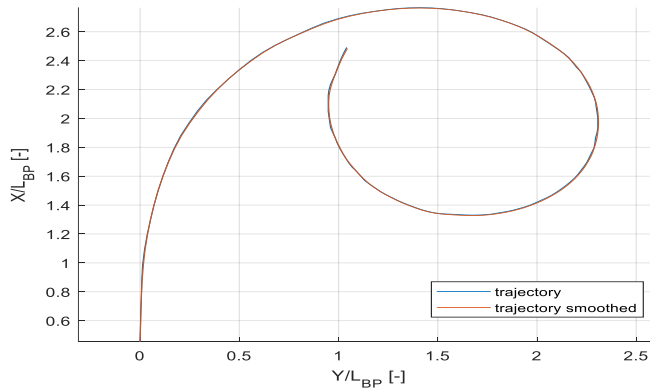
enhanced using the aforementioned crude filtering technique. As far as the trajectory is concerned, the filtering does little difference to the eventual outcome.

Figure 5. *Vessel's speeds in time during turning circle manoeuvre.*
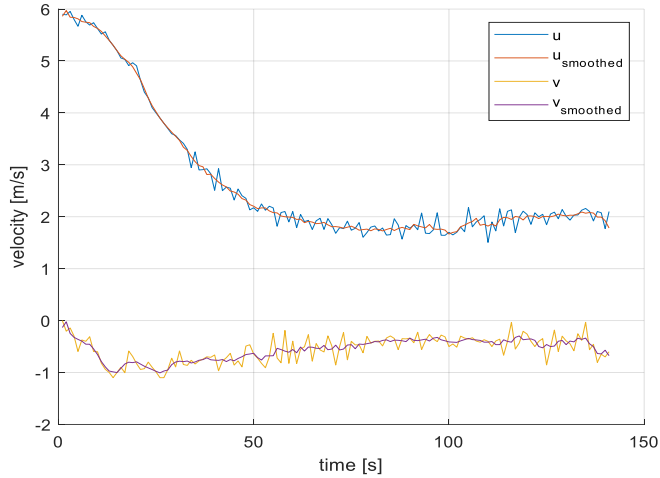
Source: *Own study.*

Figure 6. *Vessel's non-dimensional trajectory during turning circle manoeuvre.*

Source: *Own study.*

4. Calculation of Hydrodynamic Coefficients

4.1 Method

The method of obtaining a tailor-based set of hydrodynamic coefficients revolves around tuning a pre-existing set so that the cumulative error is minimised. This requires:

- having an initial set of hydrodynamic coefficients to start the optimisation process with,

- establishing error comparison framework,
- obtaining the set of indices used for error comparison,
- designing and implementing a tuning algorithm.

The initial set of hydrodynamic coefficients is not required to be sensible at all, however, the closer it is to correct one the less calculation will be done in the process. Following this premise, Yoshimura and Masumoto's semi-empirical model is generated and used as an entry point.

The error comparison framework was decided to be a sum of squares of relative errors:

$$error = \sum \left[1 - \frac{I_k^{HC}}{I_k^{ST}} \right]$$

where I_k denotes k^{th} index when either calculated using a temporary set of hydrodynamic coefficients HC or measured from sea trials ST.

After analysis of the practicality of the study the following 6 indices were chosen:

- u_S – stable longitudinal velocity after the initial turn and during steady turning,
- v_S – analogical stable lateral velocity,
- – analogical stable yaw rate,
- D – diameter of steady turning circle,
- Ad – advance, the distance travelled in the direction of the original course by the midship point of a ship from the position at which the rudder order is given to the position at which the heading has changed 90° from the original course,
- Td – tactical diameter, the distance travelled by the midship point of a ship from the position at which the rudder order is given to the position at which the heading has changed 180° from the original course, measured in a direction perpendicular to the original heading of the ship.

The last two parameters were chosen based on IMO Standards for ship manoeuvrability (IMO, 2002). The values of said indices were measured from full-scale trials for three different rudder angles and are presented in table 2 below. It is to be noted that the diameter of steady turning circle, advance and tactical diameter are presented in a non-dimensional form.

Table 2. *Manoeuvring indices measured from full-scale trials of mv Navigator XXI.*

| δ [°]	u_S [m/s]	v_S [m/s]	r_S [rad /s]	D [−]	Ad [−]	Td [−]
35	1.88	-0.43	0.048	1.2	2.2	2.2
25	2.41	-0.59	0.047	1.8	3.1	2.7
15	3.20	-0.71	0.035	3.3	3.9	3.7

Source: *Own study.*

4.2 Sensitivity Analysis and Tuning

Following the measurements of manoeuvring indices, a sensitivity analysis was undertaken. The goal was to find hydrodynamic coefficients responsible for the largest differences in a particular index value should the coefficient be subject to alteration. The sample change applied to the particular hydrodynamic derivatives was an increase of 10%. Then the sensitivity index was calculated using the following formula:

$$\%I_K = 100\% \cdot \left[\frac{I_K^{10\%} - I_K^{0\%}}{I_K^{0\%}} \right]$$

Results for the sensitivity analysis are presented in Figures 7 -12 below.

Figure 7. Sensitivity analysis results for u_S.
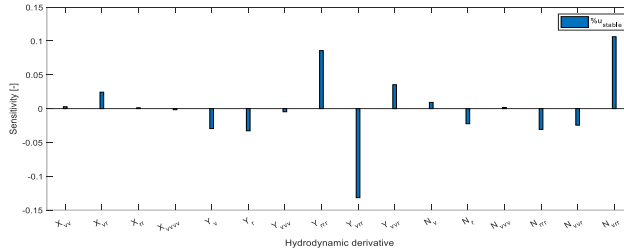
Source: Own study.

Figure 8. Sensitivity analysis results for v_S.
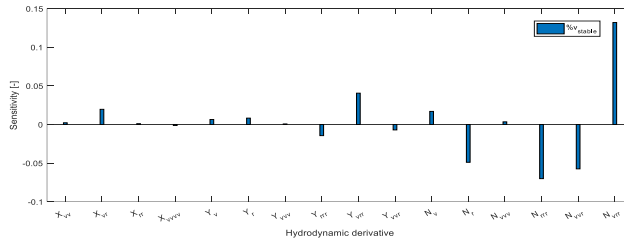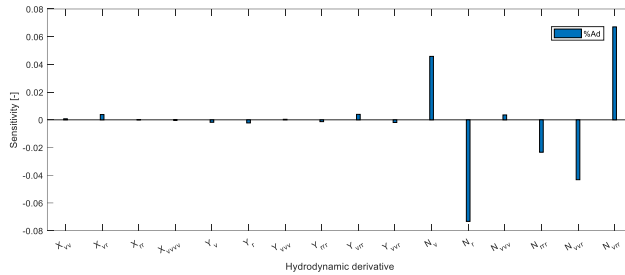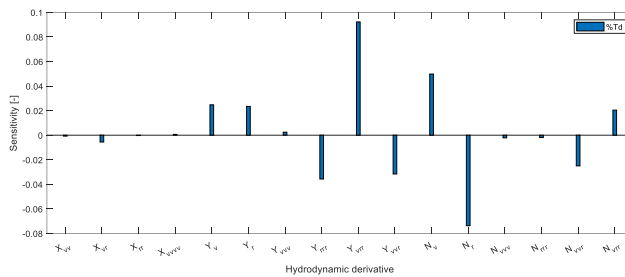

Source: Own study.
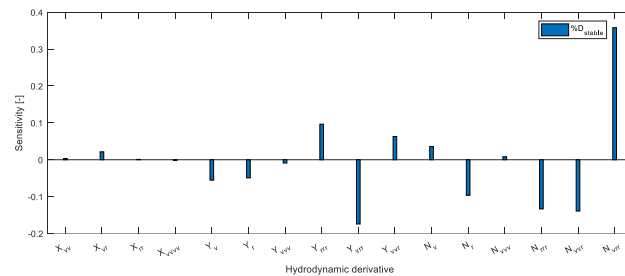
Figure 9. Sensitivity analysis results for r_S.
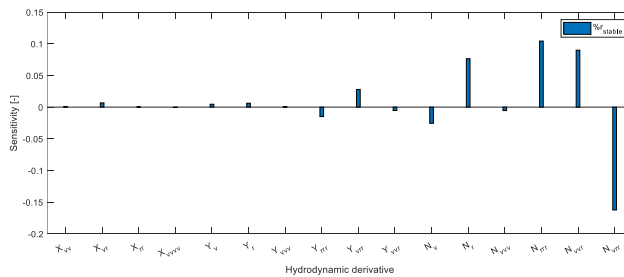
Source: Own study.

Figure 10. *Sensitivity analysis results for D.*

Source: *Own study.*

Figure 11. *Sensitivity analysis results for Ad.*

Source: *Own study.*

Figure 12. *Sensitivity analysis results for Td.*

Source: *Own study.*

The sensitivity analysis proved useful for restricting the numerical calculations in the tuning algorithm. The true optimal solution can be obtained by checking all possible combinations of hydrodynamic derivatives with a pre-defined discretisation step. Since the amount of combination is a power function of the number of elements in the set, it is reasonable to cross out as many as possible.

Thus the hydrodynamic coefficients were grouped into sub-sets of five or fewer elements that were proven to influence the particular index most. Then a multi-level loop iterated over those elements, seeking a set that would yield the smallest cumulative error. Another loop would iterate over the pre-determined sets and yet another, final, would iterate over several different values of discretisation steps,

namely 10%, 5%, 1% and a random values of $x \in [0; 0.1]$. The discretisation step would be added to and subtracted from every hydrodynamic coefficient up to 10 times, resulting in variations up to +100% and -100% of the sample value.

5. Results

The eventually tuned set of hydrodynamic coefficients yielded a cumulative error of 1.348 whereas an initial set of semi-empirical values yielded an error of value 4.273. Obtained values are presented in Table 3 below, initial values of Yoshimura and Masumoto semi-empirical formulas is provided as a comparison for readers discretion.

Table 3. *Obtained hydrodynamic derivatives as compared to semi-empirical set.*

| Hydrodynamic derivative	Current study	Yoshimura & Masumoto
X'_{vv}	-0.0521	-0.0613
X'_{vr}	0.0759	0.0893
X'_{rr}	-0.0007	-0.0008
X'_{vvvv}	0.3487	0.4103
Y'_v	-0.0579	-0.3094
Y'_r	0.1605	0.0620
Y'_{vvv}	-0.3446	-0.5771
Y'_{vvr}	-0.1642	-0.0510
Y'_{vrr}	-1.4168	-0.7191
Y'_{rrr}	-2.0690	-0.7500
N'_v	-0.1588	-0.1050
N'_r	-0.0764	-0.0457
N'_{vvv}	-1.1181	-0.2529
N'_{vvr}	-0.0242	-0.0302
N'_{vrr}	-1.2894	-0.6000
N'_{rrr}	-0.3686	-0.2737

Source: *Own study.*

This huge improvement error rate can be observed in Figure 13 displaying results of trajectory for initial Yoshimura & Masumoto approximation, full-scale measured path and a simulated one using tuned set of hydrodynamic derivatives. In Figures, 14-16 surface plots of non-dimensional forces are presented. They were calculated using the obtained set of tuned coefficients and non-dimensional sway velocity and yaw rate ranging from -0.5 to 0.5.

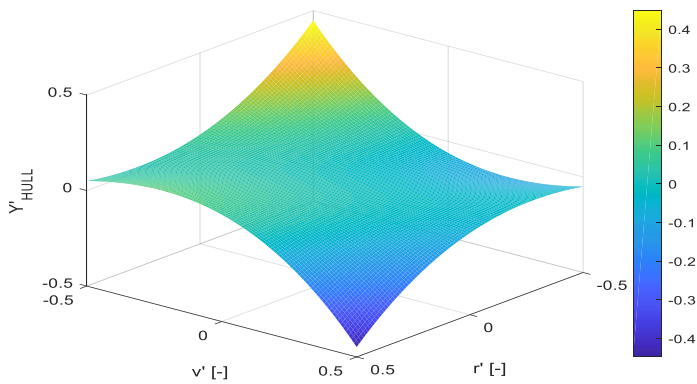**Figure 13.** *Experimental and simulated trajectories for mv Nawigator XXI.*
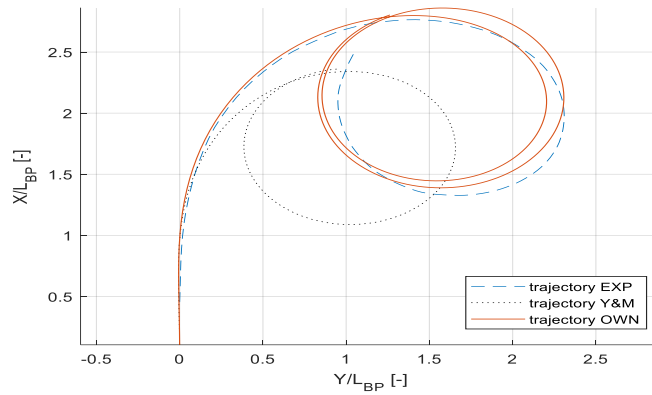
Figure 14. *Surface plot for non-dimensional hull hydrodynamic longitudinal force.*
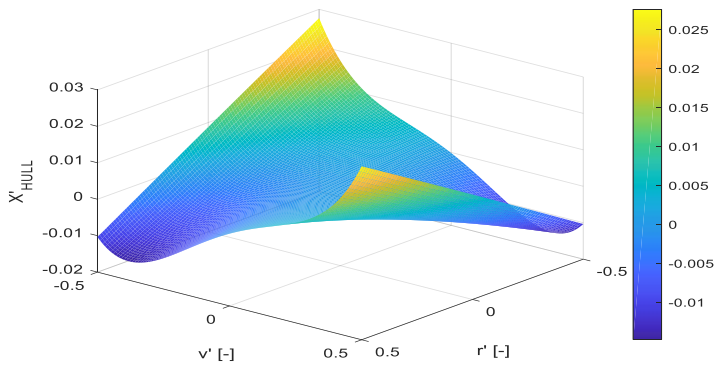
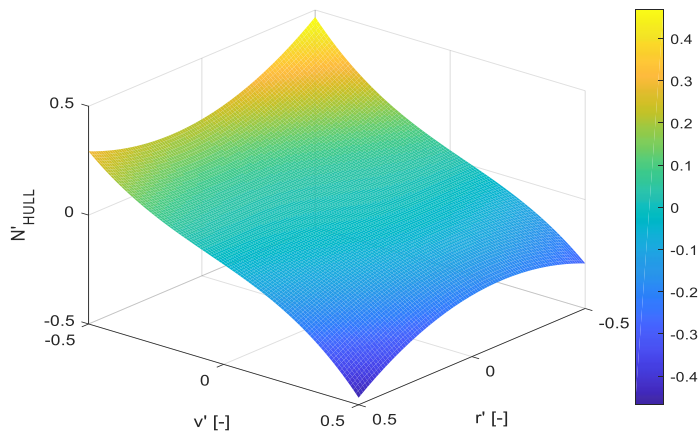**Figure 15.** *Surface plot for non-dimensional hull hydrodynamic lateral force.*

Figure 16. *Surface plot for non-dimensional hull hydrodynamic yawing moment.*

Source: Own study.

5.1 Model Validation

The final 35° to starboard circulation was simulated to present the positions of the ship when the heading is altered by a consequent right angle. This is presented in figure 17, while in figure 18 a time series of all ship linear and rotational velocities are presented. All simulations were executed using a Runge-Kutta 4th order numerical discretisation algorithm. However, it is worth noting that choosing different solvers for simulation does not impact the results to a meaningful extent. The full set of parameters required for a successful simulation of mv Nawigator XXI manoeuvre is collected and presented in Table 4 below.

Table 4. *Full set of coefficients and parameters of mv Nawigator XXI used in the simulation.*

|			
ℓ'_R	-0.71	η	0.624
ε	1.09	m_X	$0.033m$
κ	0.5	m_Y	$0.9m$
f_α	2.75	J_{ZZ}	$0.9I_{ZZ_G}$
k_2	-0.1385	t_R	0.39
k_1	-0.2753	a_H	0.31
k_0	0.2931	x'_H	-0.46
x_G	0.00	C_1	2.0
R_0	-0.0138	$C_2(\beta_P > 0)$	1.6
x'_R	-0.500	$C_1(\beta_P < 0)$	1.1
t_P	0.2	$\gamma_R(\beta_P < 0)$	0.4
w_{P0}	0.24	$\gamma_R(\beta_P > 0)$	0.64

Source: Own study.

Figure 17. *The trajectory of mv Nawigator XXI turning circle with vessel outline every 90° heading change.*
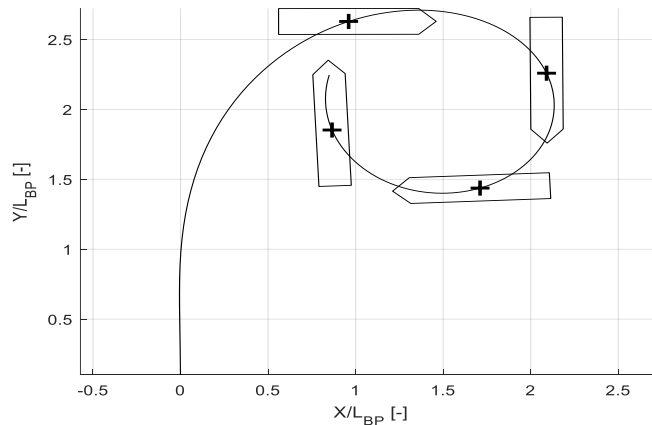
Source: Own study.

Figure 18. *Time series of linear and rotational velocities of mv Nawigator XXI during turning circle.*
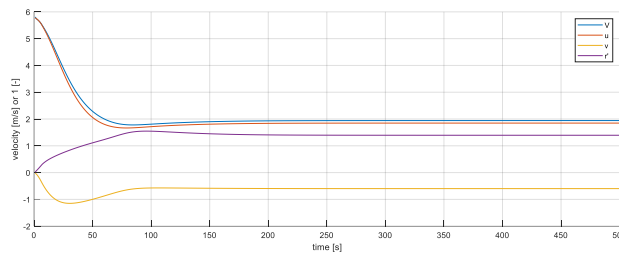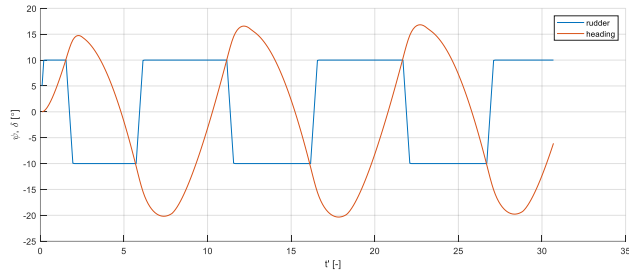

Source: Own study.

The eventual simulation for the model validation purposes is carried out using a modified zig-zag manoeuvre. Generally speaking, the test is the manoeuvre where a known amount of helm is applied alternately to either side when a known heading deviation from the original heading is reached. Originally, either a 10°/10° or 20°/20° manoeuvre is executed. Simulations of mv Nawigator XXI carrying out said manoeuvres for each side first is presented in figures 19 – 22. The X-axis displays time in a non-dimensional form $t' = tV_0/L$.

However, in full sea trials, a modified version of 10°/20° was carried out. The results of this experiment compared to the simulation can be seen in figure 23 where the heading change and the rudder angle value can be seen in a non-dimensional time series. Moreover, in figure 24 the trajectories of both the measured experimental trial and a simulated run can be seen as a comparison.
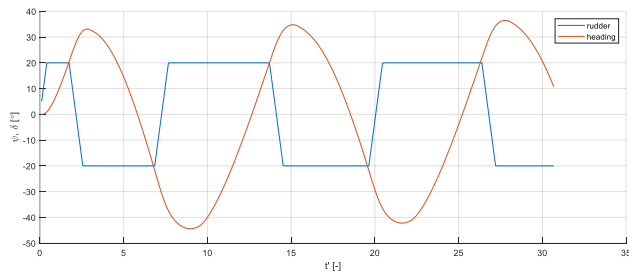
Figure 19. *10°/10° Zig-zag manoeuvre starboard first.*

Source: *Own study.*

Figure 20. *10°/10° Zig-zag manoeuvre port first.*
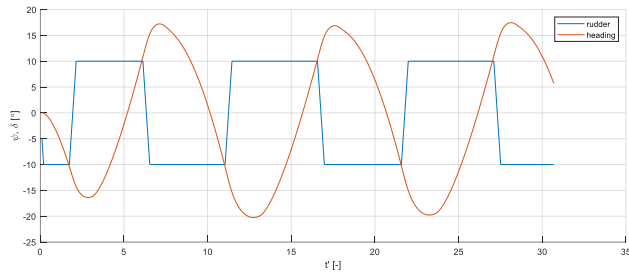
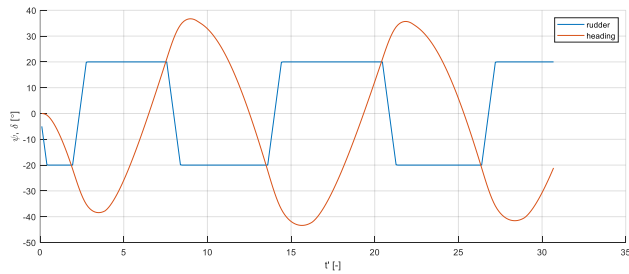**Source:** *Own study.*

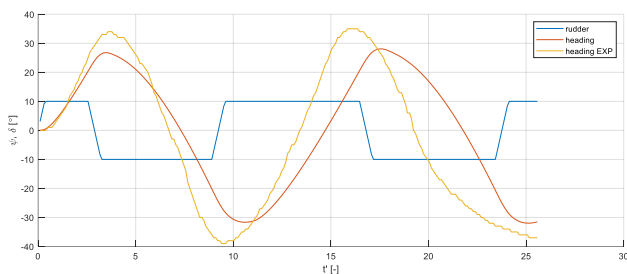**Figure 21.** *20°/20° Zig-zag manoeuvre starboard first.*
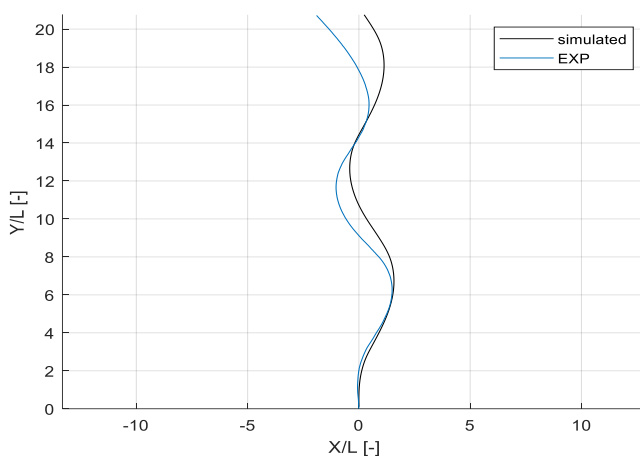

Source: *Own study.*

Figure 22. *20°/20° Zig-zag manoeuvre port first.*

Source: *Own study.*

Figure 23. *Modified 10°/20° Zig-zag manoeuvre starboard first.*

Source: Own study.

Figure 24. *The trajectory of modified 10°/20° Zig-zag manoeuvre starboard first.*

Source: Own study.

The summarised results for all zig-zag manoeuvres and turning circle has been presented in table 5. They are for comparison purposes of both experimental data, where available, and simulation generated indices.

Table 5. *Comparison of selected manoeuvring indices*

Manoeuvre	Index	Experimental	Simulation
Zig-zag 10°/10° starboard first	1st OSA	-	12.5
	2nd OSA	-	8.7
Zig-zag 10°/10° port first	1st OSA	-	8.5
	2nd OSA	-	12.4
Zig-zag 20°/20° starboard first	1st OSA	-	30.5
	2nd OSA	-	20
Zig-zag 20°/20° port first	1st OSA	-	20.7
	2nd OSA	-	28.3
Zig-zag 10°/20° starboard first	1st OSA	19	21
	2nd OSA	19	11
	Ad	2.7	2.79

	Td	2.2	1.94
	D	1.2	1.2
Turning to starboard 35°	u_S	1.88	1.82
	v_S	-0.43	-0.54
	r_S	0.048	0.051

Source: *Own study.*

6. Conclusions

The set of hydrodynamic derivatives can be obtained in one of many ways, which gives room for comparison. Since all mathematical models of ship manoeuvrability are designed to, nomen omen, model an existing or imaginative, yet realistic, object, it is the difference between the full-scale trials that give the best index of accuracy of the said model.

Because of that a model of a research vessel was established, comprising of a pre-existing MMG mathematical frame and initially supplied with values obtained from semi-empirical approximative formulas. Such a model referred to in the partial results as a Y&M differ noticeably from experimental data received from sea trials of mv Nawigator XXI.

It could be further argued that the approximation received via the employment of the regression formulas provides a sufficient and satisfactory approximation, especially when regarding an initial design of a vessel or when her manoeuvring capabilities are unknown. That notwithstanding, the Y&M model was established as a convenient starting point for further tuning of the hydrodynamic derivatives so that the manoeuvring indices mentioned by IMO resolution (IMO, 2002) could be minimalised.

Since the tuning was carried out over a set of 16 coefficients it is to be noted that to execute a mathematically proper optimisation by brute force algorithm, a number of the iterations would reach several alterations per coefficient to the power of 16. To omit this gargantuan amount of calculation a sensitivity study was carried out and the coefficients were grouped in subsets of up to four. Then a semi optimisation algorithm was executed, aiming to minimalize the summarised error of the simulated indices.

The algorithm was looped until the results received were satisfactory. Interestingly, the model based on the Taylor series approximation can be only as accurate as of the order of the series expansion. The received set, labelled as N7, is presented in the paper and provides results that accurately reflect the actual manoeuvring capabilities of the investigated ship. The simulation model used a set of approximated parameters, all of which has been derived from fellow studies and provided in the paper. A full set of hydrodynamic coefficients, along with said parameters is provided within the paper and can be used for successful simulation of manoeuvres of the investigated ship.

References:

Abkowitz, M.A. 1964. Lectures on ship hydrodynamics – steering and manoeuvrability. Lyngby, Denmark.

Hajizadeh, S., Seif, M.S., Mehdigholi, H. 2016. Determination of ship manoeuvring hydrodynamic coefficients using system identification technique based on free-running model tests. Scientia Iranica, B, 23(5), 2154-2165.

Kołodziej, R., Hoffman, P. 2021. Numerical estimation of hull hydrodynamic derivatives in ship manoeuvring prediction. Polish Maritime Research, 2(110), 28, 46-53.

Ogawa, A., Koyama, T., Kijima, K. 1977. MMG report-I, on the mathematical model of ship manoeuvring. Bull Society of Naval Architects of Japan, 575, 22-28.

Pietrzykowski, Z., Borkowski, P., Wołejsza, P., Uriasz, J. 2012. Marine navigational decision support system. International symposium Information on Ships, Hamburg.

IMO. 2002. Standards for ship manoeuvrability. Resolution MSc., 137(76).

SNAME. 1950. Nomenclature for treating the motion of a submerged body through a fluid. Technical and Research Bulletin, No 1-3.

Yasukawa, H., Yoshimura, Y. 2014. Introduction of MMG standard method for ship manoeuvring predictions. Journal of Maritime Science and Technology, 20, 37-52.

Yoshimura, Y., Masumoto, Y. 2012. Hydrodynamic Force Database with Medium-High Speed Merchant Ships Including Fishing Vessels and Investigation into a Manoeuvring Prediction Method. Journal of the Japan Society of Naval Architects and Ocean Engineers, 14, 63-73.