
Assessment of the Sector of Public Vocational Universities in Poland from the Point of View of their Efficiency

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Mariusz Pyra¹, Mieczysław Adamowicz²

Abstract:

Purpose: The aim of the article is to present the results of efficiency studies using the Data Envelopment Analysis (DEA) method, as well as the Tobit model using examples of 29 public vocational universities operating in Poland.

Design/Methodology/Approach: The research methodology is the DEA model in variants CCR-0, CCR-I, BCC-0, BCC-I. These are models based on the radial reduction of inputs in an input-oriented model. Taking into account the degree of differentiation of ineffective DMUs, it was decided to use the SE CCR-I model. Continuing the analysis and drawing conclusions from the above-mentioned DEA methods, the Tobit model was used in the next stage.

Findings: From the point of view of the aim of the research, the SE DEA CCR-I model turned out to be the most useful. First, thanks to the removal of the limitation related to the maximum value of the evaluation of the efficiency of the studied DMU, a broader and more precise characterization of the efficient facilities was obtained. It became possible to distinguish among facilities classified as efficient, going beyond the mere statement that in their case there is efficiency at level 1. Second, the model was not so much focused on differentiating inefficient facilities, showing a picture of facilities with efficiency scores above 1, without compromising the quality of evaluating facilities with efficiency below 1.

Practical Implications: The DEA method basically allows one to determine whether the object studied converts its expenditures into results in an optimal way, i.e., it allows for determining the effectiveness of the object under study. Determining the effectiveness of the functioning of public vocational universities is important for the future of this sector.

Originality/value: The application of the DEA method to research the effectiveness of universities is not as often used as in the case of entities operating for profit. This is due to the complexity of the selection of inputs and outputs. The most common analyses using DEA methods relate to academic universities. Hence the need to extend such research to public vocational universities, that matter in the transfer of human resources to the labor market, is very important.

Keywords: Efficiency assessment, education sector, model DEA.

JEL classification: C14, I23.

Paper Type: Research article.

¹Ph.D., Faculty of Economic Sciences, Pope John Paul II State School of Higher Education in Biala Podlaska, Poland, m.pyra@dydaktyka.pswbp.pl;

²Prof., the same as in 1, adamowicz.mieczyslaw@gmail.com;

1. Introduction

Efficiency of organizations' functioning is becoming increasingly important. Individual areas of activities are analyzed in order to indicate those efficient or those that need improvement. The applied methods of measuring efficiency are used in research, including hospitals (Jacobs, 2001), banks (Berger and Humphrey, 1992; Yue, 1992), farms (Alene, Manyong, and Gockowski, 2006), military facilities (Sun, 2004), non-profit organizations (Joo, Stoeberl, and Kwon, 2007), and universities (Bates, 1993; Chakraborty, Biswas, and Lewis, 2001; Feng, Lu, and Bi, 2004; Johns, 2006).

Efficiency is one of the key concepts in economics. Its assessment concerns the allocation of scarce resources between alternative uses. This takes place not only in economic enterprises, but also in public sector entities (Baran and Pietrzak, 2015). The situation in higher education, including public vocational universities, related to increasing competition, forces them to constantly improve the efficiency of their functioning (Ćwiakła-Małys and Nowak, 2009). And here comes the issue of measuring efficiency, it very often causes a lot of controversy. It is emphasized in the literature that it is difficult to indicate such measures that would not raise methodological doubts (Pietrzak, 2015). This is related to the fact that the area of university activity largely concerns the sphere of intangible assets and only in a certain sphere of material values (Morawski, 1999). However, despite the specific activity of universities, a measurement of their efficiency, as well as analyzes in this field, should be conducted. The more that the entities are subsidized by the state. Thus, the efficiency and rationality of spending public funds is important (Wolszczak-Derlacz, 2013).

When analyzing the literature on the subject, economic efficiency is determined in many ways, efficiency is the maximization of production resulting from the proper allocation of resources, with given limitations of supply (costs incurred by producers) and demand (consumer preferences) (Kamerschen, McKenzie and Nardinelli, 1991), such use of resources that contributes to the achievement of the maximum level of satisfaction possible with the given inputs and technologies (Samuelson and Nordhaus, 2004). Efficiency can also be understood as an assessment of the obtained benefits (effects) and the degree of use of the available inputs (Cooper, Seiford and Tone, 2007). Research on the efficiency of the named technical input was conducted by M.J. Farrell, he proved its connections to the economic theory, operations research, and management science (Farrel, 1957). Research in this area was organized by Førsund and Sarafoglou (2002), arranging information on technical efficiency based on Farrel's achievements.

Any assessment of the efficiency of the functioning of a university requires the ability to measure it. Various measurement methods are used, including parametric and non-parametric. Parametric methods (e.g., SFA - *Stochastic Frontier Approach*, DFA - *Distribution Free Approach*, TFA - *Thick Frontier Approach*) are applicable

for models with a strictly defined structure and require assumptions regarding the form of the production function. In the case of universities, it is more difficult to define the mathematical function of production and its interpretation becomes troublesome. Consequently, non-parametric methods, characterized by higher flexibility, have greater application value. It was shown by empirical studies of 40 universities located in Thailand. They were analyzed using the DEA and Tobit model. It allowed for the identification of areas requiring improvement (Liu, Wongchai and Peng, 2012). It is not an easy task, but it can provide interesting conclusions regarding the functioning of the surveyed entities. The aim of the article is to present the results of efficiency studies using the Data Envelopment Analysis (DEA) method, as well as the Tobit model, as exemplified by 29 public vocational universities operating in Poland.

2. Materials and Methods

2.1 Methods Used

The DEA method is one of the non-parametric methods of testing the efficiency of facilities. In 1978, the authors of the DEA method, i.e., Charnes, Cooper and Rhodes, based on the concept of productivity formulated by Debreu and Farrell, defining the productivity measure as the quotient of a single effect and a single input, implemented it in a multidimensional situation. In this case, you have more than one input and more than one effect.

The DEA method has several features in common with the production function. With both methods it is possible to analyze the relationship between outputs and inputs. The result of this analysis is a dependence curve of effects and inputs. But for production functions, knowledge of functional relationships between inputs and outputs is required. Unlike the DEA method, the non-parametric nature of which exempts you from this type of requirement, because the efficiency curve is defined not based on parameter estimation, but on based on empirical data of each facility. Thus, we are dealing here with relative efficiency, i.e. measured in relation to other facilities (Pasewicz and Świtłyk, 2010).

The CCR-O model (Charnes, Cooper, Rhodes model - a model of technical efficiency oriented at maximizing the effects at unchanged inputs and without considering the effect of scale) is the standard DEA model, along with the CCR-I model (input oriented). The use of the CCR model makes it possible to:

- establish efficient facilities and inefficient facilities (in terms of Farrell's efficiency),
- rank inefficient facilities,
- establish optimal technologies and benchmarking formulas for inefficient facilities,
- establish surpluses of inputs and deficits of results in inefficient facilities,

- establish economies of scale type,
- establish "target" (optimal) technologies for inefficient facilities,
- establish the structure of optimal technologies,
- establish characteristics of the sensitivity of the task to changes in inputs and results (dual ratings) and the weights of the facilities' function (simplex criteria) (Guzik, 2009).

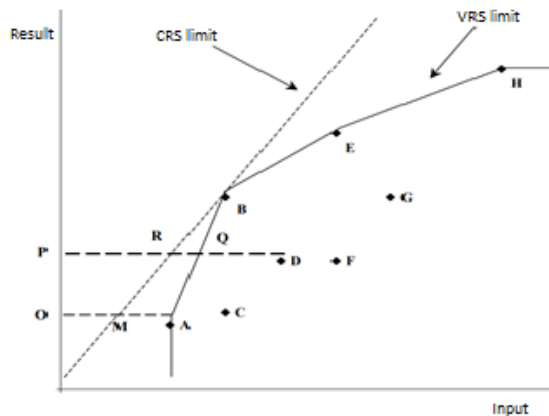
The BCC model is a modification of the CCR model first described in 1984 by R.D. Banker, A. Charnes and W.W. Cooper. The model could also be oriented towards results (BCC-O) and inputs (BCC-I). In fact, the modification consisted in adding an additional constraint condition in the form:

$$\sum_{j=1}^n \gamma_{jo} = 1$$

The introduction of the modification made it possible to consider variants with variable returns to scale (VRS - *Variable Returns to Scale*). In the classic CCR model, the postulate of the convexity of the set of production possibilities was adopted (Wagenvoort and Schure, 1999), which leads to the creation of a limited optimal efficiency associated with the constant effect of scale (the limit being the tangent radius to the VRS limit, passing through the origin of the coordinate system - Figure 1).

However, thanks to the above modification, in the BCC-I model, the VRS boundary is formed, it is always concave (Thanassoulis, 1999), with the point of contact with the radius (CRS limit), which is the unit's optimum point. There is a purely technical efficiency as well as in terms of production scale. The remaining points marked in Figure 1, included in the VRS curve, are efficient, however, they are not efficient from the point of view of the scale effect. In the CCR-I model, the CRS curve contains efficient facilities that maintain a constant scale effect, using the least inputs to produce a unit result, while operating in the optimal scale region. In the BCC-I model, the efficiency limit allows variable economies of scale (section AB - increasing economies of scale, sections BE and EH - decreasing economies of scale).

From the point of view of the BCC-I model, the facility at point B will be efficient and uses the least inputs to generate a unit result, however, it does not have to operate in the optimal area of economies of scale. Thus, the optimum of the BCC-I model is not better than the optimum of the CCR-I model. An example can be facility A in Figure 1, as it lies on the border of the VRS efficiency. The BCC-I model considers it efficient, while the CCR-I model considers it inefficient because it is not in the optimal area of economies of scale. Facility A is characterized by the so-called *Pure Technical Efficiency* (PTE), calculated as the distance between A and the VRS limit. The remaining facilities below both limits are not considered efficient in any of the models.

Figure 1. Efficiency limit in the CCR and BCC models

Source: Domagała A., *Application of the Data Envelopment Analysis method to study the efficiency of European stock exchanges. Doctoral dissertation, UE in Poznan, Poznan 2009, p. 64.*

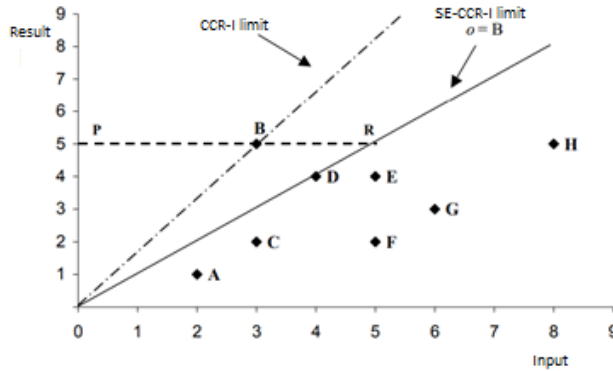
To sum up, the BCC-I model distinguishes pure technical efficiency and efficiency of scale from global technical efficiency. This enables detection in the tested facilities:

- whether the facility is characterized by pure technical efficiency and is in the optimal area of economies of scale, i.e., in the area of permanent economies of scale;
- whether the facility is characterized by pure technical efficiency, but does not operate in the optimal area of economies of scale;
- is the facility inefficient both technically and in terms of scale (Domagała, 2009).

The CCR-I model focuses on global technical efficiency (with a constant scale effect - CRS), and the BCC-I model on pure technical efficiency (with a variable effect of scale - VRS).

Another modification of the CCR model (CCR-I) is the Super Efficiency (SE) model. Simply put, the limitation for the efficiency of the tested facilities is removed, saying that it cannot be greater than 1. This means that the conditions limiting the tested facility are excluded, and the limits of efficiency are constructed based on other facilities. Consequently, just as in the CCR-I model it was possible to accurately rank inefficient facilities (they were precisely distinguished), in the SE-CCR model it becomes possible to accurately rank the efficient facilities. This modification was proposed by Andersen and Petersen in 1993. This model distinguished facilities that were super-efficient or over-efficient, and therefore were "more efficient" than the best of the other respondents. The efficiency ratio of such a facility is then greater than one (Figure 2).

Figure 2. Efficiency limit in the CCR-I and SE-CCR-I models.



Source: Domagala A., Application of the Data Envelopment Analysis method to study the efficiency of European stock exchanges. Doctoral dissertation, UE in Poznan, Poznan 2009, p. 68.

Using facility B from Figure 2, this facility is at the efficiency limit of the CCR-I model. On the other hand, the efficiency limit resulting from the SE-CCR-I model passes through point D. It arises when facility B is excluded from the set of DMU facilities. Facility B turned out to be "more efficient" (it is above the SE-CCR-I limit) than facility D, being the best facility in the set of facilities (after excluding B facility). Consequently, facility B will be given a value greater than 1 because efficiency is measured as the ratio of the distance from the limit to the Y axis to the distance of the test point from that axis. This model can also be presented by writing down its canonical form, which looks like this (Andersen and Petersen, 1993):

$$\min \theta_{SE-CCR-I, \theta}$$

with restrictions:

$$(a) \theta_{SE-CCR-I, \theta} x_{i\theta} = \sum_{j=1 \neq \theta}^n x_{ij} \lambda_{j\theta} + s_{i\theta}^{-}, \quad i = 1, \dots, m$$

$$(b) y_{r\theta} = \sum_{j=1 \neq \theta}^n y_{rj} \lambda_{j\theta} - s_{r\theta}^{+}, \quad r = 1, \dots, s$$

$$(c) \lambda_{j\theta}, s_{i\theta}^{-}, s_{r\theta}^{+} \geq 0, \quad \forall j, i, r$$

Both the CCR, BCC and SE-CCR models are radial models. They result from the assumptions of the CCR-I model related to proportionality, i.e., the radial reduction of inputs in the input-oriented model. In the CCR model (and all radial), the slacks are treated marginally - in the CCR-I model a facility is considered efficient when its result is 1 and all slacks are 0, while when they are not equal to zero, poor efficiency. However, the facility is still considered efficient, as the mentioned slacks do not ultimately affect the value of the efficiency indicator. This also applies to

inefficient facilities. It has to do with the distance measure. In radial models, it is equal to the distance from the point of intersection of the efficiency limit and the line derived from the center of the coordinate system towards the tested facility. In this way, the principle of proportional cost reduction is maintained. Many variants of the DEA model have been described in the literature, they reject this assumption and thus are classified as non-radial models. They were not used in this study.

The Tobit model was proposed in 1958 by J. Tobin as a way of describing the censored dependent variable. In a sense, it is a generalization of the Tobit model, because of including regression analysis in the description of the observed part of the dependent variable. J. Tobin's original model is currently classified as type I (there are extended models, classified as type II and III). The Tobit model has been widely used in practice - e.g., for the analysis of spending on luxury goods or for labor market analysis. It is also called the selection model.

Tobit models are used to describe the dependent variable subject to censorship or non-random selection with censorship. The key elements for the model are (Kostrzewska, 2011):

- Y^* - a latent variable, i.e., a variable whose values are usually not observed or are observed depending on certain conditions called observability criteria;
- Y - a truncated, censored or non-randomly selected variable taking the same values as Y^* if observed, or arbitrary values when Y^* is not observable / intentionally ignored. The conventional values are thresholds a or c ;
- n - number of elements in the sample;
- Z^* - variable on which the observability of the variable Y^* depends. It does not have to be directly observable. It is assumed that the variable created based on the hidden variable Z^* can be a dichotomous variable, truncated, censored or a variable that is not a limited variable (the variable values are not available). The designation I for a binary variable is taken, otherwise the designation Z is used:

$$I_i = \begin{cases} 1 & \text{if } z_i^* > a \\ 0 & \text{if } z_i^* \leq a \end{cases} \text{ or } z_i = \begin{cases} z_i^* & \text{if } z_i^* > a \\ a \text{ or none} & \text{if } z_i^* \leq a \end{cases}$$

where:

a - value for variable Z^* .

The type I Tobit model is used to describe the censored dependent variable and can be represented as:

$$y_i = \begin{cases} y_i^* & \text{if } y_i^* > c, \quad i = 1, \dots, n_1 \\ c & \text{if } y_i^* \leq c, \quad i = (n_1 + 1), \dots, n, \end{cases}$$

where:

$y_i^* = X_i\beta + u_i$ – regression equation;

$X_{i-(k+1)}$ – element vector (levels) of values of explanatory variables from the set $X = \{X_1, \dots, X_k\}$, recorded for the i observation (facilities), where the first element of vector X_i is equal to one;

$\beta = (\beta_0, \beta_1, \dots, \beta_k)$ – vector (vertical) of the regression equation parameters;

u_i – the equation random components, independent and derived from the same distribution with the expected value equal to 0 and the constant variance σ_{u_i} . Most often it is assumed that the random components are subject to the normal distribution, as was assumed in this study.

The Tobit model for left-hand censoring, with a threshold value of $c = 0$ and assuming a normal distribution of random components, is called the standard Tobit model. The following software was used to perform the calculations: MS Excel, R package with appropriate libraries.

2.2 Empirical Data

State higher vocational schools began to be established in the last years of the 20th century under the Act on Higher Vocational Schools (Journal of Laws 1997, No. 96, item 590), adopted on June 26, 1997. The development of public vocational universities in Poland was related to the pan-European process of transformation of higher education, known as the Bologna process (Bologna Declaration of 19 June 1999). Currently, state-owned higher vocational schools constitute a significant sector of higher education in Poland. They are in different parts of the country and there are currently 34 such centers. The main axis of their activity is didactics, educating tens of thousands of students. Scientific activity is also conducted through specialized laboratories and research centers. Public vocational universities are also an important entity influencing local and regional development, mainly due to the cooperation with local government facilities and business. Taking the above into account, it seems reasonable to evaluate the sector of state-run higher vocational schools from the point of view of their efficiency. Table 1 lists the universities covered by the research together with the DMU markings assigned to them.

Table 1. List of the studied universities with their assigned DMU numbers

DMU 1	State Higher Vocational School Angelus Silesius in Wałbrzych	DMU 11	Szymon Szymonowic State University in Zamość	DMU 21	Masovian Public University in Płock
DMU 2	Hipolit Cegielski State Higher Vocational School in Gniezno	DMU 12	State Higher Vocational School Witelon in Legnica	DMU 22	State Higher Vocational School in Racibórz
DMU 3	Jan Amos Komieński State School of Higher Vocational Education in Leszno	DMU 13	State Higher Vocational School in Chełm	DMU 23	Stefan Batory State University in Skierniewice

DMU 4	Jan Grodek State University in Sanok	DMU 14	Ignacy Mościcki State Vocational University in Ciechanów	DMU 24	State Higher Vocational School in Tarnów
DM U5	President Stanisław Wojciechowski State Higher Vocational School in Kalisz	DM U15	State Higher Vocational School in Elbląg	DM U25	State Higher Vocational School in Wałcz
DM U6	prof. E. Szczepanik State Higher Vocational School in Suwałki	DM U16	State Higher Vocational School in Głogów	DM U26	State Vocational University in Włocławek
DMU 7	prof. Stanisław Tarnowski State Vocational University in Tarnobrzeg	DMU 17	State Higher Vocational School in Konin	DMU 27	Podhale State Vocational University in Nowy Targ
DMU 8	Captain Witold Pilecki State University of Małopolska in Oświęcim	DMU 18	State Higher Vocational School in Koszalin	DMU 28	Karkonosze State School of Higher Education in Jelenia Góra
DM U9	Carpathian State University in Krosno	DM U19	State Higher Vocational School in Nowy Sącz	DM U29	State Higher School of Pope John Paul II in Biała Podlaska
DM U10	State Stanisław Staszic University in Piła	DM U20	State Higher Vocational School in Nysa		

Source: own study.

In the case of resources from the conceptual set of variables, the following were selected: the number of teaching / research staff, real estate (teaching facilities expressed in m²) and the number of branches of the university. Due to the update of databases (POL-on, RAD-on system) and their adaptation to the requirements of the Act of 20 July 2018 - Law on Higher Education and Science, the data on the number of teaching / research staff was not available. An assumption was made about the correlation between the number of teaching / research staff and the number of students. Thanks to this, it is possible to correct the number of employees based on the analysis of data for the 2017/2018 academic year and data for the 2018/2019 academic year (number of students). It was assumed that each university maintains the ratio of the number of teaching staff to the number of students at the level of the previous academic year (i.e., 2017/2018).

On the other hand, in the case of properties that were to reflect the teaching facilities, their usable area was adopted as a measure based on data from the POL-on / RAD-on system. In the case of effects, three variables were selected from the conceptual set: number of students, number of graduates, and the educational offer. The data was obtained from the POL-on / RAD-on system and the Central Statistical Office. Numbers of students and graduates were assumed as in 2018, in accordance with the methodology of the Central Statistical Office. The didactic offer was accepted as in 2019, according to POL-on. Part of the data was verified by reviewing and analyzing the educational offer of selected universities. All faculties and specializations at a given university were considered. Variables describing both inputs and effects were

developed and verified with statistical methods in order to eliminate variables with excessive correlation.

3. Results of Empirical Research

3.1 DEA CCR-O and CCR-I models

Taking into account the specific situation related to the availability of data and the decision to extrapolate data on one of the inputs, based on the relational dependence of the said input (number of teaching / research staff) to one of the effects (number of students), it is justified to consider the possibility of using a model oriented to inputs instead of an effect-oriented model. The table shows the results of both models, for the CCR-O and CCR-I forms. As shown in the table below, in practice, the equivalence of the models is postulated in the literature, which means that the determined efficiency in both models is identical. Consequently, a change in the approach to the application of the input-oriented model does not change the results of the efficiency assessment in the case of the application of the basic CCR model.

Table 2. *Summary of the results of the CCR-O and CCR-I models*

DMU	CCR-O	CCR-I	Lambda				
			DMU 2	DMU 6	DMU 7	DMU 22	DMU 26
DMU 1	0.966154703	0.966154703	0	0	1.400912	0.255204	0
DMU 2	1	1	1	0	0	0	0
DMU 3	0.808061973	0.808061973	0	0.058802	0.920116	0.714380	0
DMU 4	0.573558325	0.573558325	0	0.389123	0.878562	0	0
DMU 5	0.775584756	0.775584756	0	0.521114	2.944674	0.189282	0
DMU 6	1	1	0	1	0	0	0
DMU 7	1	1	0	0	1.000000	0.000000	0
DMU 8	0.995091045	0.995091045	0	0	2.526665	0.030850	0
DMU 9	0.806684768	0.806684768	0	0	2.894153	0.172758	0
DMU 10	0.718584985	0.718584985	0	0.422219	1.029198	0	0
DMU 11	0.743864307	0.743864307	0	0	1.001306	0.075314	0
DMU 12	0.65007659	0.65007659	0	0.761087	0.495881	0.912510	0
DMU 13	0.575087768	0.575087768	0	0	1.641721	0.383799	0
DMU 14	0.795055172	0.795055172	0	0.024308	1.087075	0.506066	0
DMU 15	0.626412847	0.626412847	0	0.197364	0.904045	0.334886	0
DMU 16	0.658545912	0.658545912	0	0	0.594994	0.245018	0
DMU 17	0.730871995	0.730871995	0	0.641443	1.199964	0	0
DMU 18	0.566021881	0.566021881	0	0	0.863693	0.063090	0
DMU 19	0.799401012	0.799401012	0	0.418210	3.654037	0.161433	0
DMU 20	0.754890671	0.754890671	0	0	1.693772	0.526491	0
DMU 21	0.875770781	0.875770781	0	0.346092	2.059489	0.193449	0
DMU 22	1	1	0	0	0	1	0
DMU 23	0.908612178	0.908612178	0	0	0.436114	0.219820	0.504682

DMU 24	0.917816764	0.917816764	0	0	4.975852	0.352477	0
DMU 25	0.556561086	0.556561086	0	0.529412	0	0	0
DMU 26	1	1	0	0	0	0	1
DMU 27	0.638657005	0.638657005	0	0.084032	1.951484	0.411771	0
DMU 28	0.498669314	0.498669314	0	0	1.166423	0	0
DMU 29	0.6811635	0.6811635	0	0.474717	1.891060	0.285593	0

Source: Own study.

The conducted research shows that DMU 2, 6, 7, 22 and 26 were found to be efficient in the entire group of DMUs tested, in both models. This suggests that the efficiency of the facilities is the best in the entire study group in terms of global technical efficiency.

3.2 DEA BCC-O and BCC -I models

Using the BCC model (pure technical efficiency) in the input-oriented variant (Table 3) and the effect-oriented variant (Table 4), with a variable scale effect, the results of the technical efficiency assessment of the studied DMUs are slightly different than in Table 2. There are already 12 efficient DMUs and only 5 are no efficient. It perfectly illustrates the difference between global technical efficiency and pure technical efficiency of the surveyed facilities among the universities covered by the analysis.

Table 3. Summary of the results of the BCC-I model

DMU	BCC-I	Lambda											
	Efficiency	DMU 2	DMU 5	DMU 6	DMU 7	DMU 8	DMU 12	DMU 19	DMU 21	DMU 22	DMU 24	DMU 25	DMU 26
DMU 1	0.973748349	0	0.000000	0.000000	0.329058	0.416667	0.000000	0.000000	0.000000	0.240174	0.000000	0.000000	0.014102
DMU 2	1	1	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
DMU 3	0.847056644	0	0.000000	0.190379	0.126894	0.000000	0.000000	0.000000	0.000000	0.519957	0.162770	0.000000	0.000000
DMU 4	0.603186194	0	0.000000	0.330667	0.523183	0.000000	0.000000	0.000000	0.130613	0.000000	0.015537	0.000000	0.000000
DMU 5	1	0	1.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
DMU 6	1	0	0.000000	1.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
DMU 7	1	0	0.000000	0.000000	1.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
DMU 8	1	0	0.000000	0.000000	0.000000	1.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
DMU 9	0.881588559	0	0.000000	0.000000	0.000000	0.497922	0.000000	0.000000	0.000000	0.001662	0.277674	0.000000	0.222742
DMU 10	0.831328783	0	0.000000	0.302833	0.303366	0.000000	0.000000	0.000000	0.284128	0.000000	0.000000	0.000000	0.109673
DMU 11	0.744269791	0	0.000000	0.000000	0.876997	0.049173	0.000000	0.000000	0.000000	0.073831	0.000000	0.000000	0.000000
DMU 12	1	0	0.000000	0.000000	0.000000	0.000000	1.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
DMU 13	0.626549659	0	0.000000	0.000000	0.000000	0.369357	0.000000	0.000000	0.000000	0.304514	0.075233	0.000000	0.250896
DMU 14	0.834229982	0	0.000000	0.137232	0.386006	0.000000	0.000000	0.000000	0.000000	0.330392	0.146370	0.000000	0.000000
DMU 15	0.664006241	0	0.000000	0.432467	0.239538	0.237853	0.000000	0.000000	0.074474	0.000000	0.000000	0.000000	0.015669
DMU 16	0.772046742	0	0.000000	0.127691	0.780547	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.091762	0.000000
DMU 17	0.936609235	0	0.000000	0.501800	0.000000	0.000000	0.049730	0.235146	0.000000	0.000000	0.000000	0.000000	0.213324
DMU 18	0.608136691	0	0.000000	0.007738	0.948877	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.043385	0.000000

DMU 19	1	0	0.000000	0.000000	0.000000	0.000000	0.000000	1.000000	0.000000	0.000000	0.000000	0.000000	0.000000
DMU 20	0.790754829	0	0.000000	0.000000	0.229208	0.064284	0.000000	0.000000	0.000000	0.451627	0.254880	0.000000	0.000000
DMU 21	1	0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	1.000000	0.000000	0.000000	0.000000	0.000000
DMU 22	1	0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	1.000000	0.000000	0.000000	0.000000
DMU 23	0.942719053	0	0.000000	0.141271	0.156869	0.112206	0.000000	0.000000	0.000000	0.032100	0.000000	0.000000	0.557553
DMU 24	1	0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	1.000000	0.000000	0.000000
DMU 25	1	0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	1.000000	0.000000
DMU 26	1	0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	1.000000
DMU 27	0.740134832	0	0.000000	0.000000	0.005661	0.123858	0.000000	0.000000	0.783711	0.000000	0.005238	0.000000	0.081532
DMU 28	0.506364412	0	0.000000	0.000000	0.895221	0.104779	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
DMU 29	0.899359849	0	0.624375	0.222222	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.153403

Source: Own study.

Table 4. Summary of the results of the BCC-O model

DMU	BCC-O	Lambda											
	Efficiency	DMU 2	DMU 5	DMU 6	DMU 7	DMU 8	DMU 12	DMU 19	DMU 21	DMU 22	DMU 24	DMU 25	DMU 26
DMU 1	0.9770155	0	0.0000	0.0000	0.2938	0.4387	0.0000	0.0000	0.0000	0.2435	0.0000	0.0000	0.0238
DMU 2	1	1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
DMU 3	0.8855351	0	0.0000	0.5065	0.0000	0.0000	0.0000	0.0000	0.0000	0.2243	0.2228	0.0000	0.0462
DMU 4	0.6852389	0	0.0000	0.4229	0.0000	0.0000	0.0000	0.0000	0.3524	0.0000	0.0727	0.0000	0.1517
DMU 5	1	0	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
DMU 6	1	0	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
DMU 7	1	0	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
DMU 8	1	0	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
DMU 9	0.9373623	0	0.0000	0.0000	0.0000	0.3162	0.0000	0.0000	0.0000	0.0000	0.3735	0.0000	0.3101
DMU 10	0.8684536	0	0.0000	0.3094	0.0955	0.0000	0.0000	0.0000	0.4223	0.0000	0.0000	0.0000	0.1727
DMU 11	0.7585704	0	0.0000	0.0000	0.6205	0.2587	0.0000	0.0000	0.0000	0.0818	0.0000	0.0000	0.0388
DMU 12	1	0	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
DMU 13	0.7692307	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.3657	0.0000	0.6342
DMU 14	0.8441199	0	0.0000	0.1963	0.2476	0.0000	0.0000	0.0000	0.0000	0.3388	0.2171	0.0000	0.0000
DMU 15	0.7511919	0	0.0000	0.4922	0.0000	0.0000	0.0269	0.1352	0.0000	0.0000	0.0798	0.0000	0.2657
DMU 16	0.6638530	0	0.0000	0.0000	0.5125	0.1042	0.0000	0.0000	0.0000	0.3598	0.0232	0.0000	0.0000
DMU 17	0.9801200	0	0.0000	0.4224	0.0000	0.0000	0.1075	0.2345	0.0000	0.0000	0.0000	0.0000	0.2355
DMU 18	0.6099725	0	0.0000	0.0187	0.5486	0.2500	0.0000	0.0000	0.0594	0.0000	0.0000	0.0000	0.1230
DMU 19	1	0	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
DMU 20	0.7992419	0	0.0000	0.0000	0.0357	0.0234	0.0000	0.0000	0.0000	0.5445	0.3962	0.0000	0.0000
DMU 21	1	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000
DMU 22	1	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000
DMU 23	0.9546917	0	0.0000	0.1613	0.0907	0.1238	0.0000	0.0000	0.0217	0.0000	0.0000	0.0000	0.6023
DMU 24	1	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
DMU 25	1	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000
DMU 26	1	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
DMU 27	0.8483820	0	0.0000	0.0443	0.0000	0.0000	0.0610	0.5428	0.0000	0.0000	0.0000	0.0000	0.3517
DMU 28	0.5662501	0	0.0000	0.0000	0.2748	0.7134	0.0000	0.0000	0.0000	0.0000	0.0116	0.0000	0.0000
DMU 29	0.955639	0	0.7105	0.0984	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1910

Source: Own study.

As shown by the results of research on the efficiency of the set of the studied DMUs, the classification (ranking) of DMUs looks slightly different in terms of global technical efficiency and pure technical efficiency (Table 5). Moreover, facilities characterized by pure technical efficiency, and not falling within the scope of economies of scale, are precisely those facilities that the CCR-I model deemed inefficient. A similar situation in this respect can be noticed in the results-oriented models (Table 6).

Table 5. Summary of the results of the BCC-I and CCR-I models

DMU	CCR-I	PTE	Scale	PTE+SCALE	PTE	Scale
		BCC-I				
DMU 1	0.966155	0.973748	0.992202	NO	NO	NO
DMU 2	1	1	1	YES	YES	YES
DMU 3	0.808062	0.847057	0.953965	NO	NO	NO
DMU 4	0.573558	0.603186	0.950881	NO	NO	NO
DMU 5	0.775585	1	0.775585	NO	YES	NO
DMU 6	1	1	1	YES	YES	YES
DMU 7	1	1	1	YES	YES	YES
DMU 8	0.995091	1	0.995091	NO	YES	NO
DMU 9	0.806685	0.881589	0.915035	NO	NO	NO
DMU 10	0.718585	0.831329	0.864381	NO	NO	NO
DMU 11	0.743864	0.74427	0.999455	NO	NO	NO
DMU 12	0.650077	1	0.650077	NO	YES	NO
DMU 13	0.575088	0.62655	0.917865	NO	NO	NO
DMU 14	0.795055	0.83423	0.953041	NO	NO	NO
DMU 15	0.626413	0.664006	0.943384	NO	NO	NO
DMU 16	0.658546	0.772047	0.852987	NO	NO	NO
DMU 17	0.730872	0.936609	0.780338	NO	NO	NO
DMU 18	0.566022	0.608137	0.930748	NO	NO	NO
DMU 19	0.799401	1	0.799401	NO	YES	NO
DMU 20	0.754891	0.790755	0.954646	NO	NO	NO
DMU 21	0.875771	1	0.875771	NO	NO	NO
DMU 22	1	1	1	YES	YES	YES
DMU 23	0.908612	0.942719	0.963821	NO	NO	NO
DMU 24	0.917817	1	0.917817	NO	YES	NO
DMU 25	0.556561	1	0.556561	NO	YES	NO
DMU 26	1	1	1	YES	YES	YES
DMU 27	0.638657	0.740135	0.862893	NO	NO	NO
DMU 28	0.498669	0.506364	0.984803	NO	NO	NO
DMU 29	0.681164	0.89936	0.757387	NO	NO	NO

Source: Own study.

Table 6. Summary of the results of the BCC-O and CCR-O models

DMU	CCR-O	PTE	Scale	PTE+SCALE	PTE	Scale
		BCC-O				
DMU 1	0.966155	0.977016	0.988884	NO	NO	NO
DMU 2	1	1	1	YES	YES	YES
DMU 3	0.808062	0.885535	0.912513	NO	NO	NO
DMU 4	0.573558	0.685239	0.837019	NO	NO	NO
DMU 5	0.775585	1	0.775585	NO	YES	NO
DMU 6	1	1	1	YES	YES	YES
DMU 7	1	1	1	YES	YES	YES
DMU 8	0.995091	1	0.995091	NO	YES	NO
DMU 9	0.806685	0.937362	0.86059	NO	NO	NO
DMU 10	0.718585	0.868454	0.82743	NO	NO	NO
DMU 11	0.743864	0.75857	0.980613	NO	NO	NO
DMU 12	0.650077	1	0.650077	NO	YES	NO
DMU 13	0.575088	0.769231	0.747614	NO	NO	NO
DMU 14	0.795055	0.84412	0.941875	NO	NO	NO
DMU 15	0.626413	0.751192	0.833892	NO	NO	NO
DMU 16	0.658546	0.663853	0.992006	NO	NO	NO
DMU 17	0.730872	0.98012	0.745696	NO	NO	NO
DMU 18	0.566022	0.609973	0.927946	NO	NO	NO
DMU 19	0.799401	1	0.799401	NO	YES	NO
DMU 20	0.754891	0.799242	0.944508	NO	NO	NO
DMU 21	0.875771	1	0.875771	NO	NO	NO
DMU 22	1	1	1	YES	YES	YES
DMU 23	0.908612	0.954692	0.951734	NO	NO	NO
DMU 24	0.917817	1	0.917817	NO	YES	NO
DMU 25	0.556561	1	0.556561	NO	YES	NO
DMU 26	1	1	1	YES	YES	YES
DMU 27	0.638657	0.848382	0.752794	NO	NO	NO
DMU 28	0.498669	0.56625	0.880652	NO	NO	NO
DMU 29	0.681164	0.955639	0.712783	NO	NO	NO

Source: own study.

Thus, the application of the BCC model allowed for a better classification of inefficient facilities, also with the distinction of PTE. However, there is still the problem of a more precise classification of efficient facilities. Because of the above observations and conclusions, it seems reasonable to use the SE CCR-I model. For the reasons already mentioned and the purpose of further research, attention will be focused only on the input-oriented model.

3.3 DEA SE CCR-I model in the CRS and VRS variants

In order to maintain the degree of differentiation of inefficient DMUs, it was decided to use the SE CCR-I model in two variants (Table 7).

Table 7. Basic parameters of the applied variants of the SE CCR-I model

Variant number	1	2
Description of the basic parameters	Superefficiency Structure – Convex Returns to scale (RTS) – constant Distance – Radial Oriented – Input	Superefficiency Structure – Convex Returns to scale (RTS) – VRS Distance – Radial Oriented – Input

Note: Distance – Radial – Proportional cost reduction.

Source: Own study.

The first option provides for constant economies of scale, and thus is a typical reflection of the assumptions of the CCR model. The results will therefore be comparable to those of CCR-I. However, the second option allows for variable scale effects, thus the results will be compared with the results of the BCC-I model. Table 8 presents a comparison of the results of the SE CCR-I (1) and (2) model and the CCR-I and BCC-I models.

Table 8. Comparison of the results of the SE CCR-I model and CRS and SE CCR-I and VRS with the results of the CCR-I and BCC-I models

DMU	DMU 1	DMU 2	DMU 3	DMU 4	DMU 5	DMU 6	DMU 7	DMU 8	DMU 9	DMU 10
SE CCR-I (1)	96.62 %	110.41 %	80.81%	57.36%	77.56%	158.41 %	153.73 %	99.51%	80.67%	71,86%
CCR-I	0.966155	1	0.808062	0.573558	0.775585	1	1	0.995091	0.806685	0,718585
SE CCR-I (2)	97.37 %	119.42 %	84.70%	60.32%	113.30 %	193.02 %	227.16 %	136.69 %	88.16%	83,13%
BCC-I	0.973748	1	0.847057	0.603186	1	1	1	1	0.881589	0,831329
	DMU 11	DMU 12	DMU 13	DMU 14	DMU 15	DMU 16	DMU 17	DMU 18	DMU 19	DMU 20
SE CCR-I (1)	74.39 %	65.01%	57.51%	79.51%	62.64%	65.85%	73.09%	56.60%	79.94%	75,49%
CCR-I	0.743864	0.650077	0.575088	0.795055	0.626413	0.658546	0.730872	0.566022	0.799401	0,754891
SE CCR-I (2)	74.43 %	112.60 %	62.65%	83.42%	66.40%	77.20%	93.66%	60.81%	104.36 %	79,08%
BCC-I	0.744	1	0.62655	0.83423	0.66400	0.77204	0.93660	0.60813	1	0,79075
	DMU 21	DMU 22	DMU 23	DMU 24	DMU 25	DMU 26	DMU 27	DMU 28	DMU 29	
SE CCR-I (1)	87.58 %	112.16 %	90.86%	91.78%	55.66%	116.45 %	63.87%	49.87%	68.12%	
CCR-I	0.875771	1	0.908612	0.917817	0.556561	1	0.638657	0.498669	0.681164	
SE CCR-I (2)	102.26 %	112.20 %	94.27%	big	105.13 %	285.05 %	74.01%	50.64%	89.94%	
BCC-I	1	1	0.94271	1	1	1	0.74013	0.50636	0.89936	

Note: big – a unit remains efficient with an arbitrarily large increase in input.

Source: Own study.

As expected, the application of the SE variant of the CCR-I model allowed for a more accurate identification of the efficient facilities and determination of individual efficiencies for them, allowing for their differentiation. Thus, the use of the SE model turns out to be much more useful from the point of view of ranking the studied DMUs due to the achieved efficiency, in the context of the research being conducted and the application of the next stage, consisting in the analysis of the impact of factors determining efficiency.

Comparison of the results of both variants of the SE CCR-I model with the results of the CCR-I and BCC-I models shows that in the case of inefficient facilities, the assessment has not changed and is identical. Therefore, the desired effect was achieved in the form of an assessment of the efficiency of the 29 DMUs studied, in a sufficiently accurate manner and allowing for their clear differentiation, which provides us with the basis for the further research.

3.4 Tobit Model

Continuing the analysis and drawing conclusions from the previous DEA methods, the second stage of the research was started, consisting in the use of the Tobit model with cut-off at the bottom (negative values for the efficiency of the studied universities are not observed). The efficiency determined by the DEA SE CCR-I method in the CRS variant was assumed as the dependent variable in the model (below). In the VRS variant, a non-numerical value of the evaluation was generated for DMU 24, so the data would also require cutting off in advance, which implies the need for using a different class of the Tobit model, and this is beyond the scope of the adopted research concept. What is more, research focused on models with constant scale effect.. As for independent variables, limited access to up-to-date and reliable data limited the set of variables to those already possessed and used in previous models. Consequently, the linear model is of the following form:

$$Y_i = \alpha + \beta_1 Z_i + \beta_2 P_i + \beta_3 O_i + \beta_4 S_i$$

where:

Y_i – efficiency, Z_i – back-office in the form of space, P_i – staff in the form of research / teaching staff, O_i – didactic offer, S_i – number of students, α – intercept, β_{1-4} – coefficients of independent variables.

Using the R environment and the "ggplot2", "GGally" "VGAM" packages - the vglm function from the VGAM package, the data was analyzed, and the TOBIT 1 model was solved.

The analysis of independent variables (Table 9) shows that the median for real estate is 19,278 m², for the number of employees it is 109, for the didactic offer it is 17, for the number of students it is 1,369, with the median for the efficiency equal to 7756 (77.56%).

Table 9. Overview of the Tobit Model Input Data 1

ID	Real estate	Number of research employees	Didactic offer	Number of students	Efficiency
Min.	2672	39	7	407	4987
1st Q.	9500	86	14	1080	6501
Median	19278	109	17	1369	7756
Mean	18367	128.1	17.21	1590	8322
3rd Q.	23560	153	21	2013	9178
Max.	56500	305	26	3891	15841

Note: efficiency expressed in% was multiplied by 100 and written as a number (100% = 10,000). This is the result of how the input file for R was developed.

Source: Own study.

However, the correlation analysis (Table 10) showed, as expected, a strong correlation between the number of students and the number of employees.

Table 10. Correlation matrix of independent variables

ID	Real estate	Employees	Offer	Students
Real estate	1.0000000	0.4117618	0.5058577	0.5942305
Employees	0.4117618	1.0000000	0.7225259	0.9033264
Offer	0.5058577	0.7225259	1.0000000	0.7930923
Students	0.5942305	0.9033264	0.7930923	1.0000000

Source: Own study.

The correlation between the efficiency values indicated by the model and the actual values from the set was (r) 0.8117525. Thus, the value of R^2 (coefficient of determination) was 0.6589421. This means that 65.89% of the variability (variance) of efficiency (the variable explained in this case) coincides with the correlations with the variables included in the model. It is an accepted value that reflects the extent to which the adopted predictors (independent - explanatory variables) describe the variability of the explained variable (efficiency) in relation to the set of input data (efficiency of 29 DMUs studied).

However, this suggests the possibility of conducting further, in-depth research focused on trying to select such predictors to achieve a greater fit of the model (then it is worth using the R^2 corrected value to assess the fit of the models and assess the benefits of introducing a new descriptive variable). However, this requires access to a wider amount of up-to-date data, which, as it was explained in the beginning of this article, is currently not possible.

Table 11. Results of the TOBIT 1 model solution (lower cut-off = 0)

Pearson residuals:	Min.	1st Q.	Median	3rd Q.	Max.
mu	-1.174	-0.6864	-0.1990	0.3593	3.476
loglink (sd)	-0.706	-0.6304	-0.4071	-0.1769	7.835
Coefficients:	Estimate	Std. Error	z value	Pr (> z)	
(Intercept):1	8441.23518	972.96221	8.676	< 2e - 16 ***	
(Intercept):2	7.33315	0.13133	55.839	< 2e - 16 ***	
Real estate	-0.17094	0.03568	-4.791	1.66e - 06 ***	
Employees	-69.52301	10.52523	-6.605	3.97e - 11 ***	

Offer	237.74106	83.47751	2.848	0.0044 **
Students	4.93036	1.04524	4.717	2.39e - 0.6 ***
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1. Names of linear predictors: mu, loglink (sd). Log-likelihood: -253.8105 on 52 degrees of freedom. Number of Fisher scoring iterations: 5. No Hauck-Donner effect found in any of the estimates.				

Note: The log-likelihood value for the model was - 253.8105 (it is mainly used to compare models with each other).

Source: Own study.

Analyzing the results of vglm from the VGAM package (Table 11), it can be seen that:

- the value of the constant (intercept) from the linear model was estimated at 8441.23518;
- the Z statistic shows that all explanatory variables have values above the mean assuming a normal distribution;
- the model indicates that the explanatory variables have different significance for shaping efficiency;
- the significance level for most variables is equal to $\alpha = 0$, with p higher than α (significance level). The p-value for the variables remains below 0.05, which suggests that the model is not biased.

The interpretation of the impact on the efficiency of the descriptive variables is as follows:

- an increase by one unit of a real estate resource causes a decrease by 0.17094 in the expected value of efficiency (i.e., 0.0017%);
- an increase by one resource unit in the form of employees causes a decrease by 69.52301 of the anticipated value of efficiency (i.e., 0.6952%);
- an increase by one unit of "effect" (from the point of view of the DEA model, this was an effect. In the TOBIT model, it appears as an explanatory variable, i.e., a factor determining the value of the efficiency predicted by the model) in the form of an offer increases the anticipated value of efficiency by 237.74106 (i.e., 2.3774%);
- an increase by one unit of the "effect" in the form of the number of students causes an increase by 4.93036 in the anticipated value of efficiency (i.e., 0.0493%).

The use of the Tobit model on the data set used to evaluate the efficiency with the DEA methods allowed for the creation of a linear model with a 65.89% fit, with the following form:

$$Y_i = 8441,23518 - 0,17094 Z_i - 69,52301 P_i + 237,74106 O_i + 4,93036 S_i$$

The fit of the model is not great, but it gives an overview of the impact of individual explanatory variables on the overall efficiency of the studied universities.

4. Discussion and Conclusions

When attempting to accurately assess the efficiency of universities included in the basic set - public vocational universities, operating in the Polish system of higher education, it is worth using a variant of the DEA model known as Super Efficiency (SE). Models of this group enable precise differentiation of efficient facilities. Thus, the characteristics of the examined entities, included in the group of efficient ones, are more precise from the point of view of individual assessments of the efficiency of the examined entities (DMU).

The application of the DEA-BCC model has shown that the models can provide more detailed information on inefficient facilities. However, there are still limitations for the maximum value of the efficiency assessment, which does not allow for differentiating the facilities assessed as efficient. Thus, in the case of focusing attention only on inefficient facilities, the use of this type of model would be more justified.

From the point of view of the aim and scope of the research, the SE DEA CCR-I model turned out to be the most useful. Firstly, thanks to the removal of the limitation related to the maximum value of the evaluation of the efficiency of the studied DMU, a broader and more precise characterization of the efficient facilities was obtained. It became possible to distinguish among facilities classified as efficient, going beyond the mere statement that in their case there is efficiency at level 1. Secondly, the model was not so much focused on differentiating inefficient facilities, showing an expanded picture of facilities with efficiency scores above 1, without compromising the quality of evaluating facilities with efficiency below 1.

The results of the efficiency studies with the DEA SE CCR-I model, considering the variable effect of scale (VRS) and SE DEA CCR-I with the assumption of the constant effect of scale (CRS), showed that the values obtained for individual DMUs are very similar in both cases. Thus, the significance of the variable scale effect does not seem to be so significant. The analysis of differences between the obtained efficiency assessments for the tested DMUs, in the case of variants of models focused on effects and on inputs, also showed that from the point of view of the purpose of the research, one can focus only on input-oriented models. Consequently, the SE DEA CCR-I CRS was selected as the base model in this study.

An attempt was made to analyze the impact of factors determining efficiency, taking into account the restrictions in the study related to access to reliable and up-to-date data, using the Tobit model censored from the bottom, for factors (explicit explanatory variables): space, staff in the form of the number of research / teaching employees, didactic offer, number of students, enabled the identification of those that have the greatest impact on the assessment of the efficiency of entities in the base group. Estimation of the parameters of the constructed Tobit model enabled the determination of its linear form. The model reached the level of fit (R^2) equal to

0.6589, with the significance of all explanatory variables higher than the threshold (i.e., 0.05) - the variables adopted for the model were therefore statistically significant. The model indicated that the variables contributing to the increase in the assessment of efficiency include the didactic offer and the number of students; and those contributing to the decrease in the efficiency rating include the number of employees (lower degree of impact) and area of the property (higher degree of impact). Consequently, it can be concluded that:

a) In order to increase efficiency, universities should focus on expanding their educational offer. Changes in this area translate best into the assessment of efficiency. It should be emphasized, however, that increasing the didactic offer requires a better use of resources in the form of employees and real estate or, where that is impossible, increasing the resources, which results in two variants of decision-making:

- one of the solutions to this problem may be the opening of a branch office, which will have a negative effect of increasing the real estate stock on the overall efficiency. Of course, assuming that the resources of teaching / research staff will allow it;

- Employing new employees with previously absent competences allows you to start a new field of study / expand the current educational offer. In this case, there will be a more noticeable negative effect in the area of efficiency (lower rating with each new employee) than in the case of increasing real estate resources;

- Optimizing the use of resources by eliminating less popular elements from the didactic offer and replacing them with new (with more than the liquidated ones) elements. This is related to a better use of resources, which may also be reflected in the ratio of the number of students per a researcher / teacher.

b) In order to increase efficiency, universities should focus on increasing their number of students. Changes in this respect translate positively into the assessment of efficiency. The scale and strength are lower than in the case of increasing the educational offer.

c) The outflow of students related to the liquidation of an element from the didactic offer can be more easily compensated than the loss of one element from the didactic offer (in terms of the assessment of efficiency and its impact on it by this fact).

d) A new element in the educational offer means attracting new students. Depending on the efficiency of functioning, the profitability threshold for conducting classes for the new didactic offer may be below the equivalent of 49 students indicated above. However, this is not a concern as both positive effects in such a situation occur together.

e) A positive effect for increasing the assessment of efficiency related to the increase in the number of students can be achieved by increasing the ratio of the number of students to the number of teaching / research staff. As the analysis showed, its

average value for all 29 universities surveyed was 13.13 (with the minimum value - 5.8 and the maximum value - 30.2). This is probably the easiest and cheapest way for universities to raise their efficiency assessment.

However, it has its limit, resulting from many factors (the organization of the university, resources in the form of rooms enabling classes for larger groups, the type of legal relationship with the teaching staff, etc.). In the face of recent events (the COVID-19 pandemic), this path of increasing productivity is extremely difficult. While a university is considering conducting stationary classes (i.e., at the school premises), the restrictions related to e.g., maintaining social distance rather force the creation of smaller groups. This limits the number of students who can physically participate in classes. Thus, it may be necessary to extend classes (e.g., dividing the current group of students into two), which involves research / teaching staff. On the other hand, efficient organization of remote classes makes the teaching / research staff much more flexible and gives the opportunity to "service" a larger number of students, while ensuring appropriate technical possibilities - e.g., remote classes for 100 people instead of stationary classes for 30 (limitations resulting from the size of the room).

f) Expanding the didactic offer may also take place by joining external universities with complementary competences. This creates opportunities for sharing resources (real estate, research / teaching staff). This allows you to achieve the following:

- expanding the didactic offer, when providing access to research and teaching staff with appropriate competences in the dimension necessary to implement classes in a given field of study;
- increasing the number of students by using external teaching staff and / or access to the real estate of a partner university;
- optimizing the use of the resources of both universities, which increases the overall efficiency (simultaneous improvement in many areas).

Summing up, it is necessary to indicate those facilities that achieved the best results in terms of operational efficiency. The Hipolit Cegielski State Higher Vocational School in Gniezno, prof. E. Szczepanik State Higher Vocational School in Suwałki, prof. Stanisław Tarnowski State Vocational School in Tarnobrzeg, the State Higher Vocational School in Racibórz and the State Vocational University in Włocławek should be reference points for other facilities. However, it is worth remembering that each university functions in a different geographic environment, and has immanent, specific features that may hinder and complicate the benchmarking analysis.

The above conclusions should be treated as recommendations - especially in the area related to the merger of universities, as no research has been conducted on the actual effect of merging universities. Issues in the context of changes taking place among the entire sector of public vocational universities are certainly an interesting direction for future research.

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