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## Reliability Measures in GNSS Multi-constellation Systems

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Maciej Gucma<sup>1</sup>, Renata Boć<sup>2</sup>, Mariusz Wąż<sup>3</sup>

**Abstract:**

**Purpose:** The purpose of this article is to analyse methods and factors that describe reliability in modern GNSS systems.

**Design/Methodology/Approach:** This analysis was conducted for several branches of human activity that rely on technical systems for assuring safe and reliable operations. Transportation, logistics, geodetical measurements, or development are just few professional activities dependent on satellite positioning. The reliability of the whole transport or technological chain is often combined with the singular reliability of satellite monitoring systems. Global Navigation Satellite Systems (GNSS) are present where human activity must be backed with precise positioning and timing information.

**Findings:** This paper will provide a review of researches in the field of GNSS on the side of the multi-constellation receiver concerning reliability issues.

**Practical Implications:** A combination of technical criteria to mention safety, maintainability, quality, cost, risk, or availability will be addressed as a comparative analysis.

**Originality/Value:** The results of the study reflect the applicability and effectiveness of used indicators that can contribute to many human activities. Consequently, the fundamental strength of proposed factors can contribute to many business branches where accuracy and standard measures of reliability used in single constellation (like GPS) receivers differ then multi-constellation ones where more complex reliability measures must be provided.

**Keywords:** GNSS, reliability of system, positioning of transport devices.

**JEL codes:** R41.

**Paper type:** Research article.

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<sup>1</sup>PhD. DSc. Eng, Prof. AM, ORCID: 0000-0002-7501-5632, Maritime University Szczecin, Navigation Faculty, e-mail: [m.gucma@am.szczecin.pl](mailto:m.gucma@am.szczecin.pl),

<sup>2</sup>MSc. Eng, ORCID: 0000-0003-3480-5897, Maritime University Szczecin, Navigation Faculty, e-mail: [r.boc@am.szczecin.pl](mailto:r.boc@am.szczecin.pl),

<sup>3</sup>PhD. DSc. Eng, Prof. AMW, ORCID: 0000-0003-3093-1255, Polish Naval Academy e-mail: [m.waz@amw.gdynia.pl](mailto:m.waz@amw.gdynia.pl)

## 1. Introduction

GNSS receivers can be found in almost every professional human activity where precise localization of singular positioning and timing data is required. Cost and effectiveness combined with the availability of several different satellite systems inside a single receiver create a very wide market. Receivers themselves are only the user-end subsystem of every available GNSS, and as such satellites and control subsystems are indispensable. In the article, only user end criteria and measures are taken into consideration since they are strictly combined and referred to as a GNSS further. The classical approach to the single type GNS system reliability modeling is based on the following measures are used:

1. geometric dilution of precision (GDOP) in satellite geometry distribution;
2. weighted matrix estimation in weighted least squares (WLS);
3. autonomous integrity monitoring (RAIM).

Extensive popularization and application of the oldest working GNS system i.e., Global Positioning System (GPS), as well as restoration of the Russian Federation, operated GLONASS system provides global coverage for many applications. EU countries declared and build modular and very enhanced concerning GPS, namely Galileo GNS system, which along with the youngest BeiDou (BDS) GNS system can contribute to precise, fast, and reliable services. Since setting up BDS and Galileo to operational status all 4 GNSS's can provide about 120 satellites available to users (Li *et al.*, 2015). Besides, two more GNS systems are reachable locally i.e., Japanese QZSS and Indian NavIC, but here they will not be taken into consideration.

The receiver resolves the pseudo-range measurements from each visible satellite, and since multi-constellation GNSS can be tracked by the receiver simultaneously not all SV's (satellites) are usable for single position solution. It's since the low passing (small value of elevation) SV signal suffers from larger ionospheric delay, and thus increases the total value of pseudo-range measurement error. Real-time transmission error and signal quality can contribute to an increase of total estimation error since they are not taken directly to weighted matrix estimation (Meng, Wang, and Zhu, 2015).

Another observed problem is dynamic applications. Since typical positioning problems and related to its reliability issues in a static application (like geodetical measurements) can be addressed with a longer observation period, dynamic application (like the positioning of transport devices) demands another approach. Two basic techniques of increasing reliable dynamic positioning are used: relative kinematic positioning (also referred to as real-time kinematic - RTK) and precise point positioning (PPP) (Lambiel and Delaloye, 2004). RTK is a short-range GNSS support system that can contribute to lowering the total error of positioning to around a few centimeters. The main disadvantage is the need for delivering ionospheric delay correction for a limited area. The corrections can be sent to the network (like the

Internet or Wide Area Network) for ease of delivering the corrections but still monitoring stations with high precision receivers, capable to compute errors (total displacement) are desired in the vicinity of end-user measurement (Guo *et al.*, 2009). The measurement itself is relative, and as such reliability of the cumulated solution depends on the reliability of another correction subsystem, apart from GNSS.

Since technically Differential GPS (or more precise DGNSS) is very similar to the concept of RTK, namely displacement correction is provided additionally to the system receiver, solution based on PPP differs. Here, absolute positioning solution is provided in a global reference system with only one end-user GNSS receiver. And the multi-constellation system was proposed and introduced in the multi-GNSS experiment (MGEX) by International GNSS Service (IGS) described in (Rizos *et al.*, 2013) and (Tegeodor, Øvstedal, and Vigen, 2014). Researchers have found that long convergence time can be the main PPP error source that sets reliable position back to the beginning of measurement and around 15-20 min is required to obtain the desired accuracy of a system (Li *et al.*, 2015). Another step was taken by Su *et al.* (2019) where a quad GNSS solution (based on GPS, Galileo, Glonass, and BDS) dynamic PPP model for rapid displacement determination based on velocity estimation approach was proposed. Velocity and acceleration of body (receiver) are integrated to obtain Dynamic PPP MGEX solution.

Another approach presented in Meng *et al.* (2015) does not include the full satellite information to the receiver computer but shows that preselection of SV's is desired in such complex system. Proposed algorithm assumes fast satellite selection algorithm, depending on both Newton's identities and optimal satellite geometric distribution. Meng *et al.* algorithm takes into account real-time signal quality, i.e., satellite carrier-to-noise ratio, receiver thermal noise, and atmosphere transmission delay.

## 2. Reliability Measures of GNS System

The time-dependent measure we can use for GNSS is availability defined as a probability of correct system functioning at any time (Specht, 2007). Availability by its theoretical assumption is a probability that the system will provide position information to the end-user at the border of its working range, in the other words there will be enough SV's to resolve positioning equations ('Global Positioning System Standard Positioning Service Performance Standard' 2008), and can be expressed as (Specht, 2007):

$$A_{exp}(t) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu} e^{-(\lambda + \mu)t}$$

Where:  $\lambda$  - the intensity of system malfunctions,  $\mu$  - the intensity of system recoveries,  $A_{exp}(t)$  Availability of GNSS with exponential distributions models of work and recovery times. Availability in a standard can be combined with an accuracy factor where the displacement of certain accuracy is available at a certain amount of time

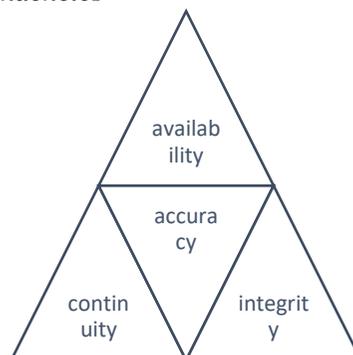
(‘Global Positioning System Standard Positioning Service Performance Standard’ 2008). The availability factor has limited functioning for multi-constellation system receivers since more SV’s are visible to end-user.

Reliability of the GNS system is treated as a probability that the system (consisting of many components - subsystems) will remain to function (Ross, 2014). Series systems will function only assuming that all components are functioning, whilst parallel will function if only one is functioning. In terms of redundancy, a parallel is way better, but in a real environment, systems will have mixed architecture (series-parallel). In other words, reliable functioning is a level of trust for the information delivered by the navigational system.

Another research led to the concept of integrity since classic concept technical reliability wasn’t suitable for direct measurement of GNS system fitness. Integrity is defined in (‘Minimum Operational Performance Standards for Global Positioning System/Wide Area Augmentation System Airborne Equipment’ 2006) (Ober, 2003) as a measure of trust which can be placed in the correctness of the information supplied by the total system. It includes the ability of the system to provide timely warnings to users when the system should not be used for navigation (Rife *et al.*, 2009). Total system was originally interpreted utilizing operator center design in avionics and SoL (Safety of Life applications).

It can be easily stated that these factors are overlapping each other and no clear distinction is set between these terminological values. Additionally, support systems are introduced and used to extend the operational capabilities, to mention, Ground-Based Augmentation Systems (GBAS), Wide Area Augmentation Systems (WAAS), and others. Researchers find also other measures like the continuity that measures the likelihood of the GNS system persisting in a non-failure state. Figure 1 presents the relation of all measures that can be interpreted as a base of accurate and reliable positioning using GNSS. Accuracy is a core factor surrounded by the others although connected. Assessment of one without knowledge of others is pointless.

**Figure 1.** Triangle of dependencies



**Source:** Own elaboration.

Generally, for the multi-constellation receiver, the same concept is correct with the additional assumption that all factors must be calculated as a set of subsystems and its  $i$ -th components is functioning through  $x_i$  variable:

$$x_i = \begin{cases} 1, & \text{if the } i\text{th GNSS component is functioning} \\ 0, & \text{if the } i\text{th GNSS component fails} \end{cases}$$

If we assume the state vector of functioning and failure components by  $\mathbf{x}=(x_1, \dots, x_n)$ , and introducing the function representing the structure of the system it can be noted:

$$\phi(\mathbf{x}) = \min(x_1, \dots, x_n) = \prod_{i=1}^n x_i$$

Since series system are identified by structure-function of:

$$\phi(\mathbf{x}) = \min(x_1, \dots, x_n) = \prod_{i=1}^n x_i$$

And parallel:

$$\phi(\mathbf{x}) = \max(x_1, \dots, x_n)$$

Here, useful system  $k$  of the  $n$  that functions only when  $k$  out of  $n$  components functions properly. Denoting  $\sum_{i=1}^n x_i$  as a number of functioning GNSS components we can obtain the structure function  $k$  out of  $n$ :

$$\phi(\mathbf{x}) = \begin{cases} 1, & \text{if } \sum_{i=1}^n x_i \geq k \\ 0, & \text{if } \sum_{i=1}^n x_i < k \end{cases}$$

Introducing the cut vector  $\phi(\mathbf{x}) = 0$  and  $\phi(\mathbf{y}) = 1$  for any  $y > x$  we can figure out the  $\mathbf{x}$  as a minimal cut vector. And minimal cut set as a  $C = \{i: x_i = 0\}$  that will describe a minimal set of GNSS components that will bring the whole system to the failure state. Furthermore for  $C_1, \dots, C_k$  minimal GNSS cut set we can define indicating function  $I_j(\mathbf{x})$  of  $j$ -th minimal cut set as a:

$$\beta_j(\mathbf{x}) = \begin{cases} 1, & \text{if one component of } j\text{-th minimal cut set is functioning} \\ 0, & \text{if all component of } j\text{-th minimal cut set is not functioning} \end{cases}$$

That denotes  $\beta_j(\mathbf{x}) = \max_{i \in C_j} x_i$  as not functioning of the whole GNSS system (if all the components of at least one minimal cut set are not functioning), and as such structure-function becomes:

$$\phi(\mathbf{x}) = \prod_{j=1}^k \beta_j(\mathbf{x}) = \prod_{j=1}^k \max_{i \in C_j} x_i$$

This structure-function can easily adapt the parallel structures into an arbitrary system consisting of a series arrangement. Adding the  $X_i$  as a state of  $i$ -th independent component, and treating it as a random variable that is true for complex systems build of independent blocks (like GNS system) it follows:

$$r = P\{\phi(\mathbf{X}) = 1\} \quad P\{X_i = 1\} = p_i = 1 - P\{X_i = 0\}$$

where:  $\mathbf{X}=(X_1, \dots, X_n)$  state matrix,  $p_i$  is the reliability of  $i$ -th component i.e. value of probability that  $i$ -th component of GNSS is functioning,  $r$  – reliability of the GNS system. Having in mind that GNS subsystems are independent  $r$  becomes a function of subsystem reliabilities.

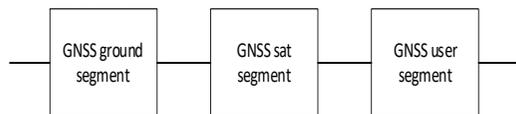
$$r = r(\mathbf{p})$$

where:  $\mathbf{p}=(p_1, \dots, p_n)$ . In such stated environment we can assume that GNSS is  $k$  out of  $n$  system;  $p_i = p$  for  $i = 1, \dots, n$  we get reliability function:

$$r(p) = P\{\phi(\mathbf{X}) = 1\} = P\left\{\sum_{i=1}^n X_i \geq k\right\} = \sum_{i=k}^n \binom{n}{i} p^i (1-p)^{n-i}$$

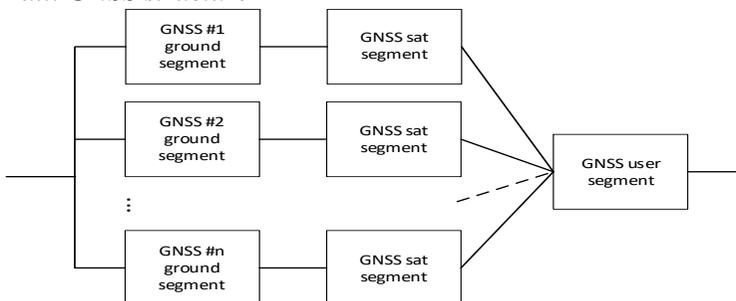
For GNSS based on the single constellation, the system will have the following presentation - Figure 2, while for multi-constellation – Figure 3.

**Figure 2.** Single GNSS structure



Source: Own elaboration

**Figure 3.** Multi GNSS structure



Source: Own elaboration.

Since every GNS system has its control segment, affecting quality parameters of it and own SV's orbiting the crucial, element of the system is a receiver (user segment). The receiver is the only element that the end-user can affect. The basic probability of failure for SVs has been investigated in (Milner, Macabiau, and Thevenon, 2016) and can be assumed as  $P=2.2 \times 10^{-5}$  for GNSS constellation at 99.9% confidence level for single satellite per hr. of failure. This is only measured for the onset of the event, and in most cases is related to the clock failures (21 of 28) (Milner, Macabiau, and Thevenon, 2016).

For Glonass satellites this value is given as a  $P=3.1 \times 10^{-4}$ , and for combined probability regarding the new constellations (Beidou and Galileo) Derived Failure Probabilities vs. Constellation Operational Time for 6 months period is calculated as independent sample period  $P=4.5 \times 10^{-3}$  (Milner, Macabiau, and Thevenon, 2016). Similar results for the Autonomous RAIM technique have been achieved by (Walter *et al.*, 2016). Treating this probability as a combined probability of failure on the control segment and sat segment can be justified.

### 3. GNSS Measurement

Since pure data about accuracy like fault ellipsoid is not suitable to assess the state of a system, some special measures were introduced in GNSS's. Dilution of precision is a nondimensional factor computed during displacement assessment inside receiver basing on geometry and visibility of SVs. Thus, a measurement can be performed accurately and the DOP factor can answer how errors in the measurement will affect the final state estimation. DOP's is a measure of accuracy in GNSS receivers (Specht, 2007).

The geometry of the SV visible to the end-user affects position error and is usually referred to as geometric dilution of precision (GDOP). DOP was utilized in ground-based radio navigation systems (pre-GPS era) and it is roughly interpreted from there, as the ratio of position error to the range error. GDOP estimation for the multi constellation was presented in (Dutt *et al.*, 2009) but since then (2009) larger number of SV's is available. DOP in GNS systems can be expressed as several factors (detailed calculation of its to be found in (Grewal, Andrews, and Bartone, 2020)):

1. HDOP – horizontal dilution of precision;
2. VDOP – vertical dilution of precision;
3. PDOP – position dilution of precision;
4. TDOP – time dilution of precision;
5. GDOP – geometric dilution of precision.

$$HDOP = \sqrt{\sigma_x^2 + \sigma_y^2}$$

$$VDOP = \sqrt{\sigma_z^2}$$

$$PDOP = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2}$$

$$TDOP = \sqrt{\sigma_t^2}$$

$$GDOP = \sqrt{TDOP^2 + PDOP^2}$$

Where:  $\sigma_x$ ,  $\sigma_y$ ,  $\sigma_z$  – mean square position errors for directions (x,y,z),  $\sigma_t$  – time measurement error. The better geometry the lower the DOP value is obtained in displacement estimation.

GDOP represents total error obtained during the cycle of estimation inside the GNSS receiver but an additional measure that combines a weighted matrix of measurement (called Weighted GDOP – WGDOP) was introduced and overall assumptions for this measure can be found in Pan *et al.* (2017), and Teng *et al.* (2018).

Since DOP measures relate more to the question of how accurate displacement is (or is not) there is a need to monitor also the rising question of how reliable it is? And in GNSS techniques it is referred to as integrity monitoring where some researchers combine assurance of integrity, reliability, and accuracy in one factor (Meng, Wang, and Zhu, 2015). It gives simplification to the end-user just to provide a one-dimensional factor of a complete solution. The key technique here is RAIM introduced in 80'ties of the XX century for GPS in series of technical papers, among them (Parkinson and Axelrad, 1987; 1988). RAIM technique assumes that the user obtains redundant pseudo-range measurement from multiple satellites, that it is not indispensable in computing position, but it's verified against internal consistency. Every pseudo-range that significantly differs from the expected value (in statistical means) may indicate:

- The faulty signal delivered by SV;
- Low visibility of satellite;
- Higher than expected dispersion of signal (like ionospheric delay).

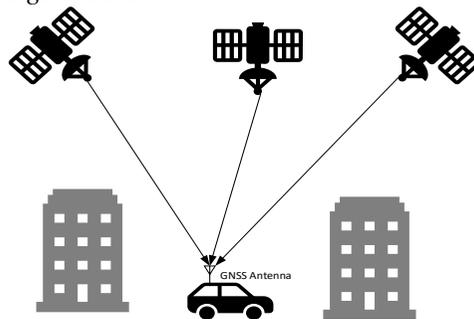
While overall assessment of receiver status may include problems with the receiver itself and its subsystems. In RAIM the least-squares residual is used for non-coherent measurement (so-called: outlier) identification. The algorithm seeks if the sum of squared residuals follows the chi-squared distribution, then the outlier is treated as a non-coherent measurement and falls out of a set of measurements. In the opposite observations are verified for outlier and then the observation with the higher residual error is (or are) considered as the outlier.

Although in some cases it is also possible that a wrong estimated position or exclusion of a wrong observation may lead to a successful chi-square test, and some solution to this problem was presented in (Akram *et al.*, 2018). This concept assumes preserving best fit observations and maintaining the integrity of the solution.

#### 4. Research Results Analysis

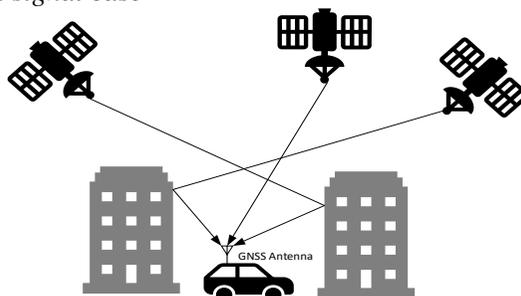
The planning of the experiment has been based on multiple passes in a real urban environment in the vicinity of buildings (maximum 5 floors) in Szczecin City, Poland. Obstacles that appear along the horizon depending on the position of the vehicle at the selected location for GNSS measurements were considered. Maximum vehicle speed was 20km/h and stops at crossings were performed. Obstacles (buildings) induce multipath phenomena to the receivers (Figures 4 and 5) when GNSS signal is reflected from obstacles and elevation cut-off angle doesn't allow direct measurement. Some software was created to plan the experiment and desired cut-off values (Trimble Studio Software) and since it's the perfect tool for static measurement using it with the dynamic environment requires very precise position estimation. This situation is not achievable in moving vehicles and results were not satisfying. Using this type of software also requires a very detailed DTM model (digital terrain model), that usually merges elevation data from the earth surface not assuming the buildings.

*Figure 4. Direct path signal case*



*Source: Own elaboration.*

*Figure 5. Multipath signal case*

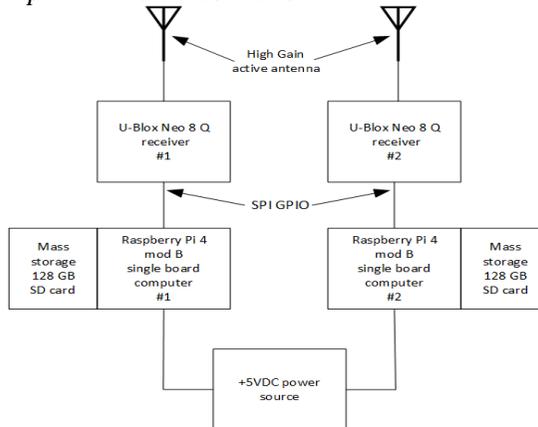


*Source: Own elaboration.*

In the experiment, two receivers of the same type have been used so the maximum computation time and processing of the signal can be treated as a similar, non-affecting measurement itself. Registered parameters were the same (all diagnostic parameters of the receiver) and among them, the GDOP factor is computed inside the receiver values and fed in for comparison. In the experiment, OEM receivers have been used namely u-Blox neo-8q. This type of receiver is a high-performance GNSS multi-constellation receiver, 72-channel GNSS capable of working in GPS L1C/A, SBAS L1C/A, QZSS L1C/A, GLONASS L1OF with base sensitivity of -164 dBm equipped with a built-in ceramic high gain antenna. The receiver itself is configurable and multiple data can be retrieved in hexadecimal format (of type UBX) via SPI (serial port) interface.

Data gathering has been realized in the Linux environment over the Raspberry Pi single-board computers and for the sake of peripherals problems with SPI interface 2 data collector tracks have been used. A schematic of the experiment setup has been presented in Figure 6.

**Figure 6.** Setup of experiment data collection.



*Source:* Own elaboration.

Ublox Neo 8 family can receive an SBAS type augmentation system where every tracked geostationary satellite is treated as a single channel GNSS SV. A list of SBAS satellites is provided in Table 1.

**Table 1.** List of SBAS SV's

Identification	Position	GPS PRN	SBAS Provider
Inmarsat 3F2 AOR-E	15.5° W	120	EGNOS
Artemis	21.5° W	124	EGNOS
Inmarsat 3F5 IOR-W	25° E	126	EGNOS
AMR	98° W	133	WAAS
Pan Am Sat Galaxy	133.0° W	135	WAAS
TeleSat Anik	107.3° W	138	WAAS
MTSAT-1R	140.1° E	129	MSAS

MTSAT-2	145° E	137	MSAS
Inmarsat-4F1/IOR	64° E	127	GAGAN
GSAT-10	83° E	128	GAGAN

*Source:* Own elaboration.

For our consideration SBAS satellite correction has been gathered via EGNOS SV service, and visibility of this satellite can also be obstructed by buildings.

It has been decided that a cut of values for SVs visibility must be set. For the first pass (date: 12.02.2021 time: 2215 UTC) the cut-off value has been set to 45° and for such value, it was nearly impossible to achieve any reasonable solution in GPS mode, thus comparison was could not be done. Second measurement campaign (date: 16.02.2021 time: 2105 UTC) where the cut of value has been reduced do 20° and acquisition of over 15k samples was successful. All gathered data were transferred from SD cards to the processing computer. Since the receivers started nearly simultaneously only simple time comparison and adjustment have been done. Collected samples have been processed and statistics of GDOP have been presented in Table 2.

**Table 2.** Statistics of GDOP factor in measurement 15137 samples (observations) for 2 u-Blox neo 8 receivers

Type/mode	mean	minimum	maximum	std. dev
GPS only	2,58	1,3	26,5	1,51
Multiconstellation	2,34	1,6	5,9	0,54

*Source:* Own elaboration.

Multiconstellation mode enabled receivers to increase in a significant way the reliability of received GNSS data in means of GDOP factor. Values of GDOP factor for the multi-constellation observation has been presented at Figure 7. For only GPS constellation the same period measurement is presented at Figure 8. For both GDOP distributions exponential scale has been adapted with 0,41788 in multi-constellation and 0,388 for GPS only.

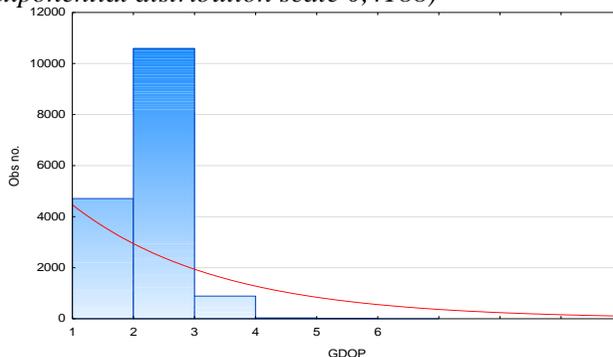
## 5. Summary and Conclusion

In many human activities there is a need to monitor, control and manage position related data. Business of many kinds requires comprehensive input data and modern GNSS can contribute to it. Monitoring important reliability measures in GNS systems is a demanding and complex issue. Even small compromises can lead to the loss of quality parameters. Autonomous techniques that can monitor set warning state if any loss of reliability occurs have been presented in the article and most available to the user (i.e. GDOP monitoring) have been measured and validated for multi-constellation GNSS.

Today's situation allows using more signals from SV that may introduce some idea that users are always safe, and quality is timely assured only based on this feature. Provided features where more systems are introduced affects only the space segment

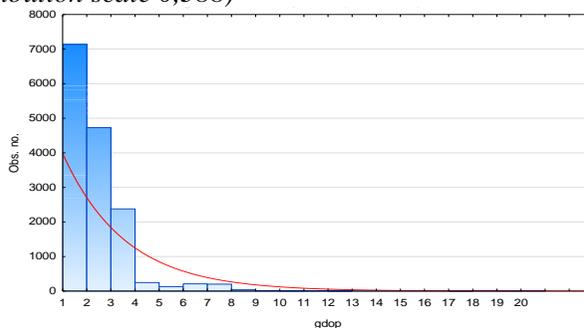
and user segment since we still got a single device with multifrequency receiver do not let increase redundancy. Also, calculations of reliability and minimal cut-set presented in the article allow modeling the reliability.

**Figure 7.** GDOP histogram for multiconstellation static observation (16218 measurements exponential distribution scale 0,4188)



*Source: Own elaboration.*

**Figure 8.** GDOP histogram for GPS static observation (16218 measurements exponential distribution scale 0,388)



*Source: Own elaboration based on a research.*

## References:

- Akram, Muhammad, Peilin Liu, Yuze Wang, and Jiuchao Qian. 2018. GNSS Positioning Accuracy Enhancement Based on Robust Statistical MM Estimation Theory for Ground Vehicles in Challenging Environments. *Applied Sciences*, 8(6), 876. <https://doi.org/10.3390/app8060876>.
- Autonomous GPS Integrity Monitoring Using the Pseudorange Residual. 1988. *Navigation, Journal of the Institute of Navigation*, 35(2), 255-274.
- Dutt, I., Rao, B., Babu, S.R., Goswami, R., Kumari, U. 2009. Investigation of GDOP for Precise User Position Computation with All Satellites in View and Optimum Four Satellite Configurations. In: *Global Positioning System Standard Positioning Service Performance Standard*, 2008. US.gov.
- Grewal, M.S., Angus, P.A., Bartone, C. 2020. *Global Navigation Satellite Systems. Inertial Navigation, and Integration*, Fourth Edition. Hoboken: Wiley.

- Guo, Lijie, Jinji Gao, Jianfeng Yang, and Jianxin Kang. 2009. Criticality Evaluation of Petrochemical Equipment Based on Fuzzy Comprehensive Evaluation and a BP Neural Network. *Journal of Loss Prevention in the Process Industries*, 22(4), 469-76. <https://doi.org/10.1016/j.jlp.2009.03.003>.
- Lambiel, C., Reynald, D. 2004. Contribution of Real-Time Kinematic GPS in the Study of Creeping Mountain Permafrost: Examples from the Western Swiss Alps. *Permafrost and Periglacial Processes*, 15(3), 229-241. <https://doi.org/10.1002/ppp.496>.
- Li, Xingxing, Xiaohong Zhang, Xiaodong Ren, Mathias Fritsche, Jens Wickert, and Harald Schuh. 2015. Precise Positioning with Current Multi-Constellation Global Navigation Satellite Systems: GPS, GLONASS, Galileo and BeiDou. *Scientific Reports*, 5(1). <https://doi.org/10.1038/srep08328>.
- Meng, Fanchen, Shan Wang, and Bocheng Zhu. 2015. GNSS Reliability and Positioning Accuracy Enhancement Based on Fast Satellite Selection Algorithm and RAIM in Multiconstellation. *IEEE Aerospace and Electronic Systems Magazine*, 30(10), 14-27. <https://doi.org/10.1109/MAES.2015.140024>.
- Milner, C., Macabiau, C., Thevenon, P. 2016. Bayesian Inference of GNSS Failures. *Journal of Navigation*, 69(2), 277-294. <https://doi.org/10.1017/S0373463315000697>.
- Minimum Operational Performance Standards for Global Positioning System/Wide Area Augmentation System Airborne Equipment. 2006. RTCA.
- Ober, P.B. 2003. Integrity prediction and monitoring of navigation systems. PhD Thesis, Delft University of Technology.
- Pan, L., Cai, C., Santerre, R., Zhang, X. 2017. Performance Evaluation of Single-Frequency Point Positioning with GPS, GLONASS, BeiDou and Galileo. *Survey Review*, 49 (354), 197-205. <https://doi.org/10.1080/00396265.2016.1151628>.
- Parkinson, Bradford W., Penina Axelrad. 1987. A Basis for the Development of Operational Algorithms for Simplified GPS Integrity Checking, 269-276. Colorado Spring.
- Rife, J., Pullen, S., Gleason, S., Gebre-Egziabher, D. 2009. Aviation Applications. In *GNSS Applications and Methods*, 245-268. Artech House Norwood, MA.
- Rizos, C., Montenbruck, O., Weber, R., Weber, G., Neilan, R., Hugentobler, U. 2013. The IGS MGEX Experiment as a Milestone for a Comprehensive Multi-GNSS Service., 289-295. Honolulu, Hawaii.
- Ross, S. 2014. Reliability Theory. Introduction to Probability Models, 559-606. Elsevier. <https://doi.org/10.1016/B978-0-12-407948-9.00009-8>.
- Specht, C. 2007. System GPS. Pelplin: Wydawnictwo Bernardinum.
- Su, K., Shuanggen J., Yulong, G. 2019. Rapid Displacement Determination with a Stand-Alone Multi-GNSS Receiver: GPS, Beidou, GLONASS, and Galileo. *GPS Solutions*, 23(2), 54. <https://doi.org/10.1007/s10291-019-0840-4>.
- Tegedor, J., Øvstedal, O., Vigen, E. 2014. Precise Orbit Determination and Point Positioning Using GPS, Glonass, Galileo and BeiDou. *Journal of Geodetic Science*, 4(1). <https://doi.org/10.2478/jogs-2014-0008>.
- Teng, Y., Wang, J., Huang, Q., Liu, B. 2018. New Characteristics of Weighted GDOP in Multi-GNSS Positioning. *GPS Solutions*, 22(3). doi:10.1007/s10291-018-0740-z.
- Walter, T., Blanch, J., Joerger, M., Pervan, B. 2016. Determination of Fault Probabilities for ARAIM. In: 2016 IEEE/ION Position, Location and Navigation Symposium (PLANS), 451-461. Savannah, GA: IEEE. <https://doi.org/10.1109/PLANS.2016.7479733>.