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Optimization of Logistics and Distribution of the Supply Chain, Taking into Account Transport Costs, Inventory and Customer Demand

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Łukasz Gołąbek[1], Józef Stokłosa[2], Jacek Dziwulski[3], Joanna Wyrwisz[4]

Abstarct:

Purpose: *The aim of the article is to develop an algorithm to optimize logistics and distribution of the supply chain, taking into account transport costs, inventory and customer demand.*
Design/Methodology/Approach: *To solve the problem, the class of optimization problems and the traveling salesman problem were used. The boundary conditions and the objective function were determined. The optimization criterion was to minimize the total transport costs, kilometers traveled and the time needed to complete the task.*
Findings: *As a result of the analyzes and calculations performed, the optimization task was performed. With the help of the prepared software, a map of goods delivery points was determined, the total route and the route with individual points marked, and various solution methods were tested.*
Practical Implications: *The model presented in the article can be used in a supply chain application to optimize routes, costs and delivery time.*
Originality/Value: *A novelty is the preparation of algorithms and universal software using the R language, which was used in the application of an intelligent IT system in a distributed model, controlling the supply chain, enabling personalization and identification of products in real time.*

Keywords: *Supply chain, travelling salesman problem, optimization, objective function.*

JEL codes: *C61, R42.*

Paper type: *Research article.*

[1]*Corresponding author, Netrix Group sp. z o.o. , Lublin, Poland,*
e-mail: lukasz.golabek@netrix.com.pl
[2]*University of Economics and Innovation in Lublin, Lublin, Poland;*
e-mail: jozef.stoklosa@wsei.lublin.pl
[3]*Faculty of Management, Lublin University of Technology, Lublin, Poland,*
e-mail: j.dziwulski@pollub.pl
[4]*Faculty of Management, Lublin University of Technology, Lublin, Poland,*
e-mail: j.wyrwisz@pollub.pl

1. Introduction

The supply chain is a fast and flexible system connected and guided by the customer selection mechanism, the aim of which is to achieve the highest satisfaction and profit for the companies making up this chain. Integrating and coordinating logistic systems of enterprises is today considered to be the essence of modern logistics management. The main factors influencing the direction and dynamics of changes in the field of logistics are the growing needs and requirements of the client. It is these requirements that most significantly affect the new way of managing the supply chain. The supply chain can be described by pointing to such features as: process (object of flow), structure (entity structure), goals - functional scope and areas of cooperation of the participating entities (Szymonik, 2014).

The subject scope of the logistics chain consists of raw materials, auxiliary materials and cooperating elements purchased on the supply market in accordance with the demand and directed to the production process, as well as finished products transferred for sale (Sołtysik, 2003).

Depending on the configuration of the chain, its links may be various types of mining, processing, service and trade companies. Their place along the supply chain results from the division of labor at the subsequent stages of production and sale of products. Due to their role as shippers and recipients of cargo and the accompanying information and financial streams, their basic role in the functioning of supply chains is unquestionable. Important links in the supply chains are also the service functions of the enterprise, which include transport and forwarding and logistics companies, brokerage firms dealing only with information intermediation, waste disposal and storage plants (Witkowski, 2010).

Today's competitive business environment is leading to many changes in production and distribution systems. One of the important changes in competition between supply chains instead of companies. An efficient and flexible supply chain helps companies meet two vital customer needs, including short delivery times and low price. The supply chain is a network of suppliers, manufacturers, warehouses and retailers organized to produce and distribute goods in the right quantities, at the right locations and at the right time, to minimize overall costs while meeting service level requirements (Simchi, Kaminsky, and Simchi-Levi, 2003).

As a traditional goal, supply chain network performance is the primary focus of researchers and practitioners in the design and optimization of supply chain networks. Typically, network performance is translated and modeled as cost minimization or profit maximization in the supply chain network design literature (Dasci and Verter, 2001; Amiri, 2006). Therefore, the aim of the article is to develop an algorithm to optimize logistics and distribution of the supply chain, taking into account transport costs, inventory and customer demand.

2. Methodology

The class of optimization problems in which all limiting conditions and the objective function are linear is called linear programming, e.g., classic problems:

- selection of the optimal range of production consisting in determining the type of products and in what quantities should be produced in order not to exceed the production capacity (related to the availability of cash, raw material resources), and on the other hand, the profit (or income) from the sale of products should be as big as possible;
- choosing the optimal diet, taking into account the provision of ingredients in the required amounts;
- selection of cutting methods or production methods, taking into account the requirements related to technologies;
- choosing the optimal route between nodes or determining the method of delivering the tare from suppliers to recipients.

In each of these tasks, the objective function and constraint conditions must be defined.

Let $x = (x_1, x_2, \dots, x_n) \in \mathbb{R}^n$ be a vector of decision variables (e.g., production vector, product purchase vector, cutting methods vector), and the vector $c = (c_1, c_2, \dots, c_n) \in \mathbb{R}^n$ - vector of objective function coefficients (e.g., vector of unit revenues/profits, vector of unit costs, vector of unit waste).

We define the objective function as $\langle x, c \rangle$, where $\langle . \, , . \rangle$ denotes the dot product. When optimizing the objective function, k limiting conditions should be taken into account, for this purpose, we define a matrix of limiting conditions (e.g. matrix of raw material consumption during production, matrix of nutrient content, matrix of unit costs of transport):

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ . & . & . & . \\ a_{k1} & a_{k2} & \cdots & a_{kn} \end{bmatrix} \tag{1}$$

and $b = col(b_1, b_2, \dots, b_k)$ constraint vector (e.g., available agent amount vector, nutrient limiting vector).

We usually present linear programming tasks as:

$$\max_{x \in D} \langle x, c \rangle \tag{2}$$

where $D = \{x \in R^n : Ax \leq b\}$
or characters

$$\min_{x \in D} \langle x, c \rangle \tag{3}$$

where $D = \{x \in R^n : Ax \geq b\}$

3. Problem of the Transport of Goods from Suppliers to Customers

The Traveling Salesman Problem (TSP) is one of the oldest optimization issues related to transport. This problem comes down to looking for the best possible route to overcome (Hiller and Liberman, 1990). Therefore, it is necessary to develop a plan for the transport of any selected product from one or more sources of supply to many collection points reporting a demand for this product. In the case of delivering goods from several suppliers to many recipients, we distinguish two types of transport tasks:

- balanced tasks are tasks in which the number of items ordered by recipients is equal to the number of items at the suppliers;
- in unbalanced tasks, the number of items ordered by recipients differs from the number of items at the suppliers.

<u>*Form task*</u>

In *m* delivery points $D_1, D_2, \dots, D_m$ there are goods in the following quantities: $d_1, d_2, \dots, d_m$, which should be delivered to n-recipients $O_1, O_2, \dots, O_n$ (or e.g. the company has *m*- warehouses from which it delivers goods to *n*-recipients). Recipients need $o_1, o_2, \dots, o_n$. Let

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \cdot & \cdot & \cdot \\ a_{m1} & \cdots & a_{mn} \end{pmatrix} \tag{4}$$

will be a transport cost matrix, where a_{ij} $(1 \leq i \leq m, 1 \leq j \leq n)$ - cost of transporting a unit of goods from the supplier D_i to the recipient O_j. The transport (logistics) company must deliver the goods at the cheapest cost.

<u>*Mathematical model*</u>

Let

$$X = \begin{pmatrix} x_{11} & \cdots & x_{1n} \\ \cdot & \cdot & \cdot \\ x_{m1} & \cdots & x_{mn} \end{pmatrix} \tag{5}$$

will be a transport matrix, the quantity x_{ij} $(1 \leq i \leq m, 1 \leq j \leq n)$ means the number of commodity units transported on the road $D_i \xrightarrow{x_{ij}} O_j$. The total cost of transport (objective function) is $\langle A, X \rangle$ $\langle \cdot, \cdot \rangle$ stands for dot product).

- In the case of balanced transport tasks, the condition is met

$$\sum_{i=1}^{m} d_i = \sum_{i=1}^{n} o_j \tag{6}$$

The mathematical model is in form

$$\min_{X \in D} \langle A, X \rangle \tag{7}$$

where a set of feasible solutions

$$D = \left\{ \begin{array}{c} X \in \mathbb{R}^{m \times n}: \quad x_{ij} > 0, \quad 1 \leq i \leq m, 1 \leq j \leq n \\ x_{11} + \ldots + x_{1n} = d_1 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ x_{m1} + \ldots + x_{mn} = d_m \\ x_{11} + \ldots + x_{m1} = o_1 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ x_{1n} + \ldots + x_{mn} = o_n \end{array} \right\} \tag{8}$$

In the definition of the permissible set D, we have $n + m$ equality, because all the goods that the suppliers have goes to the recipients.

- In the case of unbalanced transport tasks, we have $\sum_{i=1}^{m} d_i \neq \sum_{i=1}^{n} o_j$. If the surplus is on the side of suppliers, then

$$\sum_{i=1}^{m} d_i > \sum_{i=1}^{n} o_j \tag{9}$$

The set of feasible solutions is defined as

$$D = \left\{ \begin{array}{c} X \in \mathbb{R}^{m \times n}: \quad x_{ij} > 0, \quad 1 \leq i \leq m, 1 \leq j \leq n \\ x_{11} + \ldots + x_{1n} \leq d_1 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ x_{m1} + \ldots + x_{mn} \leq d_m \\ x_{11} + \ldots + x_{m1} = o_1 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ x_{1n} + \ldots + x_{mn} = o_n \end{array} \right\} \tag{10}$$

In the definition of the set of permissible D we have n equalities (because the recipients' demand will be met) and m inequalities (which means that not all goods will be exported from suppliers).

- In the case of transport tasks unbalanced with the excess demand of customers, the set of acceptable solutions is defined as
-

$$D = \left\{ \begin{array}{c} X \in \mathbb{R}^{m\times n}: \quad x_{ij} > 0,\ 1 \le i \le m, 1 \le j \le n \\ x_{11} + \ldots + x_{1n} = d_1 \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ x_{m1} + \ldots + x_{mn} = d_m \\ x_{11} + \ldots + x_{m1} \le o_1 \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ x_{1n} + \ldots + x_{mn} \le o_n \end{array} \right\} \tag{11}$$

In the definition of the set of permissible *D*, we have *m* equalities (because all goods from suppliers will be collected) and *n* inequalities (customer demand will not be fully satisfied).

Note. In the case of the transport task, we determine the transport matrix $X \in \mathbb{R}^{m\times n}$, therefore, in the linear programming task, $m \times n$ variables must be taken into account, i.e. if we have 4 suppliers and 5 recipients, then the model should take into account 20 variables.

4. Problem of Optimal Export of Goods from the Supplier

The task is to deliver goods from one supplier to n recipients/customers, $n \in \mathbb{N}$.

- We assume an oversupply, ie orders placed by customers will be completed in full.
- The load capacity of the vehicles that deliver the goods is limited and amounts to *l* (e.g. tons, pallets).
- Entire trains are shipped directly to their recipients.
- Incomplete trains are grouped and then sent to their recipients.

Let $z = (z_1, z_2, \ldots, z_n)$ be the order vector (load requirement), where z_i denotes the order quantity from the *i*-th customer (e.g. the total number of pallets on which the order will be completed).

The number of whole trains (number of vehicles fully loaded) is determined as $v = \left(\left[\frac{z_1}{l}\right], \left[\frac{z_2}{l}\right], \ldots, \left[\frac{z_n}{l}\right]\right)$. Entire warehouses are shipped directly to customers. After taking into account the entire compositions, there remains the problem of delivering the rest (e.g. pallets) $r = (r_1, r_2, \ldots, r_m)$, where $r_i = \operatorname{mod}(z_i, l)$ - the remainder from dividing z_i by *l* and $m \le n$. In the vector of residuals *r* we take into account only non-zero values $r_i \neq 0$, we select only those recipients to which the load should be delivered. In order not to generate additional costs (we can only send an incomplete stock to one customer), we can accumulate the rest of the order and send one vehicle to several delivery points, previously allowing for loading. Additionally (e.g. due to the driver's working time), we can send the completed train to $k \leq m$ delivery points. Therefore, the number of possible exports is

$$\binom{m}{1} + \binom{m}{2} + \cdots + \binom{m}{k} = d \qquad (12)$$

Thus, for each export method, we define a transport vector of the form $s^i = (s_{1i}, s_{2i}, \ldots, s_{mi})$, where $s_{ji} \in \{0,1\}$, $1 \leq i \leq d$, $1 \leq j \leq m$. The quantity $s_{ji} = 1$ means that during the i -th method of export, the order is delivered to j-recipient, while $s_{ji} = 0$ means that during the i -th method of export, no transport is sent to the j-recipient. Thus, the vector s_i defines the customers (delivery points) for the i-th delivery method.

Note: When selecting the methods of loading, one should take into account the condition that the accumulated load to suppliers does not exceed the load capacity of the vehicle, $\langle s^i, r \rangle \leq l$.

Thus, we define a matrix of modes of character transport

$$S = [s^1, s^2, \ldots, s^d] = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1d} \\ s_{21} & s_{22} & \cdots & s_{2d} \\ \cdot & \cdot & \cdots & \cdot \\ s_{m1} & s_{m2} & \cdots & s_{md} \end{bmatrix} \qquad (13)$$

For the method s^i, $1 \leq i \leq d$ we define the cost of transport (or the number of kilometers traveled, which directly affects the cost of transport). Thus, $c = (c_1, c_2, \ldots, c_d)$ vector of transport costs.

Note. The method of determining the length of the route for the delivery method s^iv in the chapter entitled "the problem of determining the optimal route".

Let $x = (x_1, x_2, x_d)$ denote a vector of decision variables implementing the method of delivering goods to customers, $x_i \in \{0,1\}$ for $1 \leq i \leq d$. If $x_i = 1$ then the way s^i is the transport of the goods, if $x_i = 0$ then the way s^i and the transport is not performed.

Therefore, in the above-mentioned task, we can present a mathematical model

$$\min_{x \in D} \langle x, c \rangle \qquad (14)$$

where a set of feasible solutions

$$D = \left\{ \begin{matrix} x \in \{0,1\}^d, \\ s_{11}x_1 + s_{12}x_2 ... + s_{1d}x_d = 1, \\ s_{21}x_1 + s_{22}x_2 ... + s_{2d}x_d = 1, \\ ..., \\ s_{m1}x_1 + s_{m2}x_2 ... + s_{md}x_d = 1 \end{matrix} \right\} \tag{15}$$

For any $1 \leq k \leq m$ the restriction $s_{k1}x_1 + s_{k2}x_2 ... + s_{kd}x_d = 1$ means that only one of the vehicles will reach the *k*th delivery point.

5. Problem of Determining the Optimal Route

Transport is an indispensable element of the functioning of every enterprise on the market. The problem of determining the optimal route is becoming more and more important in the modern economy. Below, we will consider TSP related to the search for the best possible route and the development of a transport route plan between specific points. The optimization criterion is the minimization of the total transport costs, kilometers traveled or the time needed to complete the task. Determining the optimal route is one of the basic goals of logistic activities. The task is to arrange the delivery point in such a sequence that the length of the route is as short as possible, and thus - saving time and reducing costs.

Suppose the driver has to deliver the goods to n points (knots). To solve TSP, we define $C = \{c_{ij}\} \in \mathbb{R}^{n \times n}$ distance matrix between points, i.e. the magnitude c_{ij} $1 \leq i, j \leq n$ denote the length between *i*-th and *j*-th nodes. Let $X \in \{0,1\}^{n \times n}$ denote the journey matrix, where $X = \{x_{ij}\}_{1 \leq i, j \leq n}$ and $x_{ij} \in \{0,1\}$. If $x_{ij} = 1$, the driver travels the route from point *i* to point *j*, otherwise $x_{ij} = 0$ and the driver does not go from point *i* to point *j*. Each node can be visited only once by the driver, i.e. from each node there is a drive to exactly one other node. The size $\langle X, C \rangle$ denotes the distance traveled. The solution of the problem of choosing the optimal route consists in solving the linear programming problem

$$\min_{x \in D} \langle X, C \rangle \tag{16}$$

where the set of feasible solutions is of the form

$$D = \left\{ \begin{array}{l} x \in \{0,1\}^n, \\ x_{11} + x_{12} \dots + x_{1n} = 1, \\ \dots \\ x_{n1} + x_{n2} \dots + x_{nn} = 1, \\ x_{11} + x_{21} \dots + x_{n1} = 1, \\ \dots, \\ x_{1n} + x_{2n} \dots + x_{nn} = 1, \\ x_{11} = 0, \dots, x_{nn} = 0 \\ u_1 = 1, \\ 2 \leq u_i \leq n, \ for \ i \neq 1 \\ u_i - u_j + 1 \leq (n-1)(1 - x_{ij}), \ for \ i,j \neq 1 \end{array} \right\} \qquad (17)$$

The restriction $\sum_{j=1}^{n} x_{ij} = 1,\ 1 \leq i \leq n$ means that the driver leaves the point and to only one other point, the restriction $\sum_{i=1}^{n} x_{ij} = 1,\ 1 \leq j \leq n$ means that the driver will only come to point *j* from one other point. The constraint $x_{ii} = 0,\ 1 \leq i \leq n$ means that the driver will not stay at point *i*. The introduction of dummy variables $u_1, \dots, u_n$ made it possible to exclude circular routes between points *i* and *j* (i.e. exclude connections in the route from point *i* to *j* and *zj* to i). This task is called the Miller-Tucker-Zemlin (MTZ) problem (Pataki, 2003; Diaby, 2007). As a result of solving the problem, we obtain the X matrix, the elements of which describe the solution of the TSP problem. We use osmar, osrm, tmaptools, and TSP libraries to solve the task in R language (Walesiak and Gatnar, 2009; Lordan, Fernandez and Sallan, 2015). Departure from point 20-148 Lublin, ul. Uniikowa 26 and delivery of goods to the following addresses: 03-153 Warszawa, ul. Przylesie 3, 05-840 Brwinów, Moszna Parcela 29, 10-417 Olsztyn, ul. Towarowa 2, 26-600 Radom, ul. Kielecka 166, 19-300 Ełk, ul. Gdańska 31, 21-500 Biała Podlaska, ul. Sidorska 42.

Table 1. *Delivery point locations*

Id	Address	lat	lon	lat_min	lat_max	lon_min	lon_max
1	20-148 Lublin, Związkowa 26	51.26930	22.56045	51.26918	51.26947	22.56019	22.56073
2	03-153 Warszawa, Przylesie 3	52.34570	20.95364	52.34565	52.34575	20.95359	20.95369
3	05-840 Brwinów, Moszna Parcela 29	52.17672	20.74479	52.17622	52.17732	20.74427	20.74531
4	10-417 Olsztyn, Towarowa 2	53.78057	20.50166	53.78052	53.78062	20.50161	20.50171
5	26-600 Radom, Kielecka 166	51.40728	21.12293	51.40722	51.40728	21.12284	21.12293
6	19-300 Ełk, Gdańska 31	53.83026	22.35233	53.83017	53.83037	22.35198	22.35263
7	21-500 Biała Podlaska, Sidorska 42	52.02396	23.13852	52.02391	52.02401	23.13847	23.13857

Source: *Own creation.*

The Tables below show the results of the solutions.

Table 2. *Applied methods and results of solutions*

Method	Length of the route	Delivery direction
identity	1501.245	1->2->3->4->5->6->7
random	1607.210	1->7->2->3->4->5->6
nearest_insertion	1005.775	1->7->6->4->2->3->5
farthest_insertion	1005.775	1->5->3->2->4->6->7
cheapest_insertion	1005.775	1->5->3->2->4->6->7
arbitrary_insertion	1005.775	1->5->3->2->4->6->7
nn	1232.465	1->5->3->2->7->6->4
repetitive_nn	1005.775	1->7->6->4->2->3->5
two_opt	1005.775	1->7->6->4->2->3->5

Source: *Own creation.*

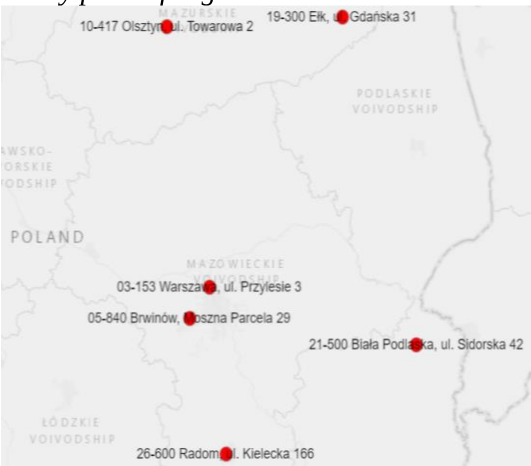
Table 3. *Delivery sequence table. The total distance is: 1005.775 km*

No	Delivery point
1	20-148 Lublin, ul. Związkowa 26
2	26-600 Radom, ul. Kielecka 166
3	05-840 Brwinów, Moszna Parcela 29
4	03-153 Warszawa, ul. Przylesie 3
5	10-417 Olsztyn, ul. Towarowa 2
6	19-300 Ełk, ul. Gdańska 31
7	21-500 Biała Podlaska, ul. Sidorska 42

Source: *Own creation.*

Figure 1 shows a map of goods delivery points. The total route can be presented in the form of the cycle shown in Figure 2. Figure 3 shows the route with the individual points marked.

Figure 1. *Map of delivery points for goods*



Source: *Own creation.*

Figure 2. *Total travel distance of the vehicle*

Source: *Own creation.*

Figure 3. *Vehicle route with individual points marked*

Source: *Own creation.*

6. Conclusions

The article presents the optimization of logistics and distribution in the supply chain. For this purpose, the class of optimization problems was used, in which the limiting conditions and the objective function are defined. The problem of delivering goods from one supplier to many recipients was analyzed. The issue related to the determination of the optimal route consisted in searching for the best possible solution and developing a transport route plan between specific points. The optimization criterion was to minimize the total transport costs, kilometers traveled and the time needed to complete the task. The problem was to arrange the delivery point in such a way that the route length was as short as possible, thus saving time and costs. As a result of the analyzes and calculations performed, the optimization task was performed. With the help of the prepared software, a map of goods delivery points, the total route and a route with individual points marked, and various solution methods were tested.

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