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## Covid-19 Death Risk Estimation Using VaR Method

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**Abstract:**

**Purpose:** The purpose of this paper is to show that the Value at Risk (VaR) method can be used to estimate the death rate from Covid-19 infection.

**Design/Methodology/Approach:** The VaR method allows for risk measurements and estimations of the highest expected loss on a portfolio at an assumed confidence level over a specified time horizon. The most important assumption affecting the calculation method is that price changes in financial markets follow a normal distribution.

**Findings:** It appears that by appropriately re-defining the concepts of assets and portfolio rates of return, we can describe the volatility in the numbers of deaths caused by Covid-19. We also confirmed using the Shapiro-Wilk and Skewness and Kurtosis tests that the rates of return distribution for the death numbers follow a normal distribution.

**Practical Implications:** The VaR method allows to estimate the number of deaths based on current trends which can be utilised to better manage available resources in order to reduce casualties. We use the data regarding the number of deaths in the Visegrad Group (V4) countries as a case study to test the effectiveness and accuracy of the VaR method in a different, non-financial domain.

**Originality/Value:** The theory we used in this paper is currently mainly applied to financial investments. We use this theory to describe social phenomenon which is the number of deaths, our approach has not been seen in the literature so far.

**Keywords:** Portfolio, Value at Risk, volatility, Covid-19 cases of deaths.

**JEL codes:** G32, C15, C22.

**Paper Type:** Research paper.

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## 1. Introduction

In the financial world, there are many mathematical models, which attempt to describe the changes occurring in the financial markets. When financial investments are involved, it is crucial to skilfully estimate the risks involved. Classic risk measures include the investment return variance, the standard deviation of that rate, the average deviation, the quarter deviation, the semivariance of the return, the semi-standard deviation, the securities beta coefficients (Jajuga, 2015).

With the development of financial markets and the emergence of more participants in these markets, along with an increase in financial instrument types a need to search for new risk measures has arisen. At the end of the 1980s, several international financial institutions introduced the concept of Value at Risk (VaR). The VaR method has gained great popularity among investors. In 1994, J.P. Morgan and Reuters created the RiskMetrics™ system (J.P. Morgan and Reuters, 1996). Since then, VaR has become a very popular risk estimation tool, mainly because of their link to Modern Portfolio Theory (Barone-Adesi and Giannopoulos, 2001). The publications of P. Jorion (Jorion, 1995; 2007; 2011) have contributed to this popularity greatly.

Currently, there is a global emergency caused by Covid-19 pandemic, which introduced the risk of death from SARS-CoV-2. In this paper we attempt to estimate this risk using models known from financial theory. Stock prices fluctuate from minute to minute, hour to hour, day to day and are time series, so the data on the number of deaths from SARS-CoV-2 can also be represented using such series. The number of deaths, its rate of change and the risk over a given time horizon in a fixed group of countries can be estimated similarly to how it is done for investment portfolios. The research object, whose four-component "portfolios" will be considered, consist of the four selected Central European countries forming the Visegrad Group (V4), namely Poland, Hungary, Czech Republic, and Slovakia. To predict the number of deaths in these countries, the previously mentioned risk estimation tool and portfolio theory will be used. This approach in death risk modelling necessitates appropriate definitions of concepts such as stock, stock value, portfolio value, portfolio return, and the risk of that return.

## 2. Value at Risk

Value at Risk (*VaR*) is currently one of the most popular risk measures and is defined as follows:

*Definition: Value at Risk refers to the maximum loss, which should not be exceeded during a specified period of time with a given probability level.*

In the portfolio theory *VaR* is a measure of the maximum potential change in value of a portfolio of financial instruments with a given probability over a pre-set

horizon (J.P. Morgan and Reuters, 1996). Using the probability concept,  $VaR$  is the loss that, with a given probability at a fixed time, will not be exceeded. Accordingly,  $VaR$  is defined as follows:

$$P(V_{T+1} \leq V_T - VaR) = \alpha, \tag{1}$$

where:

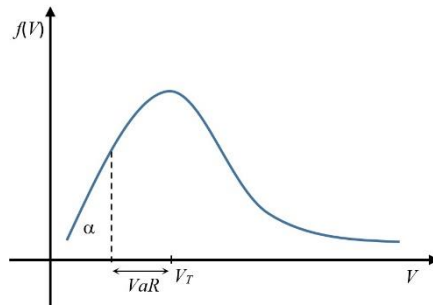
$V_{T+1}$  – portfolio value at the end of the considered period,

$V_T$  – portfolio value at initial time  $T$ ,

$\alpha$  – significance level.

From the aforementioned definition, it follows that  $VaR$  depends on two parameters: the time interval and the significance level  $\alpha$ .

**Figure 1.** Graphical interpretation of Value at Risk



**Source:** Own creation.

A graphical interpretation of the Value at Risk is shown in Figure 1 for a continuous distribution of investment values illustrated in the form of a density curve  $f(V)$ . The investment value at time  $T$  is marked on the horizontal axis along with the distance  $VaR$ , which is the distance between the  $V_T$  and the distribution quantile. Therefore, in order to determine the  $VaR$ , we need to know the cumulative distribution function of the investment value or the cumulative distribution function of the rate of returns of the portfolio along with the transformed (1):

$$P\left(\frac{V_{T+1} - V_T}{V_T} \leq -\frac{VaR}{V_T}\right) = \alpha.$$

The methods allowing the estimation of the cumulative distribution, and hence  $VaR$ , can generally be divided into two categories: parametric and non-parametric methods.

A parametric method is based on the parametric class of distribution or, alternatively, on a model in which some parameter needs to be estimated. One of the simpler parametric methods is the variance-covariance method (J.P. Morgan and Reuters, 1996). This class of market risk techniques are based on strong

theoretical assumptions and rules. They impose that the distribution of the data conforms to a known theoretical distribution. This method assumes that the return distribution of the portfolio components is normal. The normality assumption is frequently used because the normal distribution is well described. Other distributions can be used, but at a higher computational cost (Barone-Adesi and Giannopoulos, 2001).

A non-parametric approach is based on the empirical distribution. The most popular non-parametric method is historical simulation, which makes use of historical data by constructing the cumulative distribution function of financial returns over time for predicting *VaR*. A combination of these two approaches, so called semi-parametric methods can also be used. These methods and their advantages and disadvantages have been discussed in (Abad, Benito, and López, 2014; Holton, 2003; Jorion, 2007; J.P. Morgan and Reuters, 1996; Mentel, 2011). In this paper historical data and variance-covariance method are used.

If the distributions of portfolio component rates of return are normal, then the distribution of portfolio rates of return is also normal with mean  $\mu_T$  and variance  $\sigma_T^2$ , which we will write  $R_T \sim N(\mu_T, \sigma_T)$ , and then:

$$\int_{-\infty}^{r^*} \frac{1}{\sigma_T \sqrt{2\pi}} e^{-(x-\mu_T)^2/2\sigma_T^2} dx = \alpha, \quad (2)$$

where  $r^* = -\frac{VaR}{V_T}$  is a small negative number. Hence:

$$VaR = |r^*|V_T. \quad (3)$$

There are the standard mathematical properties of the normal distribution to determine the *VaR* (Grotke, 2010).

### 3. Portfolio Characteristics

Let the portfolio  $T$  consist of  $K$  assets at a fixed time. The logarithmic rates of return on the asset price  $i$  at time  $t$  are defined as follows:

$$R_{ti} = \ln\left(\frac{P_{ti}}{P_{t-1i}}\right),$$

where  $P_{ti}$  is the price of asset  $i$  at time  $t$ ,  $P_{t-1i}$  is the price of the asset  $i$  at time  $t-1$ . We assume that for each  $i$  the logarithmic rates of return have a normal distribution  $N(\bar{R}_{T,i}; \sigma_{T,i})$ , where;

$$\bar{R}_{T,i} = \frac{\sum_{t=2}^T R_{ti}}{T-1}, \quad (4)$$

$$\sigma_{T,i} = \sqrt{\frac{\sum_{t=2}^T (R_{ti} - \bar{R}_{T,i})^2}{T-2}} \tag{5}$$

are the arithmetic mean of returns rates achieved in the past based on  $T - 1$  observations and the standard deviation of returns respectively.  $T$  is the purchase date of the asset.

The average rate of return on investments in the portfolio is expressed by the formula:

$$\mu_T = \sum_{i=1}^K w_{T,i} \bar{R}_{T,i}, \tag{6}$$

Where:

$$w_{T,i} = \frac{a_i * P_{T,i}}{V_T} \tag{7}$$

are the weights being the value shares of individual assets in the portfolio,  $a_i$  is the number of asset  $i$  in the portfolio and

$$V_T = \sum_{i=1}^K a_i P_{T,i} \tag{8}$$

is the portfolio value.

The variance of the rate of return on a portfolio investment is expressed by the formula:

$$\sigma_T^2 = \mathbf{w}_T^T \mathbf{S} \mathbf{w}_T, \tag{9}$$

where  $\mathbf{w}$  is the weight vector of the form:

$$\mathbf{w}_T = \begin{bmatrix} w_{T,1} \\ w_{T,2} \\ \dots \\ w_{T,K} \end{bmatrix}, \tag{10}$$

and  $\mathbf{S}$  is the variance and covariance matrix of returns determined by the formula

$$\mathbf{S} = \boldsymbol{\sigma} \mathbf{C} \boldsymbol{\sigma},$$

where  $\boldsymbol{\sigma}$  and  $\mathbf{C}$  are matrices of the form:

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{T,1} & 0 & \dots & 0 \\ 0 & \sigma_{T,2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma_{T,K} \end{bmatrix} \tag{11}$$

$$C = \begin{bmatrix} 1 & \rho_{12} & \dots & \rho_{1K} \\ \rho_{21} & 1 & \dots & \rho_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{K1} & \rho_{K2} & \dots & 1 \end{bmatrix}. \tag{12}$$

$\rho_{jk}$ ,  $j = \{1,2, \dots, K\}$ ,  $k = \{1,2, \dots, K\}$  is the correlation coefficient of the rates of return for  $j, k$ .

#### 4. Results for Portfolio for V4

The use of VaR method in modelling the risk of death requires an appropriate re-definition of the following concepts: asset, asset value, portfolio value, rate of return on the investment in the portfolio, the risk of the rate of return and the risk of a portfolio value loss. The number of deaths will be analyzed in four chosen Central European countries forming the Visegrad Group (V4), i.e., Poland, Hungary, the Czech Republic and Slovakia. These countries will make the components (assets) of the four-component portfolio, corresponding to the indices  $i=\{1,2,3,4\}$ , respectively. Table 1 shows the population of V4 countries in 2019 (GitHub, 2021). We denote the population of the country  $i$  by  $N_i$  and we assume that these numbers will be constant over the period of our study. Certainly, the country's population changes on a daily basis, but it is impossible to verify this data.

**Table 1.** V4 countries population

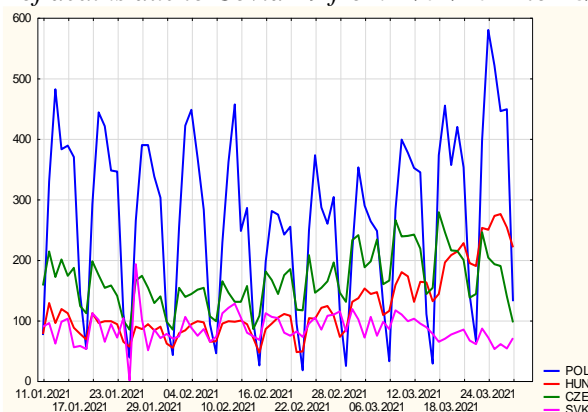
Country	$i$	$N_i$
Poland	1	37846605
Hungary	2	9660350
Czech Republic	3	10708982
Slovakia	4	5459643

**Source:** Own creation on the basis of data from (GitHub, 2021).

The number of deaths due to Covid-19 from 11/01/2021 to 28/03/2021 used in the calculation comes from the data available on the (GitHub, 2021) website. Figure 2 shows the number of deaths in Poland, Hungary, the Czech Republic and Slovakia during this period. The graph shows that the numbers of deaths in the considered countries, especially in Poland, are strongly dependent on the day of the week. Accordingly, given that the number of deaths is a reflection of the number of infections 14 days earlier, the cumulative number for 14 days of Covid-19 death cases per 100000 people,  $t = \{1,2, \dots T\}$ ,  $i = \{1,2,3,4\}$  was used as an equivalent for the price of the asset  $P_{t,i}$  in the portfolio. Therefore, the number of asset  $i$  in the portfolio at time  $T$  given by  $a_i$  in the formula (7) is defined as follows

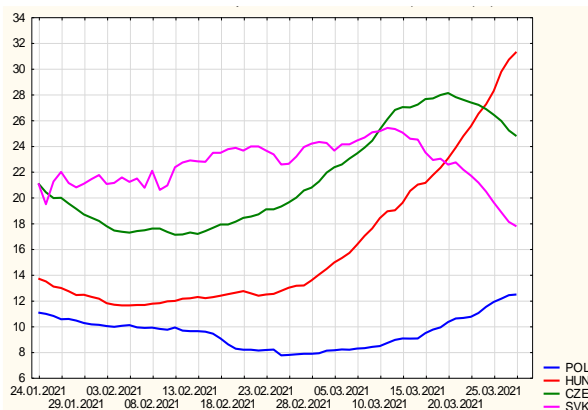
$$a_i = \frac{N_i}{100000}.$$

$N_i$  is the population of the country  $I$ , see Table 1.

**Figure 2.** Number of deaths due to Covid-19 from 11/01/2021 to 28/03/2021

*Source: Own creation.*

Two cases  $T = 62$  and  $T = 63$ ; are considered in this paper;  $t = 1$  corresponds to the period from 11/01/2021 to 24/01/2021,  $t = 2$  is the period from 12/01/2021 to 25/01/2021, ...,  $t = 62$  is the period from 13/03/2021 to 26/03/2021, and  $t = 63$  is the period from 14/03/2021 to 27/03/2021. Figures 3 and 4 show the cumulative number for 14 days of Covid-19 cases of deaths per 100000 people in Poland, Hungary, the Czech Republic and Slovakia in the period from 01/24/2021 to 03/28/2021 and the logarithmic returns of the prices of the assets in the portfolio in the period from 01/25/2021 to 03/28/2021 respectively.

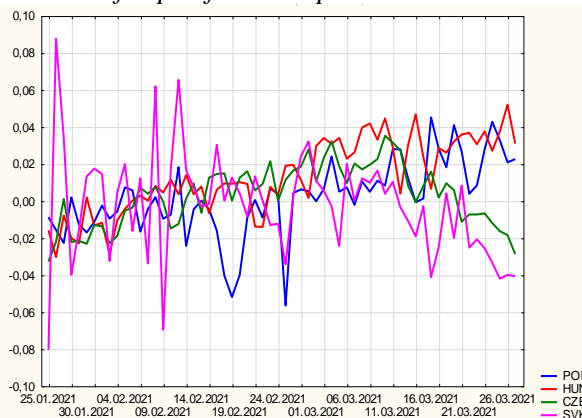
**Figure 3.** Cumulative number for 14 days of Covid-19 cases of deaths per 100000 people

*Source: Own creation.*

The situation regarding the number of infection and deaths in all considered countries is very variable. Analysing the data from the beginning of the pandemic until 28 March 2021, we observed that, in the case of Hungary and the Czech Republic, the normal distribution tests show the rates of return are distributed

normally almost regardless of the time period. In the case of the data coming from Poland and Slovakia, normality occurs only for some periods.

**Figure 4.** Relative returns for portfolio components



*Source:* Own creation.

The results of the Shapiro-Wilk and Skewness and Kurtosis tests are summarised in Tables 2 and 3 for  $T = 62$  and  $T = 63$  respectively. The  $p$  - *value* shown in Table 2 is greater than 0.05 almost in every case (except for the logarithmic rates of return for Poland for the case of  $T = 62$ ), all values of Skewness/SE and Kurtosis/SE presented in Table 3 fall into the range  $(-3.29, 3.29)$ . Based on these results, it was assumed that in both cases, for  $T = 62$  and  $T = 63$  the distributions of logarithmic rates of return are normal.

**Table 2.** Shapiro-Wilk test results for the logarithmic rate of return of portfolio components

	country	W	$p$ - value
$T=62$	POL	0.96071	0.04787
	HUN	0.97305	0.19702
	CZE	0.97937	0.39118
	SVK	0.96178	0.05412
$T=63$	POL	0.96244	0.05500
	HUN	0.97139	0.15610
	CZE	0.97892	0.3628
	SVK	0.96343	0.06169

*Source:* Own creation.

**Table 3.** The results of the Skewness and Kurtosis normality tests for the logarithmic rates of return of the portfolio components

	country	Skewness	Kurtosis	Skewness/SE	Kurtosis/SE
$T = 62$	POL	-0.40997	1.052267	-1.33857	1.742634
	HUN	-0.15319	-0.76702	-0.50016	-1.27025
	CZE	-0.17387	-0.71382	-0.56771	-1.18214
	SVK	0.169443	1.63897	0.553247	2.714259



$T = 63$	POL	-0.435722	1.023622	-1.43376	1.708063
	HUN	-0.184016	-0.769760	-0.60551	-1.28446
	CZE	-0.188617	-0.737498	-0.62065	-1.23062
	SVK	0.189715	1.508779	0.624263	2.517619

**Source:** Own creation.

The  $\sigma$  and  $C$  matrices defined by the formulas (3.8) and (3.9) based on historical observations are as follows:

For  $T = 62$ :

$$\sigma = \begin{bmatrix} 0.0204 & 0 & 0 & 0 \\ 0 & 0.0201 & 0 & 0 \\ 0 & 0 & 0.0160 & 0 \\ 0 & 0 & 0 & 0.0289 \end{bmatrix}, \tag{13}$$

$$C = \begin{bmatrix} 1 & 0.4163 & 0.1020 & -0.1766 \\ 0.4163 & 1 & 0.4023 & -0.1674 \\ 0.1020 & 0.4023 & 1 & 0.1875 \\ -0.1766 & -0.1674 & 0.1875 & 1 \end{bmatrix}.$$

For  $T = 63$ :

$$\sigma = \begin{bmatrix} 0.0204 & 0 & 0 & 0 \\ 0 & 0.0201 & 0 & 0 \\ 0 & 0 & 0.0165 & 0 \\ 0 & 0 & 0 & 0.0290 \end{bmatrix}, \tag{14}$$

$$C = \begin{bmatrix} 1 & 0.4249 & 0.0656 & -0.1947 \\ 0.4249 & 1 & 0.3585 & -0.18 \\ 0.0656 & 0.3585 & 1 & 0.2213 \\ -0.1947 & -0.18 & 0.2213 & 1 \end{bmatrix}.$$

The results obtained for the different time periods analysed in this paper revealed that the variance of the rate of change of the number of deaths in the studied countries is almost constant over time, see (13) and (14). Therefore, it was concluded that the models taking variance variation into account e.g. GARCH model or exponentially weighted moving average model recommended in many other works (Abad, Benito, and López, 2014; Mentel, 2011) are redundant.

Weight vectors determined on the basis of formulas (7) and (10) have the following form:

$$w_{62} = \begin{bmatrix} 0.407055 \\ 0.255198 \\ 0.24649 \\ 0.091256 \end{bmatrix}, \quad w_{63} = \begin{bmatrix} 0.413577 \\ 0.261476 \\ 0.237906 \\ 0.087041 \end{bmatrix}.$$

**Table 4.** The means and variances of the portfolio components and the mean, variance and value of the portfolio at time  $T$ 

	country	$\bar{R}_{T,i}$	$\sigma_{T,i}^2$	$\mu_T$	$\sigma_T^2$	$V_T$
$T = 62$	POL	0.0015253	0.000417339	0.004543	0.00017	11254
	HUN	0.0127439	0.000404361			
	CZE	0.003403	0.000259112			
	SVK	-0.001854	0.000833129			
$T = 63$	POL	0.0018628	0.000417558	0.004649	0.00017	11328
	HUN	0.013036	0.000403023			
	CZE	0.0028821	0.000271687			
	SVK	-0.002482	0.000843861			

*Source:* Own creation.

Table 4 reports the means and the variances of the portfolio components at time  $T$  calculated using formulas (4) and (5), respectively, and the mean and variance of the portfolio calculated using formulas (6) and (9), respectively, for the two cases  $T = 62$  and  $T = 63$ . Furthermore, Table 4 shows the value of the portfolio at the "time of purchase" - on the day preceding the forecast determined by the formula (8). Therefore, the  $VaR$  values determined from the formula (3) considering formulas (2), (6) and (9) are respectively:

$$VaR(T = 62) = 189.1424,$$

$$VaR(T = 63) = 189.3958.$$

Based on the results for  $T=62$  it is possible to say that with a probability of 0.05 an event will occur that at time  $T=63$ : the total number of deaths in V4 countries will be equal to or less than 11065 ( $V_T - VaR = 11254 - 189 = 11065$ ) i.e., with a probability of 0.95 the total number of deaths will be greater than 11065. The actual 14-day cumulative number of deaths at time  $T=63$  is 11328 (Table 4). Based on the results for  $T=63$ , it can be said that with a probability of 0.05 an event will occur that at time  $T=64$ : the total number of deaths will be equal to or less than 11139 ( $V_T - VaR = 11328 - 189 = 11139$ ) i.e., with a probability of 0.95 the total number of deaths will be greater than 11139. The actual 14-day cumulative number of deaths as of 03/28/2021 is 11342 (GitHub, 2021).

It is important to note that the calculation methods used report the  $VaR$  for the total number of deaths in V4 over 14 days, where for the initial 13 days, the number of deaths is known. Subtracting the corresponding values yields a death number forecast for day 14.

Note that for  $T=62$  the projected number of deaths as of 27/03/2021 is  $11065 - 10435 = 630$ , (in the 13 days to 26/03/2021 the number of deaths due to Covid-19 was 10435), which means that at least 630 deaths are projected in the V4 countries as of 27/03/2021. Actually, the number of deaths on that day was 893 (GitHub, 2021).

For  $T=63$  the projected number of deaths on 28/03/2021 is  $11139 - 10825 = 314$ , (in the 13 days until 26/03/2021, the number of deaths due to Covid-19 was 10825), which means that at least 314 deaths are projected in the V4 countries on 28/03/2021. In fact, the number of deaths on that date was 517 (GitHub, 2021).

## 5. Conclusions

The daily numbers of deaths in various countries in the world are publicly available and form time series similarly to the financial data. The paper shows that with properly defined portfolio characteristics, the VaR method can be applied to the problems beyond the financial issues. The obtained predictions of the number of deaths are interpreted as the lowest number of deaths that can occur in a given period with a fixed high probability. According to our results, based on the number of deaths in the Visegrad Group (V4) countries, the VaR method can be applied to data on the number of deaths in different regions or countries.

In the paper we assumed the normal distribution of the rates of return. While this assumption is generally not met for financial assets, it is reasonable for the rate of change in the number of deaths in the countries selected for analysis in this paper. In the studies on the VaR, it is often emphasized (Abad, Benito, and López, 2014; Mentel, 2011) that the implementation of a volatility measure, e.g. variance variation, appropriate to the considered case is a crucial part of the analysis. The implementation of the models accounting for variance variations for the issue considered in this paper is the subject of further work by the authors.

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