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## An Optimal Algorithm of Material Reserves Management based on Probabilistic Model

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**Abstarct:**

**Purpose:** In this paper a design of the probabilistic model of reserves is discussed. The objective was an elaboration of the optimal strategy of materials reserves management in the series of manufacturing.

**Design/Methodology/Approach:** The methodology of the Fundamental Power Index (FPI), the model of reserves has been verified as far as it was feasible using numerical data concerning various wood products including furniture of series production. The presented probabilistic model of reserves allows the formation of optimum strategy of the (R, Z) type of primary materials management with due consideration to profits coming from low-cost components in working and furniture industrial company.

**Findings:** The results of the study reflect that to ensure the production continuity, the reserves of different kinds of based material are generated and maintained. In the furniture industry, wide range and community of decisions, as well as, economic influence of wrong decisions concerning material reserves are of such importance, that they prove a need of overworking an optimal strategy of based material reserves management on the basis of the probabilistic model with an application of computer technique.

**Practical Implications:** The model has been verified at an attainable scale using the numerical data concerning different assortment of materials in the serial production of furniture. The proposed probabilistic model makes it possible to elaborate the optimal strategy of type R, Z in the management of reserves of based materials of the manufacturing company with large-lot production. The strategy is based on minimal expenses connected with a supply of materials.

**Originality/Value:** The constructed probabilistic model of wood reserves management has methodological value because it shows the method of working out an optimal strategy of basal reserves of based material management in the conditions of serial and poly-assortment production of furniture and other based products in special conditions of economical practice in the furniture industry.

**Keywords:** Modelling, probabilistic model, material reserves, management, strategy.

**JEL codes:** E620, E690.

**Paper type:** Research article.

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## **1. Introduction**

A functional strategy for the management of material stocks, developed on the basis of a probabilistic model, is a tool for modern, innovative management. The use of such a strategy in the management of a company's business under market conditions increases its competitiveness and operational efficiency. Efficient running of activity in an economic organization, especially in production, requires its continuity and evenness - rhythm. For this purpose, stocks of different types of basic materials are kept as they do not meet demand at certain times. In an economic organization, the economic impact of incorrect stock decisions is so important that it justifies the need for scientific management of stocks of basic materials using a mathematical model and computer technology (Korhonen and Laakso, 1986; Kang, Zhao, Li, and Horst, 2016).

In business practice, it is difficult to estimate the costs of ordering and warehousing materials and possible losses due to the lack of adequate stocks, because you need to know approximate and real cost estimates. An important issue is, therefore, the presentation of a probabilistic model of material inventory management, in which attention will be focused on the construction of the optimization criterion (objective function), which is of fundamental importance for the quantitative analysis of the stock issue and the development of an optimal material stock management strategy in an economic organization in the conditions of serial production (Carrington, Scott, and Wasserman, 1994; Flynn, Sakakibara, Roger, Schroeder, and Bates, 1990).

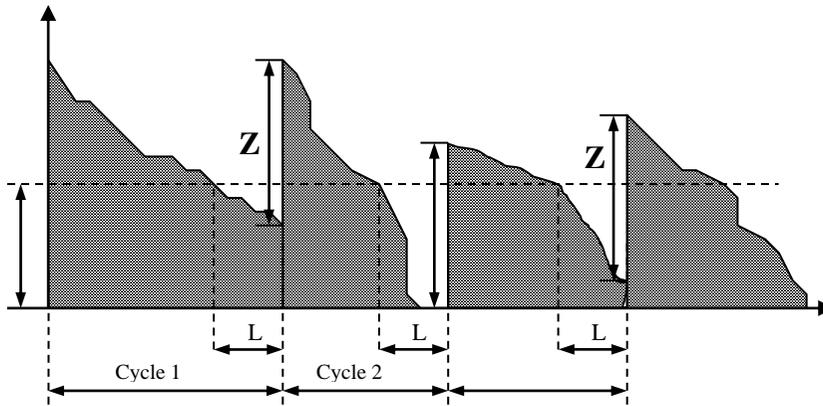
Reserves in the organization act as a buffer between the company and its suppliers and recipients. They ensure the safety of production continuity in the case of raw material supplies and guarantee the possibility of efficient execution of customer orders. However, operational security comes at a price. It is estimated that the average annual cost of maintaining inventories is 20% of their value (Guillon, Ayachi, Vareilles, Aldanondo, Villeneuve, and Merlo, 2021). Optimizing inventory in an organization is therefore not so much about the maximum reduction of their level (this is always partly at the expense of the organization's operational security), but about their adjustment (level, location, assortment) to the specificity of the company and the proper balancing of costs and the level of acceptable risk. The optimal inventory structure in an organization will always be strongly dependent on the industry, the level of production repeatability, the logistics and production structure, as well as the strategic (and sometimes operational) goals of the entire organization and the supply chain (Kannan, Khodaverdi, Olfat, Jafarian, and Diabat, 2013; Geng and Jiang, 2009).

## **2. Probabilistic Model of Material Reserves Management**

To ensure the production continuity, the reserves of different kinds of based material are generated and maintained (Stricker, Echsler, and Lanza, 2017; Zhang and Matta,

2020). In the furniture industry, wide range and community of decisions, as well as economic influence of wrong decisions concerning material reserves are of such importance, that they prove a need of overworking an optimal strategy of based material reserves management on the basis of the probabilistic model with an application of computer techniques (Farlow and Haggard, 1987).

**Figure 1.** Change in the level of material reserves in a store



$Z$  – value of order of special kind of material,  $R$  – value of safety reserve,  $L$  – period of delivery.

*Source: own elaboration.*

The level of wood material reserves in a store of furniture industry's differs during time, depending on input and output of these materials. Input of these materials depends on the amount of supplied material and time between deliveries, whereas output depends on the level of production consumption (Figure 1). A period of time between subsequent material deliveries (delivery cycle) fluctuates, as well as production consumption of different kinds of materials. The problem of based material management could be a source of opposite tendencies in a factory. There are groups of different factors (technical, organizational, economical, financial etc.) which influence high or low levels of material reserves (Tamas, Murar, 2019; Orellana, Torres, 2019). High levels of material reserves are concerned with the great costs of storage, but low levels influence costs as well, because of the lack of production consumption ensuing (the continuity not ensured). The level of based material reserves is maintained on certain rigidly determined levels. Keeping of high or low levels of materials in a store is a source of additional costs. The main objective of reserves management optimization should focus on analysis of costs of these materials, while criterion of optimization should be the minimization of value of expected medium costs of buying, storing and lack of production consumption ensuring of based materials. As a result of main (criterion) function optimization there is

to ensure the levels of based materials needed for the regular production program realization, with the lowest level of costs (Hilier and Lieberman, 1998; Kondratenko, 2015).

In order to construct the probabilistic model of reserves management such symbols are introduced:

$D$  – mean production consumption for special kind of material in certain time (e.g. medium years consumption),

$Z$  – value of order of special kind of material,

$D/Z$  – mean number of orders of special kind of material in certain time (e.g. during one year),

$R$  – value of safety reserve (for special kind of material),

$K$  – constant costs of orders,

$h$  – single cost of storing,

$p$  – single cost in case of lack of needed level of reserve for special kind of material in a store,

$L$  – period (time) of delivery (time for order realisation),

$v$  – production consumption for special kind of material in period of delivery,

$E(v)$  – wanted value of production consumption for special kind of material in period of delivery,

$g(v)$  – probability distribution of production consumption for special kind of material in period of delivery (function, of random variable density probability),

$b$  – mean level of lacking stores in period of delivery,

$B$  – mean level of lacking stores for special material in certain investigation time,

$E(B)$  – expected value of mean level of lacking stores for special material in certain time,

$A$  – uniform distribution - upper limit of function,

$E$  – operator of expected value,

$F(Z,R)$  – main function of model with decisive variables  $Z$  and  $R$ .

It should be noted, that by the end of delivery cycle expected level of reserve of certain based material in a store is  $R-E(v)$  but when a particular order is completed (on the beginning of cycle) such level is  $Z+R-E(v)$ . Expected medium level of reserves for certain kind material in the cycle (if  $v \leq R$ ) is equal to:

$$\frac{(Z + R - E(v)) + (R - E(v))}{2} = \left( \frac{Z}{2} + R - E(v) \right) \quad (1)$$

If,  $v > R$ , then medium level of lacking reserve of certain based material ( $b$ ) in a store is calculated:

$$b = \int_R^{\infty} (v - R)g(v)dv \tag{2}$$

Expected in a certain time (e.g. during one year) medium level of lacking stocks special based material (B) in a store is obtained by the equation:

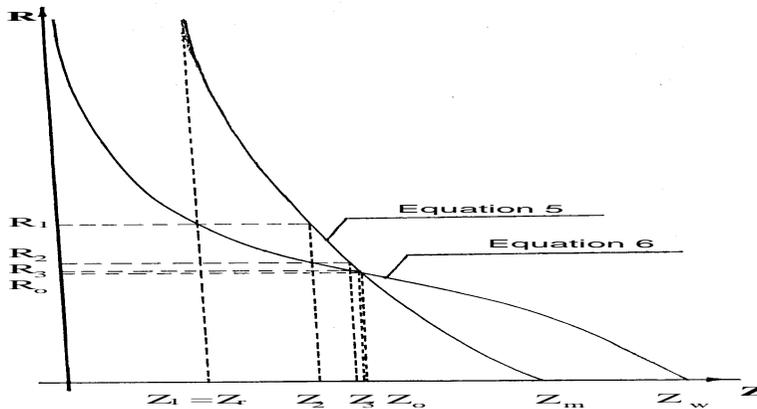
$$E(B) = b \cdot \frac{D}{Z} \tag{3}$$

Having calculated the certain levels of special material reserves, we can calculate appropriate costs, multiplying appropriate single costs h and p by the expressions (1) and (3). Thus, the main function of optimisation in the probabilistic model of material reserve management at a certain time can be expressed by the formula:

$$E[F(Z, R)] = K \frac{D}{Z} + h \left[ \frac{Z}{2} + R - E(v) \right] + p \frac{D}{Z} b \rightarrow \min \tag{4}$$

The first component of sum represents mean cost of constants, the second one is mean cost of storing reserves, whereas the third component means cost in case of lack kind of certain reserve. Changes in the level of quantities of basic values (Z and R) in the main function will be minimal when Z and R are optimal. Then, the dependencies should determine from which optimal values  $Z_0$  and  $R_0$  could be calculated. It is necessary, that for optimal values of the pair (R, Z) the first partial derivatives of the function (1) in a comparison with Z and R values will be zero.

**Figure 2.** Scheme of iteration of iteration research  $Z_0$  and  $R_0$



Source: Own elaboration.

Optimal values of order and store level R expressions could be found:

$$Z_0 = \sqrt{\frac{2D(K + pb)}{h}} \tag{5}$$

$$\int_{R_0}^{\infty} g(v)dv = \frac{hZ_0}{pD} \quad (6)$$

Should be underlined that the expressions (5) and (6) cannot be used for direct calculating of optimal values ( $Z_0$ ,  $R_0$ ). (Figure 2). Thus, an iterative method (process) of seeking  $Z_0$  and  $R_0$  values in the finite number of steps was elaborated [2]. The conditions of required convergence of the iterative method (existence of the solution of problem) satisfies the inequality as follows (Korhonen and Laakso, 1986):

$$\frac{pD}{h} > \sqrt{\frac{2D[K + pE(v)]}{h}} \quad (7)$$

i.e.,  $Z_w > Z_m$

### 3. Algorithm of Setting the Optimal Values

Beginning the iterative process with the first probable meaning of  $Z$  value equals  $\sqrt{2DK/h}$ , with the increase of iteration numbers the value of  $Z_i$  increases, when  $R_i$  value decreases. Hence, the iterative process is quickly convergent. It is recommended to use computer techniques to calculate the  $R_0$  and  $Z_0$  ( $R_0 = \lim_{i \rightarrow \infty} R_i$ ).

For this purpose, the computer software of operative scheme of  $R_0$  and  $Z_0$  was written. This program does calculations until the difference  $R_{i+1} - R_i$  values is adequately low (e.g., 0,00001). It means, that two calculated values are similar. For the optimal value  $R_0$  we use then  $R_{i+1}$  value, because  $R_0 \cong R_{i+1}$ . The optimal value of  $Z_0$  were estimated based on  $R_0(R_{i+1})$  (Figure 3, Figure 4).

It should be noted, that in case when there is even distribution of probability of production consumption of bases materials, the expressions (5) and (6) should be solved directly, i. e. optimal values of  $R_0$  and  $Z_0$  could be presented as follows:

$$R_0 = A \left[ 1 - p^{-1} \sqrt{\frac{2Kh}{D - Ah}} \right] \quad (8)$$

$$Z_0 = D \sqrt{\frac{2K}{h(D - Ah)}} \quad (9)$$

The formulas (8), (9) are based on case of even probability of production consumption of materials:

$$g(v) = \begin{cases} 0, & \text{if } v \notin ]0, A[ \\ \frac{1}{A}, & \text{if } v \in ]0, A[ \end{cases} \quad (10)$$

The integral in the expression (6) can be presented by using elementary functions. In general case, the presented simplification is not possible and iterative process of estimation  $R_0$  and  $Z_0$  should be then employed (Li, Guo, Yue, and Zhao, 2010).

#### 4. Empirical Verification of the Model (on Real Data)

The constructed probabilistic model is under empirical verification. The practical verification of the constructed model was performed for every main kind of based material used during serial production of furniture (kitchen sets, combined set, chairs, armchairs). In the calculations numerical data was used from the 6-year periods of production activities of furniture factories in Poland.

The verification showed, that worked out the iterative method of solving the model of reserves leads to estimation of optimal values of reserves  $R_0$  and order  $Z_0$  with minimal costs, and in this way leads to determination of optimal (R, Z) type strategy of based material reserves management in a factory (Mafakheri, Breton, and Ghoniem, 2011; Train, 1993).

The main rule of optimal strategy of reserves management is when the level of reserves of special based material in a store reaches value  $R_0$ , the order should be equal to the value of  $Z_0$ , so that mean costs concerned with reserves with time under consideration will be minimal.

The constructed probabilistic model of wood reserves management has methodological value because it shows the method of working out an optimal strategy of basal reserves of based material management in the conditions of serial and poly-assortment production of furniture and other based products in special conditions of economical practice in the furniture industry.

#### 5. Conclusions

The model has been verified at an attainable scale using the numerical data concerning different assortment of materials in the serial production of furniture. The proposed probabilistic model makes it possible to elaborate the optimal strategy of type R, Z in the management of reserves of based materials of the manufacturing company with large-lot production. The strategy is based on minimal expenses connected with a supply of materials. Therefore, three global conclusions can be drawn to optimize the inventory management processes in the organization.

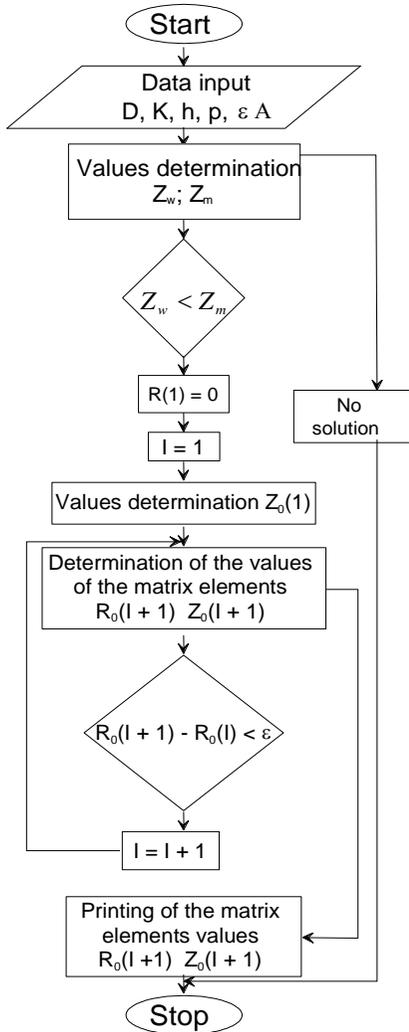


Figure 3.  $R_0$  and  $Z_0$  optimal values searching block diagram

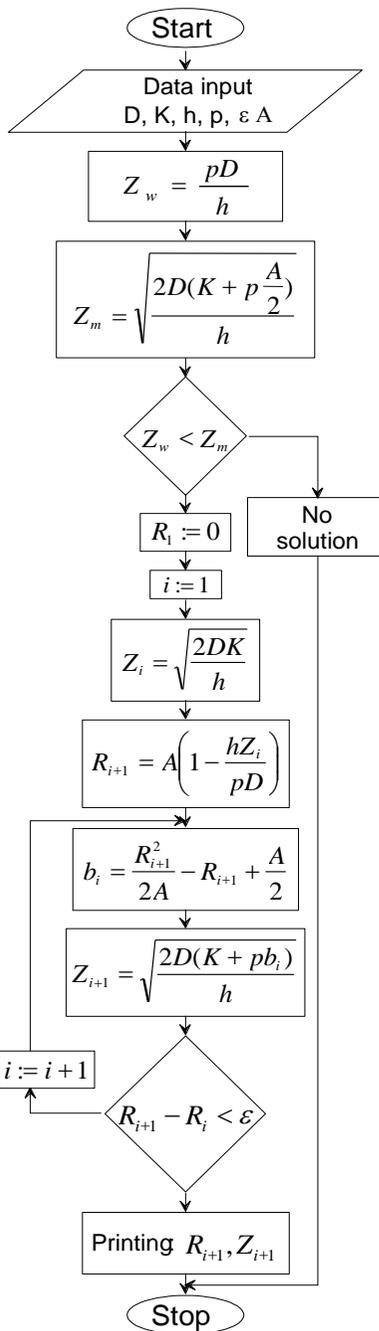


Figure 4.  $R_0$  and  $Z_0$  optimal values searching block diagram with uniform distribution

First of all, it is necessary to implement an inventory management model, the essence of which is to define the costs of maintaining all operating inventories at a minimum level that guarantees the functioning of the company in line with strategic goals. Its foundation is to define the main needs of the organization and the acceptable level of risk based on the analysis of the effectiveness of the replenishment methods, the introduction of product classifications to determine the significance of individual items, as well as the precise adjustment of operating methods.

Secondly, it is necessary to systemically manage the inventory of finished products, the task of which is to ensure and monitor the availability of the commercial offer, taking into account the structure and dynamics of market demand, the costs of a possible mismatch with the sales forecast or the effects of limiting the availability of individual products (Wołowiec, Szybowski, and Prokopowicz, 2019).

Thirdly, it is important to optimize the stocks of materials and raw materials, the task of which is to ensure their availability for production at the lowest possible cost of maintaining the necessary stocks. We achieve this goal by implementing a set of processes ensuring full control of the stocks held, defining priorities and difficulties in managing individual material items based on a developed classification system or building a knowledge base of critical parameters such as: size of optimal purchase batches, delivery time, levels and methods determining safety stocks, the frequency of ordering in line with the production strategy and many other parameters necessary in the work of the procurement department (Wołowiec and Suseł, 2010).

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