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A Variety of Processes in Decisions Making and Management

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Abstract:

Purpose: *The main goal of the article is to emphasize the role of process diversity in decision making and management. Additionally, the contribution of the random factor in the behavior of processes and the related management factor of such processes was analyzed.*
Design/Methodology/Approach: *Due to their complexity, multivariate and multivariate random processes are difficult to research, so they are more mysterious to put them into mathematical forms. The methodology of recognizing such processes used in the work (article) combines two fields of science, information theory and stochastic processes (Markov chains).*
Findings: *The methodological approach adopted by the authors to the set goal of the study, i.e., to study the diversity of Nature's processes, allowed to formulate a conclusion: one can compare the behaviors of different processes, compare their behavioral diversity in the entropy dimension, regardless of the number of their conservation states.*
Practical Implications: *The results (conclusions) obtained from the research allow for their application in practice, and in a wide range. Because you can compare the indeterminacy (entropy) of two (or several) qualitatively different random processes. So, basically to predict a more stable behavior in relation to another given random process, or less stable.*
Originality/value: *The novelty of the content of the article is, as already mentioned above, the combination of sciences and their research methods in relation to the formulated goal of the article.*

Keywords: *Diversity of processes, information, decision making, management.*

JEL classification: *C01, C18, C44, D81.*

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1. Introduction

Looking for formulated the goal in summary through the concept of analogy, we can say that the reality represented by R is one, but it has three dimensions that can be analysed (studied) separately for certain purposes. Moreover, science is also that real objects, that is, having a dimension of three, can represent certain aspects of (ownership) with a dimension (vector) greater than three, and for some real objects, that is, located in our reality have a dimension smaller than three. Whether together it all creates "kogelmogel" (chaos), or is it essentially the essence of our reality, that is, together they participate in its diversity, express it through their structures. We want to say this in the following text in a simple language (by the language). And everything together is intended to make rational decisions in the broad management, and above all this is to better understand the objects and processes of Nature and their behaviour. Because to be able to manage anything well, you need to understand the managed object first by knowing it.

Therefore, the whole text below will concern (concerns) essentially three areas (dimensions) of the issue generally related to decisions and management, but by analysing the diversity dimension of the surrounding reality in terms of learning about the structure of its objects and processes. Because only then decisions made through management can be useful. This means that our decisions should be logical (rational), that is, they are intended to serve something, some purposes (and above all, they are intended to be teleological in nature, content-oriented to something, that is, as has been said above, to some object or process). One of these goals (decision-making objects) is management. To manage well (anything), you need to have beforehand some knowledge (or just information) about what you manage. It is achieved by examining the structure (organization) of processes, their dimension (complexity), behaviour and explanation of why this and another object (process) behaves just like this, and not otherwise. And it would be the best, to be able to explain at the same time, why the process under consideration behaves differently from another and at the same time to answer the question of what the nature (complexity-structure) of such a process is.

Regarding many of them (objects and processes studied), modern science knows how to do this (that is, it gives answers to these questions). General systems theory deals with this, among many other scientific analyses. The role of this theory was analyzed in Galanc, Kołwzan, and Pieronek (2014) - in this article, the system analysis was seen in a slightly different content and formal aspect than here, that is, in this text. This explanation is also supported, and perhaps particularly, by some general, but important for the science and structure of the objects studied of their property (that is, those objects), included (expressed) in terms of a very general nature, for example such as: diversity, diversity limitation, structure, number degrees of freedom, invariability, randomness, amount of information, system status, the above mentioned General Systems Theory uses these concepts as if at a side point, because its attention is aimed at another (teleological) purpose related to the behaviour of systems (Peters, 1997;

Prigogin and Stengers, 1990) and others. It is about them and their role to make (logical) decisions important for management and generally in the field of science (knowledge), we want to say in our deliberations. It can be said that this topic is nothing new in science. After all, in our opinion, novelty of the text is a collection of these important characteristics expressed in the language of science in the form of content and formal definitions, sometimes included in a single dimension of discourse (analysis) on this subject. At the same time, we also want to emphasise here, that the inclusion of our problem is very general (in terms of the three dimensions indicated for consideration).

However, for selected (important) general terms mentioned above, they will be cited in the context of their role and importance in decision-making and management, detailed examples using learning tools such as Markov processes, random variable functions, differential equations, generative grammar, and fractals. Because according to the author of this text, they fundamentally influence the perception of our environment, which is essentially our World, through the language of science (their scientific meaning). Thus, these three distinguished areas of our analysis (diversity, logic of analysis and management) relate to each other (that is, they are not independent) primarily through the concept of diversity. In addition, the above-mentioned characteristics are also very general, but at the same time they occur in the most important processes of reality, and sometimes in conjunction all together, but most often in the set of several of them. In detail, these concepts are often expressed in a complicated language of mathematics and additionally highly abstract. In this text we will present several of them, i.e., mathematical shots such as this in footnote below.

However, our main objective is not to teach complex mathematical analyses (methods and tools of mathematics) on one or other areas of formal processes and objects in which these general concepts and dimensions of our analysis are mentioned above. Such approaches are important explications in relation to specific fields of science and specific issues analysed within them. This is important problem in optimizations processes (Wiener 1950; 1960; Wołgin 1970).

2. Continuity as a Constraint of Diversity

Continuity is a constraint of diversity, but it is not properly specific to the reference object, it is of a general nature, not to say universal, but only there, where continuity takes place, that is, when the objects under consideration have one of their characteristics of a continuity nature (regarding the dimension of dynamics when they are differential). Through the concept of continuity, you can explore different forms of process behaviour or objects in continuum, that is, this or another process can potentially have (realize) continuum of values (definiteness).

Therefore, referring to the last sentence, it should be said that this *continuity* is the strongest limitation in diversity among all others. The mathematicians' awareness of the existence in continuity, gave rise to its formal definition (a view of it in the language of mathematics-the accuracy of thinking). Continuity was linked to the concept of boundary (sequence) already existing in mathematics. Moreover, Isaac Newton, Leibnitz and Georg Friedrich Bernhard Riemann independently combined these two concepts and gave the definition of a derivative, a concept that measures the speed of change of a process (in the literature of the subject there was adopted the derivative notation given by Newton, who took it from the monk, with a symbolic figure:

$$y' = \frac{dy}{dx} \cdot$$

Based on this concept, you can create or discover and study many processes and invariabilities of nature (we will give precisely the example of a invariant, which occurs widely in various areas of human activity, but also in economics (economic processes) and takes place in nature, that is, outside human activities. This invariant appears under the name of a logistical function. And in deriving the analytical form of this function it is used precisely in continuity and other concepts related to it:

Example: A certain quantity (economic, financial, natural, ...) y is such a function y(t) that the speed of change of its value is determined by a differential equation as follows:

(1) $$\frac{dy}{dt} = ky(a-y),$$

where a is the maximum y value, while the k parameter is the proportionality factor. We are tasked with defining the function y(t), that is, to find its form. The y(t) function is the solution of the differential equation (1) with separated variables. Thus, by separating the variables in the equation (1), we get:

(2) $$\frac{1}{y(a-y)}dy = kdt.$$

It is easy to check, that

(3) $$\frac{1}{y(a-y)} = \frac{1}{a}\left(\frac{1}{y} + \frac{1}{a-y}\right).$$

Integrating equation (2) with taking into account the last equation (3), we get:

$$\frac{1}{a}\int\left(\frac{1}{y} + \frac{1}{a-y}\right)dy = k\int dt \cdot$$

From here we get:

(4) $\ln|y| - \ln|a - y| + \ln b = akt,$

where b is any positive constant.

Substituting $ak = c$ to the equation (4) and by solving this equation relative to the variable y we get:

$$y = \frac{a}{1 + be^{-ct}}.$$

This is the result of solving the differential equation (1). We are now receiving a logistical function known in many scientific applications. It is an important scientific invariant, especially in economic research (Badach and Kryński 1977). Another example of the invariant is the first formal representation of the economic model of the state given by Domar (1946), and Harrod (1939). Although Gossen (1983) had been involved in the supply and demand model much earlier, and between them he placed a man considering his behaviour. But this is not a single function model. Then, express them in the form of mathematical formulas (models) that represent – express the laws of nature physics and biology (Dröscher, 1971; Prigogin and Stengers, 1990; Thom, 1972), economics (Domar, 1946; Harrod, 1939) and important formal properties of the humanities psychology (Pavlov, 1927), linguistics (Kołwzan, 1983; Milewski, 1965) and sociology (Bokszański, Piotrowski, and Ziółkowski, 1977). That is why we will now address, in outline, very general formal tools that may represent some processes in the field of humanities. Some of them include generative grammars.

3. Operational Dimension of Diversity Forms

The examples cited above related to diversity expressed through its limitations were static, because we gave their characteristics of construction (structure). After all, the nature has an important and even basic property, which is its dynamics of behaviour (functioning and duration in time). And now we want to present just one aspect of the dynamics of diversity in the dimension of measuring its behaviour from the side of indeterminacy (entropy of its behaviour).

By measuring the diversity of the form (complexity of the structure of an object), you can simultaneously talk about its importance. However, there is one requirement that this should be handled by such a measure. This thought (idea) is worth illustrating by a content example, but also supported by numerical calculations to demonstrate the link between randomness (stochastic) and information about the diversity of forms (processes, objects, and systems). In addition, we will also try to show that randomness also has a structure (balance of behaviour of the system-object), and the information (its quantity) expresses the degree of this organization. The participation of the random factor in the implementation of a process with a clearly outlined structure (as stated above) does not cause chaos in this process. That is why we will refer to the relationship relating to these two terms in the last part of our reflections on the diversity of forms of nature with a fractional dimension. The content of that paragraph, or rather, is the conclusion that randomness does not rule determinism, but merely distorts it.

4. Diversity of Processes in Markov and Shannon Theory (terms)

We will combine in one way two concepts of taking these above-mentioned problems into an operational dimension based on events of stochastic nature. One side of this problem was formulated by Markov in the form of Markov chains (Markov 1906) and the other by C.E. Shannon, the creator of the information theory, the elements of which will be presented below, when discussing in more detail, the importance of information in our lives and its role for practice, by expressing it, unfortunately, only quantitatively. Here we will only use the measure of information that he formulated.

The A.A. Markov concept on expressing systems, that are supposed to implement various possible states, the existence of which has a non-deterministic dimension, was previously born from Claude E. Shannon's thought about measuring information. Shannon and Weaver in their book (Shannon and Weaver 1949) used the concept of the Markov chain to analyse information measurement issues and, above all, to communicate it through the information channel. As for the Markov processes (Markov chains), we will not present here their full mathematical representation, and we will focus more on their importance for practice, that is, rather, on the content dimension, and on this one, that explains the content by calculation, what the results obtained mean for the process under consideration (dynamic, that is, taking place over time). A comprehensive description of the Markov chains can be found in Fisz (1969).

On the other hand, an excellent content interpretation together with an analysis of their entropy in turn encloses the position (Ashby, 1963). And what role does the concept of information play in their (such) interpretation, or rather its measure of the behaviour of the process about the nature of the Markov chain. And such or different interpretation of Markov process together with its information measure creates the possibility of understanding the behaviour of the (dynamic) process under consideration from the form of its behaviour, that is, diversity. The examination of the system content remains at the discretion of the established teachings for this study, which will also be mentioned below. Important attention is needed here. It was said above that the theory of Markov chains would not be presented, but we meant its full dimension. Therefore, the basic structure of this theory will be presented below. So, before we start counting, we must start by explaining what (formally and in content) Markov process is. The essence of things comes down to the relationship of determinism towards indeterminism. We will illustrate this with a numerical example. There was talk above about the states and their transition to another. So, let us take into account the fact that first we have a set system with a transition matrix:

$$\begin{array}{c|ccc} \downarrow & X & Y & Z \\ \hline X & 0 & 1 & 0 \\ Y & 1 & 0 & 0 \\ Z & 0 & 0 & 1 \end{array}.$$

This is a matrix that clearly determines the state, in which the structure is located at the moment, will pass in the next step. These are deterministic transitions. This can be more clearly stated as follows:

$$\begin{array}{c|ccc} \downarrow & X & Y & Z \\ \hline & Y & X & Z \end{array}.$$

(it should be recalled here that this is only one of the possible three-state set configurations). The number of all possible pairs of the system is (3!-1), and with the identity transition, it is of course 3!.

If there is a process, whose behaviour of individual states (factors that make up its functioning, forming the structure of the system), is deterministic in nature, then it must be remembered that *Nature* as a whole has a random factor (chaos factor- causing disorders) and at some point, in time can cause some multistate deterministic system to pass into a stochastic system. How it happens is a mystery of nature, because only very easily it can all be formally illustrated. Usually, in the literature on the subject, determinism is presented as a particular (marginal) case of indeterminism. But why cannot the process be seen the other way around, which is how it was presented above.

In such cases, systems with transition matrices are considered, which, moreover, meet the assumptions, namely, that the transition probabilities of the system from one state to another are fixed (stationary) and the whole system tends to balance (states reach equilibrium probabilities, also called ergodic ones). Processes with such properties, as stated above are called homogeneous Markov chains (processes). The following text expresses them in a formal form, but also in their most basic dimension (a more complete formal representation is given - see footnote 6). Homogeneous Markov chains. Here we present how this issue looks from the formal side. Let us imagine that we have a finite or infinite series of experiments (system) and as a result of each experiment (result) there can be one and only one event (and this set is also disjunctive-deactivating pairs):

$$E_1, E_2, E_3, \cdots, E_n, \dots .$$

We will call these events *states*. When the event E_j occurs, we will say that the system is in the state E_j. We will use the symbol $E_j^{(n)}$ to indicate that the state E_j occurred in the n-th experiment (at time t in the time sense ($n = t$)). In addition, the symbol $E_j^{(0)}$ will mean that the initial state of the system is the state E_j. Let continue, $p_{ij}^{(n)}$ means the conditional probability that after the *n*-th experiment the system is in the state E_j , if after *(n-1) - this* experiment the system was able to E_i , i.e.:

$$p_{ij}^{(n)} = \left[E_j^{(n)} \mid E_i^{(n-1)}\right].$$

Definition: We say that the sequence of experiments is related to the Markov chain, if for any (i, j, n = 1, 2, 3, …) equalities:

$$p_{ij}^{(n)} = P\left[E_j^{(n)} \mid E_i^{(n-1)}\right] = P\left[E_j^{(n)} \mid E_i^{(n-1)}, E_{i_{(n-2)}}^{(n-2)}, \ldots, E_{i_1}^{(1)}, E_{i_o}^{(o)}\right]$$

occur at any

$$E_{i_{(n-2)}}^{(n-2)}, \ldots, E_{i_1}^{(1)}, E_{i_0}^{(0)}.$$

Markov chains have different properties and therefore for our purposes we will give a definition of one of the basic, but important in applications, type of Markov chain.

Definition: We say that the sequence of experiments is linked to a homogeneous Markov chain, if for (i, j = 1, 2, 3, …) probability $p_{ij}^{(n)}$ does not depend on n, i.e.

$p_{ij}^{(n)} = p_{ij}$ (n= 1, 2, 3,…).

Probability p_{ij} is called the transition probability of the system from the state E_i to the state E_j in one experiment-step (the concept of step (units, i.e. whether the step in time dimension is: second, minute, hour, day, week, ...) in a formal sense is not clearly defined and depends on the content of the problem under consideration, but it is necessary to remember that this concept of a step is very important in the described Markov's chain process).

Transition matrix: Transition probabilities p_{ij} have two indexes, just like the elements of a matrix and therefore they form together all the arrangement (U) in the form of a matrix.

Definition: Matrix whose elements are the transition probabilities p_{ij} , is called the transition matrix and we will denote it as a symbol M_1. So our system U in the matrix character has the form:

U: $$M_1 = \begin{bmatrix} p_{11} & p_{12} & p_{13} & \cdots \\ p_{21} & p_{22} & p_{23} & \cdots \\ p_{31} & p_{32} & p_{33} & \\ \vdots & \vdots & \vdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix}.$$

Let system U be in the state E_i. An event where, as a result of an experiment, the system will remain in the same state E_i, or go to any of the states $E_j \ (j \neq i)$ is a sure event. Because events turn off in pairs, for i = 1, 2, 3, ... we get the equality:

$$P\left(\left(\sum_j E_j\right) | E_i\right) = \sum_j p_{ij} = 1.$$

Thus, the sum of the probabilities in each row of the matrix M_1 is equal one. However, the sum of probabilities in a column does not have to meet this requirement (unless we understand the presented issue in columns). What constituted a novelty in the theory of the processes formulated by A.A. Markov includes the following text:

> *The works of A.A. Markov are the beginning of the study of probabilities not of individual events, but of entire processes. And it was at that time, but now also, that it was a novelty to look at the importance of probability in the capture of objects, and not just their individual elements (e.g., in the form of giving one or other probability distribution for a random discrete or continuous variable).*
>
> *What in this study of processes relies on we will present in the following text in a modest way, in terms of formal dimension? In the probability calculus, each state of the system under consideration has its probability distribution. So, Markov asked the question, and what is the distribution for the whole system, which consists of many states.*

5. Procedure for Determining Ergodic Probabilities

The probabilities of transition from state to state, as we already know, are set in the transition matrix. For the whole system, it is important to know its limit states (if the system has them). Before interpreting the results obtained from the data given below (System *X* and System *Y*), the operational procedure for obtaining ergodic probabilities, i.e., the limit for each state, must be presented. These probabilities have the property that if the system reaches these probabilities, that is, the equilibrium state, then in the next steps *n* (moments t) they remain the same, but the stationary probabilities do not change, because the system (individual elements of it) continues to move from one state to some other state or remains for some time in the state in which it is located.

These are the theorems about the ergodicity and stationarity of states in homogeneous Markov chains. But as well as based on what it is necessary to determine (adopt) the procedure for calculating the limit (ergodic) probabilities. To find the probability vector for the stationary states of the Markov process, this issue should be formulated as follows: what should be the probabilities of x, y, z of remaining at a time t in this or that state, so that the probabilities of remaining in them at the time t+1 are equal at that? The probabilities of remaining in this or that state at the time t+1 is obtained by multiplying the vector matrix $[x \quad y \quad z]$ through the matrix M_1. The calculations will be carried out on the example of the *Y*-system.

So, we should find such x, y, z, to fulfil equality:

$$[x \quad y \quad z] \cdot \begin{bmatrix} 0{,}8 & 0{,}1 & 0{,}1 \\ 0{,}1 & 0{,}6 & 0{,}3 \\ 0 & 0{,}1 & 0{,}9 \end{bmatrix} = [x \quad y \quad z] \cdot$$

As a result of the calculations, we get the result for the ergodic probabilities of the *Y*-system:

$$[x \quad y \quad z] = [0{,}1 \quad 0{,}2 \quad 0{,}7].$$

As soon as the system reaches such probability limits (or they are already given to it at the beginning of its operation) to be in this or that state, further changes in the probability (ergodic) will no longer occur; the process (system) will be stable. For the whole system, in terms of probability limits, this is a very strong reduction in diversity (limitation within entropy-related changes).

6. A Measure of Indeterminacy in Terms of Shannon's Information Theory

We will now proceed to discuss the relationship between the content of the message and the indeterminacy (information entropy). From a formal point of view, this relationship is not simple and insignificant, although its analytical representation, especially for the case of a random variable discrete *X*, is clear and understandable, because it is expressed in the following form:

$$H(X) = -k \sum_{i=1}^{n} p_i \cdot \log p_i \,,$$

where the *k* factor is the transition module from one logarithm base to another. The operational form of this module is as follows: if we only have logarithms with base a, and we want to find logarithms with base b of positive number N, then we use the formula:

$$\log_b N = \frac{\log_a N}{\log_a b}.$$

For example, $\log_2 N = 3{,}322 \log_{10} N$. The word diversity in relation to the set of distinguishable elements will be used in two meanings: as the number of different elements, i.e., as the power of the set, and as the logarithm of the a basis of this number.

> *Most often, in practice, the number two is taken as the basis for the logarithm. Then the indeterminate of the system under investigation is represented in bits of information, shortcut for* "Binary digiT" *(digit in binary numeral system).*

We will use this measure below when we talk about state entropy in Markov processes. Here, however, we will use the concept of indeterminate measured by entropy to enclose, or rather to clarify from the position of the meaning of the uncertainty concept. Especially since uncertainty is associated with another quite common and widely used term in science, which is known as risk. Nowadays one talk about making decisions in conditions of uncertainty and about taking it in a risky environment. The question arises whether these are two different qualitatively (separable) concepts, or whether they are similar and differ little. This category, i.e., uncertainty, directly involves transmitting and receiving information (messages-communications) at the same time.

Regarding this problem, Ashby (1963) writes as far as I know, that an appropriate measure of the degree of distortion (corruption) of the message due to noise has not yet been developed for individual cases. Shannon, however, introduced a measure for the channel, which communicates the information constantly. And here it is assumed, first, that both the transmitted signals and the received signals form Markov chains (because the form of information transmitted and received does not depend on the previous messages, and not in the sense of content, but only in the sense of the sent symbol). Thus, the data contained in messages (message symbols) can be represented by the frequency or probability of the appearance of all possible combinations (vector) of sent symbols and received symbols.

Example: We want to transfer a message. Each text can practically be converted (encoded) into a string of zeros and ones, economically based on the Shannon-Fano theorem. Let us assume, that zeros and ones are sent as signals, and the frequencies (relative) signals received are:

sent signals (S)	0	0	1	1
received signals (S)	0	1	0	1
$P(S)$:	0,495	0,005	0,005	0,495

From the distribution, for every thousand signals given, ten come in the wrong form-the deviation is one percent. At first glance, this one percent of inaccuracies can be understood as a natural measure of the amount of information lost to the communication. However, if the line had been interrupted during the transmission and the recipient would have threw a coin to receive the message, roughly half of the symbols would have been correct, although no message had been transmitted. Simulation plays an important role in the process research, but poor simulation can harm this simulation. The example of coin-throwing above is just such a poorly understood simulation. After all, Shannon, in his book, quoted above, convincingly demonstrated (in an operational manner) that the natural measure of information loss is the uncertainty that is calculated in the following way (and the result of this calculation will be significantly different from one percent of the loss of information). So, we will present the procedure for counting this loss of information related to the channel, where the disorders are present. First, the entropy should be calculated for all possible classes (pairs) of symbols sent and received:

$$\boxed{\begin{aligned}&-0{,}495\log_2 0{,}495-0{,}005\log_2 0{,}005-\\&-0{,}005\log_2 0{,}005-0{,}495\log_2 0{,}495.\end{aligned}}$$

This entropy is attributed to the name of the H_1. It is 1,081 bit per symbol. Now we need to calculate the entropy of the symbols received with their probabilities. They constitute an array:

Sent symbol	0	1
Probability	0,5	0,5

When the entropy recalculated, from the table above, it is seen that it is 1 bit per symbol. Let us call it H_2. When comparing these two entropies, after simple calculations, we have the conclusion that uncertainty is equal to $H_1 - H_2 = 0{,}081$ [bit per symbol]. Thus, it can be said that the actual speed of information transfer in the presence of noise is equal to the entropy of the source minus uncertainty. So, we have a result the original source entropy is 1,000 bit per symbol. Of this number, 0.919 bit passes through the information channel, and 0.081 bit is destroyed by noise (channel interference). So, the loss of information is more than 1% per symbol, because from the calculations it is seen, that it is as much as 8.1%.

It is necessary to explain at the end of these calculations, why were they used? Moreover, what did we want to demonstrate through this example? Well, any information provided to us does not have to reach us as it was sent, because in any channel of information there may be noise that distorts the form of information transmitted. And the amount of information lost may also be greater than it could result from calculations (misinterpreted loss of information). In general, however, as we have seen above, randomness distorts information and therefore we will now address the issue of the behaviour of processes (systems) in conditions of uncertainty.

7. Practical Dimension of Formal Meanings of Ergodicity and Entropy

To show how to combine, in both formal and content sense, Markov processes with measuring their diversity with Shannon's entropy theory, we will consider two such systems so that we can compare their diversity, that is, behaviour-dynamics over time. It will, in a sense, be the expression of the system through its form and content (form, as the structure of the system, and the content, in turn, constitutes its behaviour over time).

Both systems we will present together with their calculated results by specifying equilibrium probabilities (boundary, ergodic) and entropy for each state of their transition into the other (i.e., with a total probability equal to one). However, we will focus on the semantic interpretation (the content meaning of the process examined) of the formal (accounting) results obtained.

System*X*:

↓	*B*	*W*	*P*
B	0,250	0,750	0,125
W	0,750	0	0750
P	0	0,250	0,125
Entropy	0,811	0,811	1,061
Ergodic numbers	0,449	0,429	0,122

System*Y*:

↓	S_1	S_2	S_3
S_1	0,800	0,100	0
S_2	0,100	0,600	0,100
S_3	0,100	0,300	0,900
Entropy	0,922	1,295	0,469
Ergodic numbers	0,100	0,200	0,700

8. Interpretation of the Results

An analytical formula for measuring Shannon's entropy is given above. And here we will only talk about what it measures regarding Markov chains. Entropy here measures the amount of diversity, manifested at every step of the Markov chain, that is, the transition of a given state into every possible other, set by the transition probabilities. If the Markov chain has all its transitions equiprobable, then the reduction in diversity is zero here. Nature's goal (perhaps) is to provide the recipient with as much information as possible (in terms of definiteness, that is, small entropy- because, why it has equipped us with the senses, about which, in the meaning of the game, it was mentioned above).

Otherwise, there would be a lack of communication between the environment and its recipient. Each event would have the same value. In such a system, there is no variety between events (objects). And yet there are theories to pursue such a goal of happiness. You must trust science because it does not use emotions. These were short but philosophically important divagations about the meaning of information, i.e., when it exists and when it does not. In the above example about the interference in the channel of the transmitted information was said, that instead of transmitting information through the channel with the 1% disturbance, a coin can be thrown in this place. But that is not the message. That is not actual information. Although there are methods for simulating real-world processes (e.g., Monte Carlo method). Therefore, science and the tools it has created must be used in practice rationally (logically). And here we ask when simulation tools can be used in place of real-world processes and why. We will answer this question at the end of these calculations and at the same time the coin throw problem, presented above, will be explained.

Average entropy
(weighted)

Entropies derived (calculated) from different sets (states) can be combined to achieve the average (WEIGHTED) entropy for the entire system. And now we are going to do it. This combination is also used to find the entropy corresponding to the Markov chain. Each column (or row, if the matrix is written in a transposed form) contains a set of probabilities whose sum is equal to unity. Therefore, each column or row is characterized by some entropy. Shannon defines entropy for one step in the chain as the mean value of these entropies, each of which is taken with its weight (ergodic-equilibrium probability). So, we will calculate the average entropy values for both systems and see which one as a whole is more specific, or which one has more predictable behaviour.

This is determined by the weighted (average) entropy value for the entire system. And yes, the average entropy value for the system *X* is:

$$\overline{H}(X)=\frac{45\cdot 0{,}811+43\cdot 0{,}811+12\cdot 1{,}061}{45+43+12}=0{,}842\,.$$

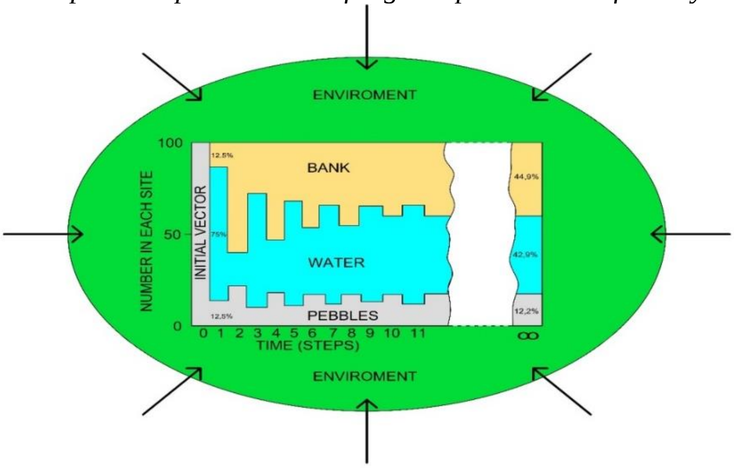
When calculating the average entropy (analogous to *X*) for the *Y*-system, we get:

$$\overline{H}(Y)=\frac{10\cdot 0{,}922+20\cdot 1{,}295+70\cdot 0{,}469}{10+20+70}=0{,}680\,.$$

The two results obtained shall be interpreted in the same way. So, we are only going to deal with the former, the X system, because the one with greater entropy is more unpredictable in terms of behaviour. Both entropies represent the behaviour of the entire system in one step. Multiple throwing of a coin creates an entropy order of 1 bit for each throw (step). Thus, the order of states occupied by one element of each of our two systems is not subject to such intense changes as coin throwing, as both average entropies are less than 1 bit of information. And this is a great advantage of this measure in the form of entropy, that it allows to compare with each other different degrees of diversity, which can be expressed, in terms of content, by different, that is, incomparable according to the raw data of the unit, but their structure of informational behaviour (diversity) can be the same or similar. In addition, it would be necessary to say what the boundary probabilities, that is, the time at which the system, and in principle its states, reach a state of equilibrium, should be further mentioned in the content interpretation of the system's behaviour of the Markov’s process. To sum up the above explications, it must be said at the end, that we use the weighted average value because of the three individually calculated (several) entropies, for each state separately, we want to get one of them, and the boundary probabilities can be interpreted in many ways, depending on what actual systems they represent.

Regarding the X system, we can understand as follows, when the system reached an equilibrium state, 45% of its components were in state B, 43% in the W state and 12 % in the P state (for Y system the interpretation is similar). Yet in other words, 45% of the system passes are done with B, 43% with W and 12% with P. And that is why 45% of his passes will have entropy (diversity) of 0.811, 43% - also 0.811, and 12% - 1.061. Lower entropy transitions (0.811) will occur more frequently than transitions with greater entropy, such as 1,061. Therefore, when determining the average value of entropy of the system, to entropies 0.811 weight 88% is given and to entropy 1,061 weight 12%. In fact, the latter analysis has a general interpretation of the Markov's content chain. The method of ergodic probabilities obtaining presented on page 10 is graphically presented in Figure 1 below.

Figure 1. *Graphical representation of ergodic probabilities for X system.*



Source: *Own study.*

Because it is still necessary here, in relation to this general approach formulated above, to add that, when the system has reached a steady state (equilibrium) and practically reached numbers (populations) in individual states, there is a sharp conflict between populations, that do not change and elements of individual populations that are in constant motion (because in the assumption of one of the properties of a homogeneous Markov chain is that the probabilities of transitions are stationary, that is, preserved until the end of the existence of the system).

9. Conclusion

To sum up the above, but from the general knowledge of these processes, only selected aspects of the amount of information obtained from the behaviour of the Markov systems and information in Shannon's perspective, it must be emphasised that these are undoubtedly significant advantages of obtaining knowledge of a given system through information theory (by measuring indeterminacy, that is, taking it into some form of definiteness). In addition, it is within the framework of information theory that concepts such as: channel, channel bandwidth, excess information (redundancy), economical information encoding (Shannon-Fano theorem), noise measurement (loss of information during its transmission through the information channel) and many other such important concepts related to information in general are defined. And knowledge (random) about the possible subsequent states of the behaviour of the test system gives the opportunity to express the possible behaviour of this system with entropy.

In addition, it can be said that although entropy does not measure the semantics (content) of the processes expressed therein, it has been evident through the examples given that it nevertheless expresses in some way the organization of this content. A well-organized system is more important than a content-like but less organized system, which has greater entropy. By the concept of analogy, this mentioned above idea of the organization of systems with similar content can be transferred to the organization of companies, States. There are nations with well-organised statehood, that is, predictable behaviour, and there are also those whose organisation does not allow a clear definition- anticipating their behaviour in relation to the same real situation (for both parties), e.g., Political.

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