Application of Grey Systems Theory in the Analysis of Data Obtained from Family Businesses

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Abstract:

Purpose: *The article presents a methodology whose aim was: first, to present the problem of the influence of conflicts upon family and business spheres in family businesses and to conduct a statistical analysis on a large sample, second: to check whether similar results can be obtained making use of small samples, randomly chosen from a large sample and analyzed using Grey Systems Theory tools.*

Design/Methodology/Approach: *The obtained data was used for correlation analyses and analyses that follow the procedure of Grey Incidence Analysis. The research outcome corroborated the convergence of results for both analytical approaches, which, according to the authors, allows the presented research methodology to be used with small samples of family businesses.*

Findings: *The obtained research results confirm the hypothesis which says that the results of conclusions drawn from a small amount of data using the GST methods are in keeping with the results of correlational research for a large amount of data defining the same phenomenon.*

Practical implications: *The results of the research indicate the usefulness of grey incidence analysis in empirical research on family business issues.*

Originality/value: *The research presented in the article is a positive verification of grey analytical models which can be extremely useful for empirical analyses conducted on small research samples.*

Keywords: *Family business, small sample, grey systems theory, grey incidence analysis.*

JEL classification: *C19, C02, C89.*

Paper Type: *Research paper.*

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1. Introduction

The main problem regarding the practice of conducting research on family entrepreneurship is the difficulty in obtaining large samples. The disadvantage of using small samples is the inadequacy of the results obtained with respect to general populations from which the sample was drawn. The indicated problem is not specific to the issue of family entrepreneurship, but in this area of knowledge it is extremely intense. This is due to entrepreneurs' reluctance to disclose sensitive information about their families (Winter *et al.,* 1998), especially about family secrets (Schulze *et al.,* 2001), not considering family criteria in public statistics such as COMPUSTAT or EUROSTAT, or low response from surveys conducted in family enterprises (Stamm and Lubiński, 2011).

The problem of small sample sizes applies to family entrepreneurship researchers regardless of the quantitative research methodology adopted. This applies both to classical studies with the statistical inference procedure as well as the use of several alternative methods, such as artificial neural networks (Scott, 2005; Trinkle and Baldwin, 2016; Rojek *at al.,* 2017), fuzzy logic (Melin *et al.,* 2017; Seifollahi and Reza, 2016; 2017), rough set theory (Cai and Silva, 2016; Hegazi, 2015), analysis of data from social research (Ban *at al.,* 2015; Sabokbar *et al.,* 2016). The reason for this is the lack of adequate methodological instruments, which would allow family entrepreneurship researchers to undertake research on small samples, the results of which can be induced on the whole population while maintaining an adequately high confidence level.

The importance of the problem of sample size in family entrepreneurship research is often indicated in source literature (Kung and Wen, 2007; Donckels and Fröhlich, 1991; Allouche, Amann, Jaussaud, and Kurashina, 2008; Owen and Winter, 1991; Barnes and Hershon, 1989; Welsch, 1991; Zellweger, 2007; Zellweger *et al.,* 2011; Carrasco-Hernandez and Sánchez-Marin, 2007). This validity is both substantive and economic. Although the problem outlined above is important, it has not yet been widely discussed. Only a few articles dealing with the problem of sample size in research on family entrepreneurship are known (De Massis and Kotlar, 2014; Comi and Eppler, 2014; Więcek-Janka and Mierzwiak, 2015; Więcek-Janka; Mierzwiak and Kijewska, 2015; Więcek-Janka, Mierzwiak, and Kijewska, 2016; Więcek-Janka, Mierzwiak, and Kijewska, 2016b). The present article is intended to fill, if only in a fragmentary way, the existing shortage. The purpose of the study is to propose a set of tools that provide the opportunity to conduct research on the phenomenon of family entrepreneurship with the use of small samples while maintaining the analogous reliability of the obtained results as in the case of large samples. The proposed tools are the set of instruments of gray system theory (GST).

The research hypothesis verified in this paper was formulated as follows, inference from a small amount of data related to the causes of conflicts using GST methods is

consistent with the results of correlation studies for a large amount of data describing this phenomenon (Delcea, 2015a; 2015b; Liu *at al.,* 2011, Liu and Zhu, 1996; Liu and Wang, 1996; Scarlat and Delcea, 2011; Wu and Chen, 2011; Yuan, Guo, and Liu, 2013; Yi, 1987; Xu and Gao, 2007; Huang, Baetz, and Patry, 1993). The current state of research on the reduction of the need to use large samples in the field of family entrepreneurship with the use of GST tools should be described as the initial one.

The intention of the authors of the article is to present that for a specific problem, which is the causes of conflicts in family enterprises, the use of GST tools allows conducting research on small samples because the results obtained are convergent for both large and small samples. At the current state of research, generalization of conclusions regarding the size of samples to address any problems of family entrepreneurship with the use of GST tools is premature. The authors, however, believe that such a situation is natural at the current stage of research development and generalizations will be able to be carried out after conducting a series of analyses on the diverse issues of family entrepreneurship. The large accumulation of individual cases will allow to draw a conclusion that the problem of the necessity of using large samples in quantitative research on family entrepreneurship has been eliminated using GST methods.

2. Grey Systems Theory

Grey Systems Theory was proposed in 1982 by Chinese mathematician Deng (1989) as another methodology of data analysis, next to already existing statistical methods, Fuzzy Logic and Rough Set Theory. GST had long remained widely unknown because the first publications on the subject were released in English in 1989, whereas its full comprehensive description in the form of a book was published as late as in 2006 (Liu and Lin, 2006; Liu and Lin, 2011). Currently, this theory is being dynamically developed, which is confirmed by an increase in publications related to its application in economic, social, and technical sciences (Yi, 1987; Xu and Gao, 2007; Scarlat and Delcea, 2011; Wu and Chen, 2011; Liu and Lin, 2010; Yuan, Guo, and Liu, 2013; Delcea, 2015b; Więcek-Janka, Mierzwiak, and Kijewska 2016a; Więcek-Janka and Mierzwiak, 2017).

Theoretical and practical research in Grey Systems Theory focuses on a few principal aspects. They can relate to diverse branches of science and economic practice. Traditional grey systems methods can be divided into (Liu *at al.,* 2016):

- Grey Incidence Analysis like regression models,
- Grey Forecast Models based on specific differential equations,
- Grey Clustering Models aimed at grouping the objects,
- Grey Decision Models similar to multi-criteria decision-making methods,
- Hybrid Models which combine Grey Systems Theory methods with data analysis methods such as Rough Set Theory or Fuzzy Logic.

All the above-mentioned methods of Grey Systems Theory may be used in the context of family business research. This appears particularly useful when the set of collected empirical data is small or if there are problems with determining distributions of the analyzed random variables. It is a typical area in which GST can be applied. GST then can be treated as a complementary set of methods and tools with respect to classical statistical analysis.

When the application of GST methods in family business research is analyzed, one can arrive at the conclusion that their use in that respect has so far been marginal. The only applications related to family business research which can be found in source literature are: Grey Incidence Analysis (GIA) (Więcek-Janka, Mierzwiak, and Kijewska 2016a; Mierzwiak and Więcek-Janka 2016) and GCA (Więcek-Janka, Mierzwiak, and Kijewska, 2016b). It does not, however, exclude the possibility of using other grey methods in family business research, which is confirmed by their extensive application in economics- and management-related aspects (Delcea, 2015b).

Grey Systems Theory methods are meant to be used in cases when a researcher has access to a limited amount of data, when mechanisms governing relationships in the analyzed systems are only partially known and in particular (Liu and Lin, 2010):

- the information of elements (or parameters) is incomplete,
- the information on structure is incomplete,
- the information on boundary is incomplete,
- the behavior information of movement is incomplete.

Grey Systems Theory may be a useful tool with due to its specific character and in many cases may enrich the apparatus set used in data analysis methods.

The application of this theory in analyses related to family businesses is worth considering as access to information is difficult, population to be studied is large and there is reluctance to share experiences connected not only with business aspects but, first and foremost, with family aspects (Więcek-Janka and Mierzwiak, 2015; Więcek-Janka, Mierzwiak, and Kijewska, 2016a; Więcek-Janka, Mierzwiak, and Kijewska, 2016b). The place Grey Systems Theory occupies in the structure of existing information analysis solutions is shown in Table 1.

Table 1. *Comparison between Grey Systems Theory, probability statistics, fuzzy mathematics and Bayesian methodology (based on Liu, Sifeng, and Lin)*

	Grey Systems Theory	Probability Statistics	Fuzzy mathematics	Bayesian methodology
Objects of study	Poor information uncertainty	Stochastic Uncertainty	Cognitive uncertainty	Stochastic uncertainty
Basic sets	Grey hazy sets	Cantor sets	Fuzzy sets	Cantor sets
Methods	Information coverage	Probability Distribution	Function of affliation	Probability function of the

				expanded a'priori distribution
Procedure	Grey series generation	Frequency distribution	Marginal sampling	The a'priori and a'posteriori probability distribution
Requirement	Any distribution	Typical distribution	Experience	Any distribution
Objective	Laws of reality	Laws of statistics	Cognitive expression	Laws of statistics knowledge about a phenomenon
Characteristics	Small samples	Large samples	Experience	Large samples and small samples

The properties of Grey Systems Theory in comparison with other data analysis methods (probability statistics, fuzzy mathematics, Bayesian methodology) do not require an assumption related to the probability distribution of the analyzed features. It is a consequence of operating on a small sample in which one allows for the emergence of insufficient and incomplete data (Table 1).

In Grey Systems Theory, the equivalent of the variable dependency analysis is Grey Incidence Analysis (GIA). Thanks to GIA, one can analyze a relationship between variables describing the studied phenomenon (problem). The theoretical basis of analyzing these dependencies is the study of geometric similarity of data vectors represented in Euclidean space. By means of GIA, one can assess similarity between two vectors and their degree of association if we know that one of them represents an independent variable and the other one represents a dependent variable (Liu and Lin, 2006; Liu and Lin, 2010; Liu, Yang, and Forrest, 2016).

3. Data Collection

To verify the set hypothesis (see: Introduction), data concerning the influence of conflicts upon family and business spheres in family businesses in Poland was collected. A general population unit, following the European Union definition of 2009, was taken to be a firm which meets one of the following four criteria (https://ec.europa.eu/growth/smes/pro-moting-entrepreneurship/we-work-for/family-business_en):

[1] Most decision-making rights are in the possession of the natural person(s) who established the firm, or in the possession of the natural person(s) who has/have acquired the share capital of the firm, or in the possession of their spouses, parents, child, or children's direct heirs.
[2] Most decision-making rights are indirect or direct.
[3] At least one representative of the family or kin is formally involved in the governance of the firm.

[4] Listed companies meet the definition of family enterprise if the person who established or acquired the firm (share capital) or their families or descendants possess 25 per cent of the decision-making rights mandated by their share capital.

For such a numerous general population, the minimum size of the sample in questionnaire-based research which allowed to generalize the analyses numbered 383 family businesses[6]. The size of the sample obtained in the study was larger than the assumed one and numbered 422 businesses. The research made use of a survey method using CATI (Computer Assisted Telephone Interview). Due to errors and incomplete answers, two questionnaires were excluded from the obtained set.

The evaluation of the variables was performed on a scale ordering the frequency of their occurrence. The statements were adopted as parameters: "it never happens to me", "it happens rarely", "it happens sometimes", "it happens often", "it always happens", and "I have no opinion".

The research questionnaire contained, among others, two questions related to the selection of the direction of profit allocation of family businesses, which related to the adopted dependent variables:

Y1 – level of conflicts concerning family-related issues,
Y2 – level of conflicts concerning business issues.

Moreover, the questionnaire contained 10 questions determining independent variables which allowed to perform a multi-faceted characterization of a studied firm. They were:

X_1 – *generation exercising authority,*
X_2 – *number of family members employed in a business,*
X_3 – *total number of employees,*
X_4 – *age of a business,*
X_5 – *bringing successors up in the entrepreneurial spirit,*
X_6 – *controlling business operation outside set working hours,*
X_7 – *level of family members' engagement in the business sphere,*
X_8 – *level of employed family members' privileges,*
X_9 – *perspective of investment planning,*
X_{10} – *degree of business innovation.*

The structure of the analyzed sample is presented in Table 2.

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[6]Minimum sample numbered 383 businesses, with α= 0.05, b=0.05 and p=0.5. (α: test significance level; b: adopted error; p: fraction of the analyzed property

Table 2. *Structure of the sample*

| Business line		Legal form	
Manufacturing	8%	Joint-stock company	1%
Manufacturing-trading-		Limited partnership	4%
services	9%	Registered partnership	8%
Manufacturing-trading	13%	Civil law partnership	13%
Trading-services	17%	Limited liability company	19%
Trading	21%	Natural person	55%
Services	32%		
Size		**Governing generation**	
Large (over 250)	2%	Fourth (and later)	1%
Medium (50-250)	10%	Third generation	6%
Small (9-49)	28%	Second generation	24%
Micro (up to 9)	60%	First generation (founders)	69%
Region of research		**Owner's gender**	
Eastern Poland	9%	Female	26%
Southern Poland	12%	Male	74%
Northern Poland	19%		
Central Poland	27%		
Western Poland	33%		

The statistical analysis of the collected material aimed at determining the strength and direction of the influence exerted by independent variables (X_1-X_{10}) on dependent variables (Y1, Y2).

4. Research Methodology

To compare the results of research conducted using traditional statistical methods and the results which can be obtained using Grey Systems Theory methods, a proceedings methodology has been developed. The fundamental principle of this methodology is to conduct a statistical analysis on a large sample (420 units), and then to check whether similar results may be obtained using small samples (30 units) randomly selected from the large sample and analyzed by means of Grey Systems Theory instruments. Regarding the specific character of data, correlational procedures were used in the statistical analysis, whereas with regard to the grey analysis, the procedure of Grey Incidence Analysis was used (Liu and Lin, 2006; Liu and Lin, 2011). Figure 1 presents a detailed way of proceedings.

Figure 1. *Adopted research methodology (own elaboration)*
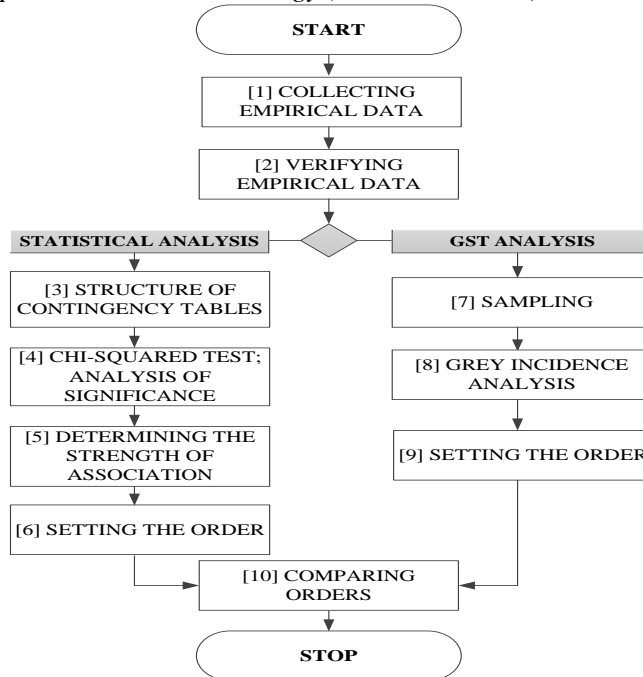
According to Figure 1, the developed methodology consists of the following eight steps:

[1] COLLECTING EMPIRICAL DATA: Collecting empirical data related to dependent variables $X_i (i = 1...10)$ and independent variables $Y_j (j = 1,2)$. A detailed description of this process can be found in the *Data collection* section.

[2] VERIFYING EMPIRICAL DATA: Checking the comprehensiveness and correctness of data coding.

[3] STRUCTURE OF CONTINGENCY TABLES: Development of contingency tables which allowed to tabulate the distribution of answers for two variables: independent X_i (rows) and dependent variable Y_j (columns). Contingency tables allow to receive answers to two questions: *are the differences in the distributions in the table statistically significant and what is the strength of association between variables?*

[3a] CHI-SQUARED TEST; ANALYSIS OF SIGNIFICANCE: Conducting a *chi-squared test* in order to verify statistical significance for a relationship between variables X_i and Y_j.

[3b] DETERMINING THE STRENGTH OF ASSOCIATION BETWEEN VARIABLES: The strength of association between variables X_i and Y_j was determined. They were expressed by means of the following coefficients:

– *C-Pearson's* contingency:

$$C = \sqrt{\frac{\chi^2}{n+\chi^2}};$$ where n-sample size and $\square^2$ is chi-squared statistic.

With the lack of dependency, coefficient C is 0, whereas the top boundary depends on the number of categories of variables. For table *n x n*, this boundary equals:

$$\sqrt{\frac{w-1}{w}};$$

where w-number of *rows* in the table.

– *Cramer's V*, which takes values from the interval <0; 1>:
–

$$V = \sqrt{\frac{\chi^2}{n(k-1)}};$$ k-smaller from the number of columns or rows.

[4] SETTING THE ORDER: Based on the indicators of the strength of association between variables, the order of the influence of particular variables X_i on variables Y_j was determined. As a result of these operations, two orders of variables X_i were set. The first one in relation to variable Y_1, the second one in relation to Y_2. The foundation for setting these orders is Cramer's V coefficient.

[5] SAMPLING: 10 samples (30-unit) were randomly selected from the entire set of empirical data (N=420). Random sampling with replacement was performed.

[6] GREY INCIDENCE ANALYSIS: The *grey relational indicator* between X_i and Y_j for each of 10 samples was calculated. To this end, the following operations were performed:

– variable *Xi = (xi(1), xi(2), …, xi(n))* was developed by means of operator D1 in the following way:
  XiD1 = (xi(1)d1, xi(2)d1, ..., xi(n)d1)
  where:
  xi(k)d1 = xi(k) - min {xi(k) / max {xi(k)} - min {xi(k)} for k=1,2, ..., n.
– variable Yi = (yi(1), yi(2), …, yi(n)) was developed by means of interval operator D1 in the following way:

$\overline{}$

YiD1 = (yi(1)d1, yi(2)d1, …, yi(n)d1)

    where:
yi(k)d1 = yi(k) - min {yi(k) / max {yi(k)} - min {yi(k)} for k=1,2, …, n.
Following the development of data by means of operator D1, the symbols of the
variables remain unchanged.
−   A sequence of data necessary for calculations for each 30-unit sample was formed
in the following way:
−

$Y_1 = (y1(1), y1(2), …, y1(30))$
$Y_2 = (y1(1), y1(2), …, y1(30))$
$X_1 = (x1(1), x1(2), …, x1(30))$
$X_2 = (x2(1), x2(2), …, x2(30))$
$X_3 = (x3(1), x3(2), …, x3(30))$
$X_4 = (x4(1), x4(2), …, x4(30))$
$X_5 = (x5(1), x5(2), …, x5(30))$
$X_6 = (x6(1), x6(2), …, x6(30))$
$X_7 = (x7(1), x7(2), …, x7(30))$
$X_8 = (x8(1), x8(2), …, x8(30))$
$X_9 = (x9(1), x9(2), …, x9(30))$
$X_{10} = (x10(1), x10(2), …, x10(30))$

−   compute the absolute degree of grey incidence.
−

$$s_{yi} = \left| \sum_{k=1}^{n-1} y_i(k) + \frac{1}{2} y_i(n) \right|$$

$$s_{xi} = \left| \sum_{k=1}^{n-1} x_i(k) + \frac{1}{2} x_i(n) \right|$$

$$s_{xi-yi} = \left| \sum_{k=1}^{n-1} x_i(k) - y_1(k) + \frac{1}{2} x_i(n) - y_1(n) \right|$$

$$\varepsilon_{yixi} = \frac{1 + \left| s_{y1} \right| + \left| s_{xi} \right|}{1 + + \left| s_{y1} \right| + \left| s_{xi} \right| + \left| s_{xi} - s_{y1} \right|}$$

[7] SETTING THE ORDER: Based on the average of the indicator *the absolute
degree of grey incidence* for 10 samples, the order of the influence of variables X_i

on variables Y_j was determined. As a result of this operation, two orders were set. The first one in relation to variable Y_1, the second one in relation to variable Y_2.

[8] COMPARING ORDERS: The orders of the influence of variables X_i on variables Y_j obtained using statistical procedures were compared with the orders obtained using grey methods. To this end, the Wilcoxon signed-rank test was used.

5. Results of Orders Analysis

The analysis of data using the classical approach by means of contingency tables for a large sample of 420 allowed to assess the significance of the influence of independent variables X_1-X_{10} as well as their strength of influence on dependent variables Y_1, Y_2. The results of the research are presented in table 3.

Table 3. *Results of the goodness of fit test and evaluation of the strength of association*

	Analysis of contingency tables for Y_1				Analysis of contingency tables for Y_2			
	Value of test statistics	df	p-value	V-Cramer	Value of test statistics	df	p-value	V-Cramer
					21.5326	12	0.04310	0.425968
X_1	21.2321	12	0.04766	0.50351	48.3443	12	0.0005	0.365162
X_2	21.4345	12	0.04436	0.51562	10.0406	12	0.62124	0.445258
X_3	19.1541	12	0.08010	0.50069	12.9174	9	0.21065	0.315528
X_4	14.084	9	0.12924	0.46787	22.3962	16	0.21305	0.589308
X_5	42.7730	16	0.00031	0.93450	20.8962	16	0.14265	0.676536
X_6	27.9684	16	0.04447	0.55402	30.9402	16	0.01748	0.475976
X_7	28.9881	16	0.03235	0.61105	32.3896	12	0.01661	0.620182
X_8	22.6251	12	0.03917	0.54333	44.2922	20	0.00005	0.94201
X_9	24.8462	20	0.23121	0.54265	34.74010	16	0.00543	0.47561
X_{10}	23.60068	16	0.12447	0.55284				

Not all independent variables covered in the research significantly affect dependent variables: Y_1 (*level of conflicts concerning family-related issues)* and Y_2 (*level of conflicts concerning business issues).* A significant influence of analyzed variables on a dependent variable was proven: Y_1: X_1, X_2, X_5, X_6, X_7, X_8. In case of Y_2 variable, the influence of variables: X_1, X_2, X_7, X_8, X_9, X_{10} was proven.

From the point of view of succession changes which in recent years have been intensely taking place in Poland, it seems of particular importance to analyze the influence of the generation that is exercising control on the emergence of family-related conflicts. As it was shown in the research, it is the authority-exercising generation (X_1) that is associated with the analyzed conflict spheres. The detailed analysis of the contingency tables allows to state that the first generation of family business owners in Poland largely gets into conflicts with family (Y_1), less often in the business sphere (Y_2). However, both in the former and latter case, the influence of this variable on Y_1 and Y_2 is statistically significant with a medium force of association.

Similar "behaviour" is observed in case of X_2 variable (number of family members employed in a firm) and X_8 (level of family employees' privileges), which affects dependent variables in a similar way. It means that the number of family members engaged in business activity (X_2) influences both family and business conflicts. Similar influence is exerted by the level of family employees' privileges (X_8) on Y_1. The analysis of Cramer's V indicator points to a medium force of association of X_2 (number of family members) on the analyzed level of family-related conflicts and low for business conflicts. Similar results are observed for X_8 variable (level of family employees' privileges) (cf. table 3). Slightly higher results of Cramer's V test were obtained for X_7 variable (level of family members' engagement in the business sphere). In this case it is 0.61 for Y_1 variable and 0.48 for Y_2 variable. The analysis of the contingency tables shows that the higher level of family members' engagement in the business sphere is, the greater family-related conflicts (Y_1) are. Such an interdependency was not noted in case of analyzing contingency tables for X_7 (level of family members' engagement in the business sphere) with Y_2 (level of conflicts concerning business issues).

The results of analyzing the significance of the influence of X_9 variable (perspective of investment planning) on Y_1 did not confirm this influence, unlike in case of Y_2 variable. Association between X_9 and Y_2 was proven, and the force of this association may be regarded as extremely high (the result of Cramer's V coefficient at the level of 0.94). The obtained result relates to the specific character of family businesses' functioning in Poland, which is based on family values more often connected with tradition than innovation. Such businesses base their structure and mission on tradition whereas the environment puts pressure on using innovative solutions both in production, service-related and trade-related processes, and company organization. In such a situation, when it becomes necessary to satisfy opposed expectations, one observes the emergence of conflicts in company structure, which relate to making decisions, allocating financial resources, being responsible for planned activities, etc. X_{10} variable (the degree of business innovation) significantly influences Y_2. The force of association between these variables is at a medium level (Cramer's V – 0.47). The analysis of the contingency tables points to lowering the assessment of conflict

intensification in the business spheres with the high assessment of business innovation.

Summarizing the qualitative character of the obtained data, one can state that the greatest influence on family-related conflicts in family businesses is exercised by bringing up future successors in the entrepreneurial spirit and the level of family members' engagement in the business sphere. The greatest influence on business conflicts is exercised by the perspective of investment planning and the level of family employees' privileges.

Next, based on the force of association, the order of variables was determined using a chi-square test. The first place was taken up by a variable for which the force expressed by means of Cramer's V coefficient was the greatest (table 5 – "chi-square test" column).
Grey Incidence Analysis was used to determine the force of association between independent and dependent variables, for small samples (30-element, step 7 in the methodology). The analysis results are presented in Table 4.

Table 4. *GIA results*

	Grey Incidence Analysis			
	Y_1 average	Y_1 standard deviation	Y_2 average	Y_2 standard deviation
X_1	0.610	0.162	0.453	0.182
X_2	0.537	0.053	0.678	0.190
X_3	0.502	0.003	0.511	0.180
X_4	0.578	0.123	0.299	0.165
X_5	0.721	0.138	0.680	0.133
X_6	0.667	0.165	0.755	0.151
X_7	0.611	0.137	0.598	0.118
X_8	0.608	0.142	0.618	0.141
X_9	0.646	0.148	0.814	0.110
X_{10}	0.843	0.144	0.701	0.110

Next, based on the grey relational index (in the range from 0 to 1, where 0 means a total lack of relationship between variables, and 1 means the maximum degree of correlation between variables) the sequence of the order was set, which is shown in Table 5.

Table 5. *Order of variables obtained in the GIA and classical statistics*

	Significance			
	GIA		**chi-square test**	
	Y_1	Y_2	Y_1	Y_2
1	X_5	X_9	X_5	X_9
2	X_7	X_8	X_7	X_8
3	X_6	X_5	X_6	X_5
4	X_{10}	X_7	X_{10}	X_7
5	X_9	X_{10}	X_8	X_{10}
6	X_8	X_3	X_9	X_3
7	X_2	X_1	X_2	X_1
8	X_1	X_2	X_1	X_2
9	X_4	X_4	X_3	X_6
10	X_3	X_6	X_4	X_4

To verify differences between the orders obtained when using classical tools and GIA, the Wilcoxon signed-rank test was used. The results obtained from the chi-squared test and GIA were treated as pairs. It was assumed in the null hypothesis that there are no statistically significant differences between the set orders. The adopted significance level in the carried-out test (separate for variables Y_1 and Y_2) was equal to 0.05.

Table 6. *Results of the Wilcoxon test for variables Y1 and Y2*

	Test statistic	p-value	Decision
H_0: Lack of differences between the order of variables X_1-X_{10} for variable **Y_1** set when using the classical approach and the order set in GIA	17	0.955	no reason to reject the hypothesis 0
H_0: Lack of differences between the order of variables X_1-X_{10} for variable **Y_2** set when using the classical approach and the order set in GIA	22.5	0.999	no reason to reject the hypothesis 0

The results of the test presented in table 6 show that the order of dependent variables obtained in the classical approach may be the same as in the case of using Grey Incidence Analysis.

6. Summary

The conducted research allowed to collate the results of the analyses carried out using classical statistical methods for a large sample with the analysis performed using the GIA method for small samples. The obtained research results confirm the hypothesis which says that the results of conclusions drawn from a small amount of data using the GST methods are in keeping with the results of correlational research for a large

amount of data defining the same phenomenon. The positive verification of the presented research hypothesis underlies a valid conclusion for researchers engaged in the issues related to family businesses, which states that when there are problems with collecting a large sample or when we want to conduct a quick pilot or preliminary study which will examine interdependencies between variables defining a given phenomenon, a new and alternative statistical research methodology to be used may be Grey Systems Theory.

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