
Long-Term Relationship Between Prices and Exchange Rates

Submitted 13/11/20, 1st revision 21/12/20, 2nd revision 11/01/21, accepted 16/02/21

Zenon Wiśniewski¹

Abstract:

Purpose: This paper aimed to prove the existence of a long-term relationship between prices and exchange rates like that derived from the theory of purchasing power parity. Particular attention has been paid to the possible reasons behind the deviations of exchange rates from the values implied by the parity and to the theoretical justification of this phenomenon.

Design/Methodology/Approach: The research was carried out by detailed analysing of the exchange rates of major currencies of the world, the U.S. dollar, the British pound and the Japanese yen related to individual quarters in the 40 years period. The formal empirical verification was made with the use of econometric tools that allow a long-term study of the relationship between stochastic processes. In analyses there were used the OLS estimator and the appropriate regression of USD/JPY/GBP relative to the logarithms of price levels in these countries. The phenomena of the cointegration of rates and prices and the stationarity of real exchange rates were also examined.

Findings: Formal verification did not allow the confirmation of the exact relationship of long-term forecast by purchasing power parity. For all the examined exchange rates, the existence of co-integrating relationship was not shown, the relationship which would be fulfilled in spite of short-term deviations from parity. Despite the fact that the study employed data concerning a forty-year time period, the cycles of deviations from the balance path take too long to be able to demonstrate the stationarity of residuals in the cointegration analysis. This hypothetically makes purchasing power parity a very long-term relationship, which in the short-term and medium-term is subject to relatively strong disturbance.

Practical implications: This paper analyses and empirically verifies the widely known long-term relationship of purchasing power parity. Moreover, it comes up with the proper economic justification of the results.

Originality/Value: The purchasing power parity theory tends to be used by institutions actively creating macroeconomic policies. The research findings may contribute for better understanding of monetary policies in environment of inflation.

Keywords: Exchange rates, purchasing power parity, cointegration.

JEL Code: F31, C12, C22.

Paper type: Research paper.

¹Nicolaus Copernicus University in Toruń, e-mail: zenwis@umk.pl;

1. Introduction

The purchasing power parity (PPP) is one of the oldest and most important theories in the economics. It was first formulated by Gustav Cassel in 1918 (Cassel, 1918). He found a relationship between the purchasing power of two currencies and their exchange rates. Since then, the purchasing power parity has been widely regarded by central banks as a guide on how to impact the value of national currencies, if they are in a state of deep imbalance. Moreover, a variety of new practical applications of the theory of purchasing power parity have been developed. For instance, PPP is sometimes used to predict future long-term balance exchange rates.

According to theory the arbitration mechanism in the goods market ought to lead to the fulfilment of the purchasing power parity. However, there is a number of factors, which may cause some deviations from the theory. The first and most important model of long-run deviations from PPP was advanced by Balassa (1964) and Samuelson (1964). They argued that empirically, when all countries price levels once translated to dollars at prevailing nominal exchange rates, rich countries tend to have higher price levels than poor. Moreover, another factors such as customs duty and transaction costs in international trade, the presence of non-tradable goods, changes in the long-term real exchange rate balance, as well as short-term deviations described by the Dornbusch model of exchange rate overshooting may have also an impact (Dornbusch, 1976). Short-term price rigidity, with the full flexibility of nominal exchange rates means that PPP should be interpreted only as a long-term balance state and not as literal dependence fulfilled for every moment of time rates (Shapiro, 1983).

The objective of this study was to verify empirically the purchasing power parity theses and to interpret the obtained results in the light of the economic theory. The research question that was posed in the work is whether the empirical data provide evidence of the actual existence of the PPP relationship. The issue of the parity is a controversial topic in the literature, since it requires far-reaching simplification of processes that have an impact on the development of exchange rates. Moreover, this theory is hampered by a number of unrealistic assumptions, the fulfilment of which, in fact, is practically impossible.

Despite its shortcomings, the theory of purchasing power parity is recognised by many economists who publish in world literature more and more empirical evidence of the impact of this theory on the observed exchange rate. The development of new econometric tools has enabled a departure from the classical methods of linear regression phenomena with a view to verifying them. The application of advanced statistics to analyse phenomena

such as, for example, the stationarity of time series or their co-integration, allowed researchers to change their approach to the study of PPP.

2. The Purchasing Power Parity Theory

Considerations on the purchasing power parity should begin by defining the exchange rate, as it is a basic concept used in the theory of open economy macroeconomics. The exchange rate is the price of one currency expressed in another currency. For example, the exchange rate of GBP against USD is interpreted as the number of pounds you get when you sell one US dollar. These prices can be observed in currency markets where transactions of sale and purchase of foreign currencies are realised. Trading currencies of different countries allows the exchange of goods between economies. Having the opportunity to exchange their own currency for foreign, residents of a country are able to import goods from abroad, paying foreign producers in their native currency.

The exchange rate is integral to the functioning of the economy and has a real impact on it. It is crucial for international trade, since a change in an exchange rate affects the competitiveness of goods abroad. Growth in the price of the domestic currency expressed in a foreign currency is called appreciation. This process causes an increase in prices of domestic products on foreign markets, other things being equal, and is disadvantageous from the point of view of exporters operating in the country (Krugman, Obstfeld, and Melitz, 2018). However, appreciation is beneficial for importers of foreign goods, because as a result of the strengthening of the national currency, you can buy these products at a relatively lower price. As a result of depreciation, *i.e.*, a decline in the value of the national currency, it is exactly the opposite situation.

Exchange rate changes impact the economy significantly: in particular, they make national export goods more competitive as a result of depreciation, or in the case of the appreciation of the domestic currency, they contribute to the decline in that competitiveness. As a result, the trade balance as well as a number of other economic indicators, such as, for instance, the inflation rate, or the volume of foreign debt, are changed in the country.

The exchange rate determined as a result of transactions in foreign exchange markets is called the nominal exchange rate. It is observed in reality and used every day in foreign exchange transactions. The value of this rate, however, does not necessarily reflect the purchasing power of the currency. To this end, the concept of the real exchange rate was introduced and is defined by the following formula (Krugman, Obstfeld, and Melitz, 2018):

$$Q^{HC/FC} = E^{HC/FC} * (P^{FC} / P^{HC})$$

where the symbols denote subsequently:

$Q^{HC/FC}$ – real exchange rate;

$E^{HC/FC}$ – nominal exchange rate (the amount of the home currency per a foreign currency unit);

P^{FC} – price levels abroad;

P^{HC} – price levels in the country.

The real exchange rate is interpreted as a measure of foreign prices expressed in domestic prices. Both the nominal exchange rate and real exchange rate are bilateral, as they describe the price of the domestic currency expressed in a selected foreign currency. If $Q^{HC/FC}$ increases, then there is a real depreciation of the home currency. However, if $Q^{HC/FC}$ drops, then a real appreciation of the domestic currency occurs, since its purchasing power increases relative to foreign goods. The most far-reaching theoretical concept of interdependence of exchange rates and prices is LOOP (the law of one price). The law of one price states that if the following conditions are met:

- goods in the economies of the two countries are tradable,
- goods produced in each of the countries are homogeneous,
- there are no restrictions imposed on international trade,
- both economies have full employment,

then the price of an identical good in the countries, expressed in a common currency, should be the same. You can also say that with the appropriate conditions of an effective market, arbitration forces cause the convergence of world prices of a homogeneous good to a uniform level. In addition, assuming no barriers to free arbitration, the conclusion resulting from the law of one price for the home country (HC) and for the foreign country (FC) can be written as follows:

$$P_i^{FC} * E^{HC/FC} = P_i^{HC},$$

where the symbols denote subsequently:

P_i^{HC} – price i – of the good in the country;

P_i^{FC} – price i – of the good abroad;

$E^{HC/FC}$ – exchange rate, *i.e.*, the amount of home currency for a foreign currency unit.

With data on product prices in the two economies, it is possible to calculate the nominal equilibrium exchange rate for the currencies of the two countries. If the above equation is not satisfied, then there is a possibility of commodity arbitrage. To sum up, when under the conditions of the efficient market we can observe that the difference between the observed rates and the rate resulting from the LOOP is noticeable, arbitration will lead to the

convergence of the rate and prices of the good in the direction of those appointed by the parity. The law of one price is subject to the restrictive assumptions that mean that LOOP most frequently remains a purely theoretical dependence. However, it has become a source of further exploration of the relationship between the nominal exchange rate and price levels in different countries.

The concept of absolute purchasing power parity was introduced based on the law of one price earlier described. If you extend the operation of the law of one price for all the goods occurring in the economy, then when calculating the ratio of the prices of two identical baskets of goods of the two economies, we are able to determine the nominal exchange rate (Eiteman, Stonehill, Moffett, 2007). The exchange rate consistent with purchasing power parity can be written as follows:

$$E^{HC/FC} = \sum_{i=1}^N \frac{\alpha_i * P_i^{HC}}{\alpha_i * P_i^{FC}}$$

where:

$E^{HC/FC}$ – the exchange rate of the country, *i.e.*, the amount of currency per unit of a foreign currency,

P_i^{FC} – price *i* of the good in the foreign market expressed in a foreign currency,

P_i^{HC} – price *i* of the good in the home market expressed in the country's currency,

α_i – share *i* of the good in the reference basket.

In a mathematical sense, the above formula is an extension of the law of one price for a modified sum of goods making up the basket. The assumption of the equal price of a homogeneous good in two countries denominated in one currency was converted into the equality of the prices of baskets of goods, or to put it in more general terms, into the equality of price levels in the two economies.

This equation determines the nominal exchange rate, for which two baskets of two different economies will cost the purchaser exactly the same in terms of a common currency. An economic interpretation leads to the conclusion that the value of the currency, which is its nominal exchange rate relative to other currencies, is dependent on its purchasing power, and thus the amount of goods and services that can be purchased per unit of the currency analysed. Thus, PPP is satisfied when for every value of the exchange rate, the purchasing power of the currency abroad is equal to the purchasing power in the country (Krugman, Obstfeld, and Melitz, 2018).

It is worth considering the impact of meeting PPP on the real exchange rate imposes that the following equation is obtained:

$$Q^{HC/FC} = 1.$$

With the fulfilled purchasing power parity in its absolute version the real exchange rate is always equal to one. Changes in the exchange rate should lead to appropriate adjustments in prices, while inflation should cause changes in the exchange rate (Thalassinos, 2007). With these adjustments the real exchange rate should theoretically be invariably equal to one.

To conclude the description of the absolute purchasing power parity we must again note the relationship linking it with the law of one price. If LOOP is met for all the goods taken into account while designing the reference basket, then, as a result the parity will be met. In turn, the behaviour of PPP is not equivalent to the fulfilment of the law of one price for all goods. The non-fulfilment of LOOP does not necessarily cause an excessive deviation of the observed prices and exchange rates from the relationship resulting from PPP (Engel and Rogers, 2001).

The absolute PPP requires meeting a significant number of far-reaching assumptions that prove to be difficult to observe in reality. However, the absolute PPP shows a weaker theorem called the purchasing power parity in a relative version. It found an extensive application, since it is believed to be true, even under the conditions of limited arbitrage opportunities. A common opinion is that the relative PPP holds when there are duties, incomplete information and transportation costs.

The dependence of the relative PPP can be derived from the absolute version hence the following pattern emerges (country (Krugman, Obstfeld, and Melitz, 2018):

$$\frac{E_t^{HC/FC} - E_{t-1}^{HC/FC}}{E_{t-1}^{HC/FC}} = \pi_t^{HC} - \pi_t^{FC}$$

where:

$E_t^{HC/FC}$, $E_{t-1}^{HC/FC}$ – exchange rates, *i.e.*, the amount of the currency of the country per unit of a foreign currency at time t , and $t-1$ respectively,

π_t^{HC} – the rate of inflation in the country in period t ,

π_t^{FC} – the rate of inflation abroad in period t .

Thus, according to the relative version of PPP, a percentage change in the exchange rate is equal to the difference in the inflation rate of two economies. The major difference between the versions of the parity lies in

the fact that the relative version is based on change rates in price levels and exchange rates. It is worth noting that for the veracity of the above relationship, fulfilling the absolute PPP is not a necessary condition, since the relationship implied by the relative version of the proposal is weaker than in the case of the absolute version. The parity in question is particularly fulfilled, if the nominal exchange rate and price levels hold the following relation:

$$E^{HC/FC} = \lambda * (P^{HC} / P^{FC}),$$

where λ is a positive constant. Just note that in the derivation of the equations of the relative PPP lambdas will be reduced while dividing the price quotients of two consecutive periods. The parameter λ can be interpreted simply as including in the model transport costs or, more generally, transaction costs and customs duties. In a particular case of the fulfilment of the absolute parity the case when $\lambda = 1$ is considered.

An important issue is the impact of meeting the relative PPP on the real exchange rate. In order to determine $Q^{FC/HC}$ one should act similarly as in the case of the absolute parity. If the premise is that the relative PPP is met, then the following holds:

$$Q^{HC/FC} = \lambda.$$

In conclusion, in the case of the fulfilment of only the relative parity, the real exchange rate remains constant in time and is equal to λ , where lambda is a positive constant. In a particular case when $\lambda = 1$, the situation is the absolute fulfilment of the PPP and the real exchange rate equals 1.

3. Empirical Evidence

The aim of the research is to verify empirically the theory of purchasing power parity in the long run. Three exchange rates of major currencies of the world were analysed: the Japanese yen (JPY), the U.S. dollar (USD) and the British pound (GBP). The study used data related to individual quarters, derived from the database of *The OECD Statistical Compendium*. Statistical data cover the period from 1970 to 2009. The period end is set as 2009 due to the financial global crisis that led to the following monetary quantitative easing undertaken by central banks, especially the US Federal Reserve System. The significant growth in the monetary base supply leads to money supply shock and may result in short and medium term disruptions in the macroeconomic equilibriums. In order to avoid the constraint of use of the extremely long data series, the data range has been trimmed accordingly.

The starting point is the relation of the PPP. Having transformed both sides into logarithms we obtain the following equation describing the model:

$$e_t = \beta^{HC} * p^{HC}_t + \beta^{FC} * p^{FC}_t + \varepsilon_t,$$

where the symbols denote the following:

e_t – logarithm of the currency exchange rates,
 p^{HC}_t – logarithm of price levels in the country,
 p^{FC}_t – logarithm of price levels abroad,
 ε_t – random component.

The above equation is an example of the issue that can be approximated by means of the OLS estimator. The absolute parity requires imposing restrictions on the model parameters. According to the original version of the parity formulated by Cassel, the model coefficients should be equal to $\beta^{HC}=1$, $\beta^{FC} = (-1)$ respectively. In addition, the absolute version excludes the existence of a constant in this model. The relative version, in turn, implies the following formula:

$$\ln(E^{HC/FC}) = \ln(\lambda) + \ln(P^{HC}) - \ln(P^{FC}).$$

This is from where we can derive a linear model describing the relative form of PPP:

$$e_t = \alpha + \beta^{HC} * p^{HC}_t + \beta^{FC} * p^{FC}_t + \varepsilon_t.$$

The symbols denote exactly the same economic variables as in the case of the previous model. The only difference caused by the change from the absolute version to the relative is that the model includes a constant α . It results from the assumption of the existence of the factor λ changing the price ratio.

The three steps estimations were carried out in order to empirically examine the parity hypothesis: the least squares method, the cointegration and stationarity of the real exchange rates. The previously derived model will be estimated on the rate logarithms and price levels using the least squares method. The estimation will be performed on the currencies examined in the present work: JPY/USD, GBP/JPY and GBP/USD. Based on the results of the regression, it will be determined whether the received values β^{HC} , β^{FC} are equal to unity in the absolute value, and whether they have opposite signs. Otherwise, the parity will not be met. The regressions were carried out relative to both the CPI and PPI. The first to be presented are the results of the regression of USD/JPY relative to the logarithms of price levels in both countries.

Table 1. *The results of the regression of the JPY/USD exchange rate logarithm*

	CPI	PPI
Ln of the Japanese prices index	0.17227 (0.10153)	0.77874 (*) (0.10687)
Ln of the American prices index	-0.85501 (*) (0.06371)	-1.14362 (*) (0.04630)
Constant	4.76595 (*)	4.76679 (*)
R ²	0.8678	0.8540
F	518.69	447.42

Note: (*) denotes significance at the 5% level, in parentheses are standard errors.

Source: Own calculations.

In the regression on the CPI logarithms the null hypothesis of the F test with a total insignificance of variables was rejected. The variable representing the logarithm of the price level in Japan for the 5% confidence interval turns out to be statistically insignificant, since its value P is equal to 9.2%. The P -value is defined as the probability that the test statistics are equal to or greater than the value obtained from the sample. According to the result, there is no reason to reject the null hypothesis of the student's t test, which says about the statistical insignificance of the price level. The R square level reaches as high as 86.7%, so the volatility of the logarithm of the exchange rate is explained in its considerable part by the changes in price levels. The model subjected to regression has all of the variables logarithmical, so the calculated regression parameter estimates are elasticities of the exchange rate relative to price levels in Japan and the United States.

The resulting parameter estimates of the consumer price indices logarithms, though different have (according to the PPP) opposite signs - the results are 0.17 for the Japanese prices and -0.85 for the U.S prices. The estimator of the Japanese prices is definitely different from unity moreover, it is statistically insignificant.

The third column provided the results obtained from an analogous procedure applying the Producer Price Index. Changing the indices results in the obtainment of the results closer to the purchasing power parity.

The values of F statistics used in the regression of the PPI logarithms indicate the total statistical significance of variables. In addition, all of the PPI indexes separately are also important, and in the previous case insignificance of the logarithm of the price levels in Japan was obtained. The R square remained at the level of 85 percent. Particular attention should be paid to the coefficients obtained in the exchange rate regression relative to the logarithm of the PPI. For the logarithm of the price level of producer goods in the United States, the value is (-1.14), which is relatively close to (-1) resulting from the PPP. The parameter for the Japanese producer goods

prices stood at 0.78, which is significantly smaller than one: however, the sign of the impact predicted by the parity is maintained.

To sum up, the change in the price indices approximated significantly the estimation results to the values derived from the theory of purchasing power parity. Due to the inclusion in the study of the goods that to a greater extent can be considered as tradable, we obtained the significance of all variables and it meant that we obtained the vector of regression parameters more consistent with the one predicted by the theory of PPP. Finally a regression of the logarithm of the GBP/JPY exchange rate on the logarithms of the price levels in Japan and the United Kingdom was calculated.

Table 2. *The results of the regression of the GBP/JPY exchange rate logarithm*

	CPI	PPI
Ln of the Japanese prices index	-0.05754 (0.11751)	-0.67418 (*) (0.02783)
Ln of the British prices index	0.70285 (*) (0.05052)	0.89605 (*) (0.10228)
Constant	-5.21925 (*)	-5.22660 (*)
R ²	0.9233	0.9227
F	950.78	913.09

Source: Own calculations.

The conclusions drawn from the regression in many aspects are similar to the previous regression. Again, for both regressions a high coefficient of determination *R* square of 92% was obtained. The variables in the two regressions are jointly statistically significant, since the *P*-values for *F* statistics calculated for them are practically close to zero. Applying for the regression the Producer Price Index has removed the statistical insignificance of the price level in Japan. The conversion of the considered CPI indexes to the Producer Price Index led to the obtainment of the results closer to the parity theory. The estimator of the impact of the Japanese price level approximated considerably to the level resulting from the absolute PPP, that is to (-1). These results confirm the assumption that purchasing power parity is closer to fulfilment in the case of the level producer goods than in the case of the regression rate relative to the CPI indexes. The coefficient of the logarithm of the British PPI also approximated to one.

Table 3. *The results of the regression of the GBP/USD exchange rate logarithm*

	CPI	PPI
Ln of the American prices index	-1.09463 (*) (0.12699)	-1.25356 (*) (0.15367)
Ln of the British prices index	0.90641 (*) (0.08701)	0.96904 (*) (0.09651)
Constant	-0.44966 (*)	-0.45940 (*)
R ²	0.6500	0.6516

F	146.72	143.09
---	--------	--------

Source: Own calculations.

In contrast to the previous cases, the coefficient of determination R square in both regressions was only 65%. Despite this, all of the variables proved statistically significant, both jointly or separately, for all statistics informing about it the P -value obtained equalled zero.

The regressions conducted are most consistent with the purchasing power parity among all that have been conducted so far. Unlike in the previous cases, the estimators of the examined impact of the CPI logarithms are close to the values predicted by the purchasing power parity. The coefficients of the price levels logarithms amounted to -1.09 for the United States and 0.91 for the UK, and so the absolute values of the parameters are approximate to unity and to each other. In addition, they have opposite signs, which confirms another thesis of PPP.

The conversion of the price index to PPI brought mixed results. Although the estimator of the British prices approximated to one, the impact of the U.S. prices departed somewhat from the minus one resulting from the PPP. It is worth noting that the case of the GBP/USD exchange rate was the only one where the relationship observed was close to the consumer price indices parity. Prior to conducting the formal statistical tests assessing the conformity of the actual data with the theory of purchasing power parity, first conclusions were drawn from the observation made:

1. The change in the price index from the one describing the prices of goods and services to the Producer Price Index led to the obtainment of the results that were related closely to the theory. It also follows from the fact that services are among the wide-ranging class of non-tradable goods. Non-tradable goods belong to the group of factors that cause deviations from the relation defined by the parity.
2. In most cases it was confirmed that a statistically significant relation occurred between the nominal exchange rates and prices. Furthermore, the signs of the estimated coefficients are consistent with the theory of PPP. The final conclusion is the observation that the coefficients are slightly different from the theoretical values. The study will present tests corresponding to the question of whether those differences are statistically significant.

Fulfilling the PPP requires imposing parameter restrictions on the model, which can be written in the form of the following hypotheses:

$$H_0: \begin{cases} \beta^{HC} = 1 \\ \beta^{FC} = (-1) \end{cases}$$

Then, based on the F statistic a verification of the validity of the null hypothesis for the three regressions with the use of the CPI was made. Examining the p -value for the three statistical tests performed will allow definitive conclusions from our earlier discussion and analysis to be drawn.

Table 4. *The test results of the hypotheses for the regression with the use of the CPI*

Regression name	F statistic value	Calculated P -value
logarithm of the JPY/USD exchange rate divided by price logarithms	137.00	0.0000
logarithm of the GBP/JPY exchange rate divided by price logarithms	48.23	0.0000
Logarithm of the GBP/USD exchange rate divided by price logarithms	103.90	0.0000

Source: *Own calculations.*

The tests clearly reject the null hypothesis that says about exact fulfilment of the relation arising from the parity for selected pairs of countries. The formal proof of the rejection of the hypotheses of fulfilling the relative PPP by these rates are the p -values calculated for them. In all of the cases, they are practically equal to zero, which results in the rejection of the correctness of the restrictions imposed on the model. The introduction of the PPI indices into the regression led to a change in the estimators of the impact of prices on those closer to the theory. It is necessary, therefore, to examine whether the restrictions imposed on the parameters will be confirmed by statistical tests.

Table 5. *The test results of the hypotheses for the regression with the use of the PPI*

Regression name	F statistic value	Calculated P -value
logarithm of the JPY/USD exchange rate divided by price logarithms	26.72	0.0000
logarithm of the GBP/JPY exchange rate divided by price logarithms	7.10	0.0011
logarithm of the GBP/USD exchange rate divided by price logarithms	111.43	0.0000

Source: *Own calculations.*

As in the previous proceedings, for all of the three pairs of countries the model restrictions consistent with the purchasing power parity were rejected. For all cases, the p -value is less than five per cent, which results in the rejection of the null hypothesis. As a matter of fact, in two cases the value of the F statistics decreased, however, it did not have any significant impact on the p -value. It is thus formally confirmed that the restrictions imposed on the model are not correct.

To sum up the statistical analysis hitherto, it can be concluded that despite the occurrence of significant correlations between rates and price levels, the relationship resulting from purchasing power parity is not exactly satisfied.

Most of the early empirical studies did not consider the difference between the essence of the short-term and long-term impact of purchasing power parity. Although it is widely believed the PPP is a long-term relationship, the tested models did not formalize these suspicions. Attempts were made to estimate the problem of the classical linear regression model in the least restrictive form of:

$$e_t = \alpha + \beta^{HC} * p^{HC}_t + \beta^{FC} * p^{FC}_t + \varepsilon_t.$$

PPP relationship disturbances were considered important, however, they were identified only with a random component ε_t .

Similar to the tests carried out in the previous subchapter, most studies suggested rejecting the dependence imposed by the PPP in its exact form. The exception, however, is the work of Frenkel (1978), in which he presented completely different empirical results. He estimated the model of purchasing power parity in a form which was very similar to the previously discussed relative version, with the exception that he assumed *a priori* an equal impact of prices on the nominal exchange rate. Frenkel estimated the following model:

$$e_t = \alpha + \beta *(p^{HC}_t - p^{FC}_t) + \varepsilon_t,$$

where the symbols retain the existing and already defined meaning. The research was conducted on the data describing exchange rates of countries affected by high inflation. Frenkel received β values close to unity, which gave reasons to conclude that in a situation where prices rise rapidly, the exchange rate behaves in accordance with the principle of purchasing power parity. Unfortunately, the work did not take into account the issues of a stationary random component, which is a prerequisite to assess the accuracy of regression.

The non-stationarity of regression residuals makes the interpretation of the results lead to false conclusions. It may turn out that the obtained dependence resulting from the OLS does not hold true, in spite of the significance of the F statistic and student's t statistics. In econometrics this phenomenon is referred to as the apparent regression and may occur when the explanatory variables selected by the researcher show a trend similar to the dependent variable of the model. A completely random correlation may cause a statistically significant relationship in the sense of OLS.

If when attempting to regress variables of a similar trend, they prove to be statistically significantly different from zero, the value of R^2 may be high, but the inferred relationship may have a completely random nature, hence the name of the apparent regression. The apparent regression can occur in the following cases (Moosa and Bhati, 1997):

- the regression with the use of random variables with a large number of samples,
- the regression observed in the realities of an economic variable changeable into a random variable,
- non-stationarity of the variables.

In this situation, it must be examined whether the results described above are a real long-term relationship, or if there is a risk that the regressions are apparent.

A stochastic process is referred to as strongly stationary, if the total and conditional probability distributions of the process are not dependent on time (Charemza and Deadman, 1997). However, the definition of weak stationarity turns out to be more practical and easier to verify when based on the data. This definition is limited to the assumptions about the expected value, variance and covariance of the stochastic process. It is assumed that both the average and the variance of this process are constant over time, and the covariance of the variable considered for two points in time depends only on the time distance between them, and not on the moments in which the observation was made. Processes that are not characterized by such behaviour and do not comply with even the weak stationarity are called 'non-stationary'.

The time series based on macroeconomic data often turn out to be non-stationary (Moosa, Bhati, 1997). Most likely, this also applies to data describing the exchange rates and price levels, but this will be examined in the later part. The issue discussed is important for statistical inference, because the omission of the issue of non-stationarity of the variables in regression generally leads to drawing dubious conclusions from the results analysis.

To eliminate the non-stationarity we can differentiate a variable, that means to generate its increments indicated by a Greek letter Δ . This allows, among other things, to eliminate the trend, whose occurrence makes the regression analysis meaningless. A formal model for the variable X_t increment is expressed by the following formula:

$$\Delta X_t = X_t - X_{t-1},$$

where X_{t-1} is a numerical realization of the process at the time of one period earlier than t . If the first increments of the variable are non-stationary, we can repeat the operation taking increments for the variable ΔX_t where we obtain the variable $\Delta(\Delta X_t) = \Delta\Delta X_t$. It happens that to achieve stationarity the series needs to be differentiated a couple of times. For this purpose, it is useful to apply the definition of the degree of integration of the series.

A time series is called ‘degree k integrated’, if the series k of these increments is stationary. The processes of this property are designated with the symbol $X_t \sim I(k)$. In particular, if the process is zero degree integrated, then it is stationary and there is no need to differentiate. It must be remembered, however, that the differentiation of the data leads to the loss of information on the long-term dependence. Even if two non-stationary series drift together following one trend, then after regression on the stationary first differences, this dependence cannot be captured. Therefore, such a method of testing the long-term dependence between variables is unreliable.

The primary and most common test to detect non-stationarity is the Dickey Fuller test, which later was transformed into an improved version named as ‘the Augmented Dickey Fuller Test’. The null hypothesis in the ADF test is its non-stationarity of a random variable, while the alternative hypothesis is its stationarity (Moosa and Bhati, 1997).

Another test that is often used in practice is the KPSS test, named after the inventors of this test statistics: Kwiatkowski, Phillips, Schmidt and Shin. The major difference between the KPSS test and the Dickey Fuller test is the opposite null hypothesis. In the case of the KPSS test, it is the stationarity of a random variable examined and the alternative hypothesis is its non-stationarity (Charemza and Deadman, 1997).

The statistical study of the non-stationarity of the variables was started with the application of the KPSS test to the logarithms of nominal exchange rates. If the non-stationarity of the rates is obtained, then the non-stationarity of successive differences will be examined and will be continued until the moment when an appropriate degree of the integration of variables is found.

Table 6. *The results of the KPSS tests conducted for exchange rates*

Logarithm	The value of the statistic	The critical value	Conclusion
JPY/USD	1.99	0.146	Non-stationarity
GBP/USD	2.98	0.146	Non-stationarity
GBP/JPY	1.89	0.146	Non-stationarity

Source: Own calculations.

The KPSS test values calculated for all of the three exchange rates indicate the rejection of the null hypothesis with the stationarity of exchange rates. Therefore, the alternative hypothesis of the non-stationarity in the exchange rates logarithms must be accepted. The rate variables will certainly be non-zero and non-negative degree integrated. In order to find the exact series the first increments of the exchange rates logarithms were examined, which will allow the exact integration series to be determined. First, the KPSS test was conducted for the first increments of the exchange rates logarithms.

Table 7. *The KPSS test results for the first differences of exchange rates*

Δ logarithm	The value of the statistic	The critical value	Conclusion
JPY/USD	.0525	0.146	Not rejected H_0
GBP/USD	.0446	0.146	Not rejected H_0
GBP/JPY	.0558	0.146	Not rejected H_0

Source: *Own calculations.*

Test results do not give the KPSS statistic reasons for rejecting the null hypothesis, that is stationary. There are, therefore, reasonable grounds for believing that the first differences examined will be of the order $I(1)$. If only the studied logarithms of price levels were of the same order, then it would be possible to carry out a correct analysis of cointegration. The KPSS test did not give grounds for the rejection of stationarity, but still there is a risk of the second kind error, *i.e.*, acceptance of a false null hypothesis. In order to eliminate this risk the Dickey Fuller test for the first exchange rates increments was carried out additionally. The null hypothesis is non-stationarity, so if for all three increments the rejection of this hypothesis is obtained, then the integration of the exchange rates of the order $I(1)$ will be proved.

Table 8. *The results of the ADF test for the first exchange rates differences*

Δ logarithm	The value of the statistic	The critical value	Conclusion
JPY/USD	-9.458	-1.950	Stationarity
GBP/USD	-9.669	-1.950	Stationarity
GBP/JPY	-8.958	-1.950	Stationarity

Source: *Own calculations.*

The test results confirm the previous assumptions regarding the stationarity of the first differences in exchange rates. The calculated values of the statistics should be viewed differently than in the case of the KPSS test. The DF absolute values are bigger than the critical absolute values which results in the rejection of the null hypotheses, and so all the increments are stationary.

The tests conducted proved the first degree integration of exchange rates. The next step consists in carrying out an analogous procedure for the time series of the logarithms of the CPI and PPI price levels for each country. For the purpose of analysis, the same tests, *i.e.*, KPSS and DF, will be applied. The study covers the following CPI and PPI logarithms: Japanese, British and American.

Table 9. *The results of the KPSS test for price levels*

Logarithm		The value of the statistic	The critical value	Conclusion
Japanese	CPI	3.29	0.146	Non-stationarity
	PPI	2.76	0.146	Non-stationarity
British	CPI	3.88	0.146	Non-stationarity
	PPI	3.59	0.146	Non-stationarity
American	CPI	3.73	0.146	Non-stationarity
	PPI	3.22	0.146	Non-stationarity

Source: Own calculations.

Table 10. *The results of the ADF test for the first price increments*

Δ logarithm		The value of the statistic	The critical value	Conclusion
Japanese	CPI	-4.785	-2.886	Stationarity
	PPI	-7.147	-2.886	Stationarity
British	CPI	-6.097	-2.886	Stationarity
	PPI	-3.661	-2.886	Stationarity
American	CPI	-4.666	-2.886	Stationarity
	PPI	-7.282	-2.886	Stationarity

Source: Own calculations.

The analysis of the stationary series of price logarithms was started with the KPSS test, which will allow possible stationarity to be examined. The KPSS test pointed to the rejection of the hypothesis of the stationarity of the studied logarithms of price indices. In order to determine the degree of integration, analogous tests for the first increments of price levels logarithms were conducted.

For the first increments of the logarithms of price indices surveyed the rejection of the null hypothesis of the ADF test was obtained. All the tested variables are stationary, which leads to the conclusion that the variables of the logarithms of price levels are order I(1) integrated.

To sum up the issue of the stationarity analysis - all variables are integrated into the order I(1). Both the logarithms of nominal exchange rates and the logarithms of price indices after the first differentiation give a stationary variable. As previously stated, all the variables examined are order I(1)

integrated which means that the results obtained in conducting the regressions cannot be interpreted in a literal way. All variables are non-stationary which necessitates further econometric analysis of data.

In connection with the series of variables integration, the first increments of variables are stationary. If an OLS analysis is to be carried out methodically, correct increment regressions of logarithms of exchange rates relative to the increments of price indices logarithms should be made. Attempts to model the increments of the logarithms of the rates under scrutiny against the logarithms of the corresponding increments of price levels ended in failure. The regression results do not provide information on the theory of parity, since the F statistic of these regressions does not provide a basis to reject the hypothesis of a total insignificance of variables, which makes that due to the procedure conducted it was impossible to obtain any information on the relationship between increments. This results from the long-term nature of the PPP relationship. Therefore, it is necessary to use the modelling tools of long-term dependence.

As was already shown, the data that follow the trend may cause difficulties in applying the empirical verification tools of the purchasing power parity theory. In addition, the calculation of further time series increments and subsequent regression fails, if the objective is to capture the long-term relation between stochastic processes. However, there is an econometric method that allows the long-term dependence to be investigated, even though in the short term a dependent variable is subject to deviations from the balance state.

Citing this method, however, requires defining the concept of co-integration, a key concept for further considerations. If two time series Y_t , X_t are integrated to the same order $I(d)$, and there is a linear combination of Y_t , X_t which is of the order $I(b)$ ($b < d$), then such series is called order d , b coordinated, which basically can be written down as:

$$\begin{bmatrix} X_t \\ Y_t \end{bmatrix} \sim CI(d, b).$$

A particularly interesting case of cointegration occurs when the variables Y_t , X_t are integrated to grade $I(1)$. You can then formulate the following equation (Charemza and Deadman, 1997):

$$Y_t = \beta * X_t + \varepsilon_t.$$

In this case, the variables are cointegrated, if $\varepsilon_t \sim I(0)$. ε_t is interpreted as a short-term deviation from the balance state which is determined by the cointegrating parameter β . Cointegration between variables ensures that they

follow the common balance state in the long run, showing some common features such as for example a common trend. The described issue of cointegration allows long-term relationships between time series to be modelled. This tool is useful in the study of the relationship of PPP, as it allows to analyse this relationship, despite the short-term deviations from the trend. The described issue of cointegration allows the method of researching PPP to be changed. Estimates relating to the equation:

$$e_t = \alpha + \beta^{HC} * p^{HC}_t + \beta^{FC} * p^{FC}_t + \varepsilon_t$$

can be treated as potential long-term relationships. If only the regression residuals are found to be stationary, then estimating the parameters of the impact of the price levels logarithms will determine the value of the long-term impact of price changes on the nominal exchange rate. However, the residuals will reflect the value of short-term deviations from the balance state designated by the parameters of the cointegration vector:

$$[1 - \hat{\beta}_{HC} - \hat{\beta}_{FC}]..$$

Table 11. *Residuals stationarity*

Source regression of residuals		ADF	KPSS	Conclusion
Logarithm of the JPY/USD rate	ln(CPI)	-2.847	0.893	Non-stationarity
	ln(PPI)	-3.020	1.22	Non-stationarity
Logarithm of the GBP/JPY rate	ln(CPI)	-3.480	0.752	Non-stationarity
	ln(PPI)	-3.248	1.13	Non-stationarity
Logarithm of the GBP/USD rate	ln(CPI)	-3.735	0.248	Non-stationarity
	ln(PPI)	-2.996	0.411	Non-stationarity

Source: Own calculations.

In order to verify whether the vectors obtained earlier actually describe a long-term relationship between the nominal exchange rates logarithms and price levels, tests on stationary residuals were carried out. If residuals turn out to be stationary, then we may say that the co-integrating dependence between the variables is proved. Table 12 below shows the results of the ADF test conducted for the residuals

For all of the examined regression residuals the tests showed non-stationarity. The critical value of the ADF test for 150 observations and three variables, among which we investigate cointegration, amounts to -3.77. All the ADF values obtained are greater than the critical value so there is no reason to reject the null hypothesis of non-stationarity. For certainty, also the KPSS statistic was calculated, whose values in all of the cases exceed the critical value of 0.146. This results in the rejection of the hypothesis of residuals stationarity.

The non-stationarity of residuals causes the rejection of the hypothesis of the existence of co-integrating relationship between the exchange rates logarithm and price indices logarithms for the studied pairs of countries formed from the following countries: Japan, the United Kingdom and the United States. Consequently, it is impossible to prove formally the existence of the balance relationship based on the purchasing power parity for the countries in the period 1970-2009.

It should be noted, however, that the values of the ADF tests are relatively close to the critical values for the 5% confidence level. This may indicate that despite the rejection when testing the hypothesis of the cointegration relationship, to some extent, however, it can be observed. In all cases, too large deviations from the balance relationship cause the rejection of the hypothesis of purchasing power parity as a co-integrating relationship. The residual series do not meet the definition of stationarity due to the excessive variance volatility over time, which made the cointegration of the data collected impossible to prove.

Rosenberg (2003) draws attention to the considerable length of the movement cycles of the nominal exchange rate of the dollar. These lengths often exceed the size of other common business cycles in the United States and around the world. The dollar cycles lasted an average of five years, in special cases, significantly exceeding even this value. At the end of each major cycle, the dollar tends to overshoot the long-term balance rate. Then we have significant deviations from the estimated value of the PPP which causes major inside and outside imbalances not only in the U.S. but also in Europe and Japan. When deviations reach a level commonly considered as critical, market forces lead to the end of the overshoot, which completes the cycle and leads to the return to the relationship arising from purchasing power parity. Although the study included observations from 40 years, the process of return to equilibrium is probably too long in relation to the analysed period, which may cause the non-stationarity of residuals and rejection of the hypothesis of cointegration.

The results of the cointegration tests are consistent with other studies described in the literature. Zhenhui Xu (2003) rejects the long-term effects of purchasing power parity, assuming a priori the exact PPP dependence on the issue of price levels. He studied selected rates of eight countries representing the most developed economies. Serletis and Gogas (2004) also found no evidence of the actual existence of a long-term PPP relationship for the dollar, the yen and the German mark. They note, however, that the use of PPI indexes rather than consumer prices indexes approximates the test values to those confirming the cointegration hypotheses.

However, there are studies confirming the existence of cointegration between prices and the actual exchange rate. McNown and Wallace (1989) analysed pairs of countries affected by high inflation in the past. Test results conducted with the use of PPI and obtained for Argentina, Chile, Brazil and Israel prove the existence of a long-term dependence resulting from purchasing power parity. At the same time they indicate that the rates overshooting as well as government policies significantly increase the return path of the exchange rate to the long-term balance consistent with PPP.

Kim (1990) attempted to resolve the inconsistency of the obtained parameters of the long-term price impact on the rate by extending the examined sample, which largely removed the burden of estimators. Analysing the sample describing the years 1900-1987, not only did he reject the hypothesis of the absence of cointegration for the studied pairs of currencies, but also he obtained parameters that were surprisingly close to the theory of purchasing power parity.

Table 12. *The results of the KPSS test conducted for real exchange rates*

Exchange rate	The statistics value KPP	The critical value of the KPSS test	Decision
the yen against the dollar	1.65	0.146	Non-stationarity
the pound against the yen	2	0.146	Non-stationarity
the pound against the dollar	0.565	0.146	Non-stationarity

Source: Own calculations.

Another way to test the purchasing power parity in the long run is to test the stationarity of real exchange rates. In the theoretical part it was proved that the fulfilment of the parity implies $Q^{HC/FC} = \lambda$, where a special case is $\lambda = 1 = Q^{FC/HC}$, when the absolute PPP occurs. If, on the basis of the data, it turned out that the real exchange rate is stationary, it would be proof of the fulfilment of the purchasing power parity relationship in the relative version in the long-term². In order to check the stationarity of real exchange rates, the KPSS test was applied, the null hypothesis of which is stationarity of the variable.

In all three cases, non-stationarity of the real exchange rates was obtained. Attempts made to confirm the exact performance of a long-term relationship imposed by the purchasing power parity failed. Similar results of the stationarity of real exchange rates were obtained in the literature examining the issue of PPP by testing real exchange rates stationarity. Meese and Rogoff (1988) while studying the exchange rates of the dollar against the German mark, the dollar against the pound and of the dollar against the yen in the period 1974-1986 were not able to reject the null hypothesis which

² To calculate real exchange rates the PPI index was used.

said that the real exchange rate is a random walk process. An alternative hypothesis was the exact fulfilment of the long-term relationship of PPP.

Recent research in real exchange rates uses very long time series, even ones exceeding seventy-year periods. Also, more sophisticated econometric methods are used, especially panel models. They allow to prove the stationarity of real exchange rates, however, they point to a very long, lasting up to 5 years period of return of the rate to the balance state (Engel., Rogers, 2001). As in the case of analysing co-integration, lengthy deviation duration causes rejection of stationarity in a relatively short time.

4. Final Remarks

Purchasing power parity is a phenomenon extremely difficult to verify empirically. Although it requires the fulfilment of a number of unrealistic assumptions, the nominal exchange rate exhibits a strong tendency to converge towards the long-term balance state determined by the relationship of prices in different countries. The observed phenomenon has its justification in the theory of open economy macroeconomics. The analyses carried out in the present work aimed to prove the existence of a long-term relationship between prices and exchange rates similar to that derived from the theory, but formal verification did not allow the confirmation of the exact relationship of long-term forecast by purchasing power parity. For all the examined exchange rates, the existence of co-integrating relationship was not shown, the relationship which would be fulfilled in spite of short-term deviations from parity. This is consistent with other studies conducted in the literature for a similar period of study (Niu , Chu, and Ma, 2016).

Despite the fact that the study employed data concerning a forty-year time period, the cycles of deviations from the balance path take too long to be able to demonstrate the stationarity of residuals in the cointegration analysis. Results of other studies indicate that analysis of very long time series can provide evidence of the dependence of the nominal exchange rate on price levels. This makes PPP a very long-term relationship, which in the short-term and medium-term is subject to relatively strong disturbance.

Regardless of the use of the indices that theoretically are conducive to PPP, formal statistical tests rejected the opportunity to influence price levels with coefficients equal in their value to the absolute unity. This meant that we had to look for dependence applying less restrictive assumptions about the form of the cointegration vector. In the subject literature, however, the prevailing opinion is that in countries affected by high inflation PPP is closer to the fulfilment in its most theoretical form. This is due to changes in the behaviour of prices in the economy. As mentioned earlier, short-term price rigidity causes deviations of the nominal exchange rate from the state

balance. Under conditions of high inflation pressure prices change faster, and, therefore, they adjust quicker to the long-term balance after the occurrence of disturbances in the economy. Tests on the stationarity of real exchange rates did not provide any evidence that they behave as a series with weak stationarity. This is a consequence of the previous inference with too long cycles of deviations from the purchasing power parity relationship, which results in the lack of a formal basis for positive verification of the theory of PPP in the analysed countries.

It should be noted, however, that in recent studies of real exchange rates very long time series are applied. These analyses also employ more sophisticated econometric methods, in particular panel models. They allow the stationarity of real exchange rates to be proved and indicate a very long, lasting up to five years, period of the return to the long-term PPP relationships after the occurrence of fluctuations.

References:

- Balassa, B. 1964). The Purchasing Power-Parity Doctrine: A Reappraisal. *Journal of Political Economy* 72, 584-596.
- Betty, D. 1986. Empirical determinants of PPP deviation. *Journal of International Economics* 21, 313-326.
- Cassel, G. 1918. Abnormal deviations in international exchanges. *The Economic Journal*, 112, 413-415.
- Charemza, W., Deadman, D. 1997. *New Directions in Econometric Practice, General to Specific Modelling, Cointegration and Vector Auto Regression*. Edward Elgar Publishing Limited, Cheltenham.
- Cheung Ling, W., Lai, K. 1993. Long - run PPP during the recent float. *Journal of International Economics*, 34, 181-192.
- Coakley, J., Fuertes, A. 1997. New panel root tests of PPP. *Economic Letters* 57, 17-22.
- Coe, P., Serletis, A. 2002. Bound tests of theory of PPP. *Journal of Banking & Finance*, 26, 179-199.
- Dornbusch, R. 1976. Expectations and Exchange Rate Dynamics. *Journal of Political Economy*, 84, 1161-1176.
- Dutton, M., Strauss, J. 1997. Cointegration tests of PPP: the impact of non - traded goods. *Journal of International Money and Finance*, 16, 433-444.
- Eiteman, D., Stonehill, A., Moffett, M. 2007. *Multinational Business Finance*. Pearson/Addison, Wesley.
- Engel, Ch., Rogers, J.H. 2001. Deviations from Purchasing Power Parity: Causes and Welfare Costs. *Journal of International Economics*, 55, 29-57.
- European Union/OECD. 2012. *Methodological Manual on Purchasing Power Parities. Methodologies and Working papers*, 2nd Edition. Luxembourg.
- Faria, R., Leson-Ledesma, M. 2003. Testing the Balassa-Samuelson effect: Implications for growth and the PPP. *Journal of Macroeconomics*, 25, 241-253.
- Frenkel, J. 1978. Purchasing power parity: Doctrinal perspective and evidence from the 1920s. *Journal of International Economics*, 8, 169-191.
- Hallwood, C.P., MacDonald, R. 2000. *International Money and Finance*. Blackwell Publishers, Oxford.

- Ito, T. 1997. The long run PPP for the yen: historical overview. *Journal of the Japanese and International Economies*, 12, 502-521.
- Johnson, P. 1991. Aggregate price indexes, cointegration and test of the PPP hypothesis. *Economic Letters*, 36, 305-309.
- Karoglou, M., Morley, B. 2012. Purchasing power parity and structural instability in the US/UK exchange rate. *Journal of International Financial Markets, Institutions and Money* 22, 958-972.
- Kim, Y. 1990. Purchasing Power Parity in the Long Run: A Cointegration Approach. *Journal of Money, Credit and Banking*, 22, 491-503.
- Krugman, P.R., Obstfeld, M., Melitz, M.J. 2018. *International Economics: Theory and Policy*. Pearson, Wesley.
- Lo, M., Wong, S. 2006. What explains the deviations of PPP across countries? International evidence from macro data. *Economic Letters*, 91, 229-235.
- Rogoff, K. 1988. Was it real? The exchange rate interest differential relation over the modern floating exchange rate period. *Journal of International Economics*, 43, 933-948.
- McNown, R., Wallace, M. 1989. National price levels, PPP and cointegration: a test of four high inflation economies. *Journal of International Money and Finance* 8, 533-545.
- Moosa, I., Bhati, R. 1997. *International Parity Conditions. Theory, Econometric Testing and Empirical Evidence*. Palgrave Macmillan, London.
- Niu, H., Chu, X., Ma, Y. 2016. Study on the Fluctuation of Purchasing Power Parity. *Open Journal of Business and Management*, 4, 67-68.
- Papell, D. 1997. Searching for stationarity: PPP under the current float. *Journal of International Economics*, 43, 313-332.
- Rosenberg, R.M. 2003. *Exchange – Rate Determination*. The McGraw – Hill, NY.
- Samuelson, P. 1964. Theoretical Notes on Trade Problems. *Revue Economic Statistics*, 46, 145-154.
- Serletis, A., Gogas, P. 2004. Long horizon regression tests of the theory of PPP. *Journal of Banking & Finance*, 28, 1961-1985.
- Shapiro, A. 2002. *Foundations of Multinational Financial Management*. John Wiley & Sons, New Jersey.
- Shapiro, A. 1983. What does PPP mean? *Journal of International Money and Finance*, 2, 295-318.
- Shively, P. 2001. A test of long – run PPP. *Economic Letters*, 73, 201-205.
- Sideris, D. 2006. Testing for long – run PPP in a system context: Evidence for the US, Germany, and Japan. *Journal of International Financial Markets, Institutions and Money*, 16, 143-154.
- Soyoung, K., Renato, L. 2009. Local persistence and the PPP hypothesis. *Journal of International Money and Finance*, 28, 570-584.
- Taylor, M. ed. 2012. *Purchasing Power Parity and Real Exchange Rates*. Routledge, NY.
- Thalassinos, I.E. 2007. Trade Regionalization, Exchange Rate Policies and EU-US Economic Cooperation. *European Research Studies Journal*, 10(1-2), 111-118. DOI: 10.35808/ersj/167.
- Wang, L., Fan, C. 2013. The Research on Purchasing Power Parity Deviates from Exchange Rate. *The Journal of Quantitative and Technical Economics*, 11, 125-142.
- Zhenhui, X. 2003. PPP, price indices and exchange rate forecasts. *Journal of International Money and Finance*, 22, 105-130.