
Is It Possible to Overfit the Algorithm? Case Study of Mass Valuation of Land Properties in Szczecin*

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Abstract:

Purpose: The article presents the mass estimation of real estate value using the Szczecin Mass Real Estate Valuation Algorithm, with various sizes of the initial sample. The aim of the article is to investigate how the change of the sampling pattern affects the results of the valuation and to capture how acceptably small the sample may be, so that the algorithm overfitting does not occur.

Design/Methodology/Approach: In the mass valuation algorithm used, correlation coefficients were used to estimate the influence of particular property attributes on the property value. Empirical research was conducted on the basis of the database of land properties in Szczecin. This database was divided into two groups: a training set and a test set. For the trainee set, appropriate correlation coefficients were calculated, and then, using the algorithm of mass valuation, the value of real estate in the test set was estimated.

Findings: In all analysed cases, the MAPE error for the testing sample was greater than for the training sample. However, the smallest difference between the errors for the training and testing sample occurred in the case of using proportional stratified sampling. When using simple randomization, increasing the sample size by 40% resulted in a decrease in the MAPE error value. On the other hand, reducing the sample size in order to reduce the costs of mass real estate valuation will result in an increase in the error value and model overfitting.

Practical Implications: Appropriate selection of the real estate sample on the basis of which the coefficients of the influence of individual attributes on the value of the real estate are calculated directly affects the cost-effectiveness of the entire mass valuation process.

Originality/Value: The smaller the sample size, the less real estate has to be individually appraised by property appraisers. In addition, the precise selection of real estate for the training sample in the algorithm affects the convergence of the obtained real estate valuations with their market value.

Keywords: Mass real estate valuation, sampling, statistical methods.

JEL classification: C40, C83, R39.

Paper Type: Research study.

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1. Introduction

Searching for the regularities of economic phenomena is a process in which, on the basis of a large number of cases, we analyse the occurring dependencies (or their lack). Conducting an audit on the basis of all the units is often pointless, impossible, or too costly. The procedure to be followed in such cases is as follows: based on the potential group of units to be analysed, with the selected method of selection, elements are selected (or otherwise selected), which are called a sample. On the basis of the sample, an exploratory study is conducted, and regularities are defined. The methods of the probability calculus allow to generalize the obtained results for the entire population, or not in the case of unfulfilled assumptions.

The problem of selecting units for preliminary (sample) testing is of particular importance in the case of property value estimation based on the Szczecin Mass Real Estate Valuation Algorithm (SAMWN). The procedure of mass real estate appraisal assumes the selection of real estate for the sample, which real estate will be individually appraised by property appraisers. Such a procedure is associated with a large financial outlay and time necessary for the valuation, and “contaminated” is an individual assessment of the property value by appraisers (Kokot and Gnat, 2019). It is therefore understandable that it is desirable that the group of properties that qualify for individual valuation is as small as possible. On the other hand, the sample should ensure the representativeness of the population and at the same time the discrepancy between the valuation error on the preliminary sample and the testing sample should not be too large.

The article presents the mass estimation of real estate value using SAMWN, with various sizes of the initial sample. The aim of the article is to investigate how the change of the sampling pattern affects the results of the valuation and to capture how acceptably small the sample may be, so that the algorithm overfitting does not occur.

2. Mass Valuation of Real Estate and the Usefulness of the Algorithm

2.1 Szczecin Mass Real Estate Valuation Algorithm

Mass real estate appraisal is defined when, for a large number of real estate, their value is determined at the same time, using a uniform approach (algorithm). It is important that the valuation process takes place using the same calculation procedure for all properties, i.e. it has no subjective influence on the result of the valuer. SAMWN is a proposition for such a procedure. The property valuation is based on the algorithm of the form:

$$w_{ji} = wwr_j \cdot pow_i \cdot w_{baz} \cdot \prod_{k=1}^K \prod_{p=1}^{k_p} (1 + A_{kpi}) \quad (1)$$

where:

w_{ji} – market (or cadastral) value of the i -th real estate in the j -th zone of location attractiveness,

wwr_j – market value coefficient in the j -th zone of location attractiveness ($j = 1, 2, \dots, J$),

J – number of location attractiveness zones,

pow_i – area of i -th real estate,

w_{baz} – estimated value of 1 m² of real estate with the worst attributes in the worst location attractiveness zone,

A_{kpi} – influence of the p -th category of k -th attribute for the i -th real estate ($k = 1, 2, \dots, K$; $p = 1, 2, \dots, k_p$),

K – number of attributes,

k_p – number of categories of k -th attribute,

N – number of real estate appraised ($i = 1, 2, \dots, N$),

R – number of representatives ($r = 1, 2, \dots, R$).

The algorithm determines the market or cadastral value of the property, not the price. The reference point for determining the value is the base value, i.e. the value of 1 m² of the property with the worst attributes, in the worst location attractiveness zone. The base value is multiplied in the algorithm by the influence of the attribute states of the valued real estate.

The influence of attribute states (A_{kpi}) can be determined on the basis of quantitative methods, that is, for example, using statistical methods. The value of a property depends not only on its attributes. The value is also influenced by factors on the demand side. Two properties, similar in terms of their attributes, may have different values if they are located in different location attractiveness zones (SAL). In the algorithm, the influence of these types of factors is considered by the market value coefficients (wwr_j). These coefficients are determined for each zone of location attractiveness and they show the influence of the location. The market value coefficient for the j -th zone of location attractiveness can be determined as:

$$wwr_j = \sqrt[n_j]{\prod_{i=1}^{n_j} \frac{w_{ji}^{rz}}{w_{ji}^h}} \quad (2)$$

where:

w_{ji}^{rz} – value of the i -th real estate in the j -th zone of location attractiveness determined by a property appraiser,

w_{ji}^h – hypothetical value of i -th real estate in j -th zone of location attractiveness,

n_j – number of representative properties in the j -th zone of location attractiveness.

Market value ratios are calculated on the basis of representative property valuations performed by property appraisers. Appraisers evaluate representative real estate on an individual basis, as a result of which real real estate values are obtained (w_{ji}^{rz}). In turn, the hypothetical values (w_{ji}^h) are calculated on the basis of (1), but without the market value coefficients:

$$w_{ji}^h = pow_i \cdot w_{baz} \cdot \prod_{k=1}^K \prod_{p=1}^{k_p} (1 + A_{kpi}) \quad (3)$$

If the values of the randomly selected or appropriately selected representative properties (w_{ji}^{rz}), the attribute states, the influence of the attribute states, the base value (w_{baz}) and areas are known, the market value coefficients can be estimated for each location attractiveness zone by calculating the geometric mean of the quotients of real values and hypothetical real estate. Knowing the market value coefficients for individual location attractiveness zones, it is possible to estimate the market (cadastral) value of each property located in it, considering the status of property attributes.

The influence of attributes on the value of 1 m² of real estate can be estimated using correlation coefficients. When examining the correlation between the attributes on the real estate market and the value of 1 m², the following problems are encountered (Dmytrów *et al.*, 2019):

- the attributes describing the properties being the subject of the valuation are very often qualitative in nature – they are measured on an ordinal scale (Walesiak, 1996).
- the differences between successive values of the attributes measured on an ordinal scale are not necessarily constant (changes are not linear).
- when examining the interdependencies between the value of 1 m² and individual attributes, the influence of other attributes, which may significantly disturb the studied correlation, should be eliminated.

Considering the above conditions, the partial Kendall coefficients were used in further calculations (Parker *et al.*, 2011; Han and Zhu, 2008). On their basis, weights were determined, i.e. the impact of individual attributes on the value of 1 m² of the property. Weights were defined as a percentage share of individual coefficients in the sum of all coefficients. Additionally, it was assumed that due to the scaling of the levels of attributes (a higher attribute level means a higher value of 1 m² of the property), all correlation coefficients should be positive. If the value of the coefficient was lower than 0, then an insignificant influence of this attribute on the property value is assumed and zero is assumed in further calculations. To determine the impact of each state of the attribute in SAMWN, the formula was used (Doszyń and Dmytrów, 2019):

$$1 + \alpha_{kp} = e^{\ln\left(\frac{w_{max}}{w_{baz}}\right) \cdot u_{kp}} \quad (4)$$

where:

w_{max} – the theoretical maximum value of the property for the best attributes in the most expensive zone of location attractiveness,

w_{baz} – base value,

u_{kp} – the state weight of the attribute k is calculated as follows:

$$u_{kp} = \frac{w_k}{k_p - 1} (p - 1) \quad (5)$$

where:

w_k – weight of k -th attribute, $p = 1, \dots, k_p$.

2.2 Assessment of the Functionality of the Algorithm

SAMWN testing consists in determining the error between the values of 1 m² of real estate estimated by property appraisers and the values estimated by the algorithm. For this purpose, the error MAPE (Mean Absolute Percentage Error) percentage was used:

$$MAPE = \frac{100\%}{n} \sum_{i=1}^n \frac{|w_i - w_{t,i}|}{w_i} \quad (6)$$

where:

w_i – value of 1 m² of the property estimated by a property appraiser,

$w_{t,i}$ – value of 1 m² of the property estimated by SAMWN.

Error (6) at a low level will indicate a good adjustment of the algorithm to real data - the valuations obtained on the basis of the algorithm will be at a level similar to individual valuations. A high error rate (6) will indicate a divergence of the algorithm from the actual valuations.

The concepts of sample overfitting, algorithm or overfitting are related to issues where certain solutions are tested on a training (initial) sample and then applied to a validation (testing) sample. Obtaining satisfactory results on the training sample does not guarantee obtaining the same results on the testing sample. It happens when the algorithm training process is too long or the training sample is not too numerous, i.e. when the algorithm used has too many parameters in relation to the sample size on the basis of which it was calibrated (Bilger and Manning, 2011).

The MAPE error was counted twice: first by comparing the algorithm's valuation results with the individual appraisal of the training sample real estate, and then for

the testing sample real estate. Significantly higher results obtained in the second case will prove the algorithm overfitting.

The obtained results of MAPE errors were compared with each other using the Mann Whitney's U test (Siegel and Castellan, 1988; Stanis, 2006). The authors used this test, because not all errors distributions were characterized by a normal distribution.

$$U = n_1 * n_2 + \frac{n_1(n_1+1)}{2} - R_1 \quad (7)$$

where:

n_1 – the size of the first sample,

n_2 – the size of the second sample,

R_1 – sum of the ranks assigned to the values of the first sample.

3. Selection of the Real Estate for the Sample

Among the partial studies, representative studies have the greatest cognitive value, because only these results can be generalized to the entire population with high probability. The credibility of the results of sample surveys depends primarily on the size and structure of the sample population (Mazurek-Łopacińska, 2005). The randomness of the sample means that the results obtained on the basis of this sample can be treated as implementations of random variables with a distribution identical to the population distribution. The sample size depends, among others on the type of sample, the analyzed statistics, the homogeneity of the population (Chuechill, 2002), the assumed confidence level, and many other factors (Ritchie *et al.*, 2003; Szreder, 2010). The basis for deriving the necessary rose size is focusing on a trait (or features) of a population, and not on its units (Pasikowski, 2015).

A random pattern is understood as a random mechanism that implements a specific sampling pattern, i.e. it is the method of sampling. The selected units (teams) are called sampling units. The drawing of a specific sample from the studied population, in accordance with the established drawing scheme, is carried out on certain materials (documents) constituting the sampling frame.

The random sample should reflect as well as possible the structure of the general community from which it was taken. The representativeness of the sample depends on two factors: the way the sample was selected and the size of the sample. The sampling schemes are divided into (Jabkowski, 2015) independent and dependent sampling, individual and team sampling, single-stage and multi-stage sampling, limited and unlimited sampling, tiered sampling.

The sampling is individual when individual, individual elements of the studied population are drawn. Unlimited individual draw with equal probability of selection

is a simple sampling. It consists in the fact that: the sampling unit is the test unit, the selection is made for each element of the population, the probability of selecting it is the same, the sample is drawn from the entire population. Unlimited individual drawing consists in drawing units to be sampled directly from the entire population. Assuming that the maximum error of estimating the mean should not exceed the value d , the necessary sample size is determined according to formula (8) (Bracha, 1996):

$$n_p = \frac{N}{1 + \frac{4 \cdot d^2 \cdot N}{u_\alpha^2}} + 1 \quad (8)$$

where:

N - size of the general population,

d - maximum allowable measurement error,

u_α - significance level.

The division into layers is made according to such a classification criterion that significantly differentiates the distribution of the variable. The principle of separation should be observed when stratifying a community. This sampling ensures that all layers are represented in the sample.

4. Empirical Research

The study used data on 318 land properties in Szczecin, intended for residential purposes, located in the northern part of the city. For the purposes of the conducted analyzes, all properties have been individually valued by property appraisers, although in the actual conditions of using SAMWN, only the properties of the training sample are subject to such valuation. All properties are described using a set of attributes: x_1 – area: large, average, small; x_2 – technical infrastructure: none, partial, complete; x_3 – neighbourhood: troublesome, unfavourable, average, favourable; x_4 – access: unfavourable, average, favourable; x_5 – physical properties: unfavourable, average, favourable. Attributes are presented on ordinal scales giving 1 to the weakest level of the attribute and subsequent numbers for the more favourable levels.

In the previous applications of SAMWN, the training sample was selected on purpose so as to ensure adequate representativeness of selected indicators (attribute levels and zones of location attractiveness) and at the same time the sample was as small as possible. Since real estate selected for the training sample must be individually valued by property appraisers, it is understandable that the sample should be as small as possible.

In the study, a 30-element sample (approx. 10%) was drawn from a database of 318 properties, which was defined as the testing sample. The remaining part of the property (288 plots) was a sampling frame – the basis for sampling, according to various criteria, learning samples. 12 sampling patterns were used. Seven of them were simple samplings with a varying number of properties drawn five patterns concerned stratified sampling.

Based on the formula (8), the necessary number of real estates was determined, which should constitute a learning sample: $np = 73$ (assuming a maximum measurement error of 10%). In order to check whether the increase in the number of properties drawn had an impact on the final results, the sample size was increased by 20%, obtaining another three sample cases with the size of 88, 102 and 117 properties. In the next step, the training sample was reduced by 20%, obtaining 58, 44 and 29 observations (cases 1-7 in table 2). The subsequent drawing patterns concerned stratified sampling. The strata were a combination of attribute states because in the SAMWN statistical approach it is important that each attribute level is represented. In the preliminary sample, no variability of the technical infrastructure attribute was noted (all properties were complete), hence this attribute was not a variant of the sampling strata. 39 strata were distinguished (table 1). The "2222" stratum means that in the preliminary sample there were 53 properties with average area, unfavourable neighbourhood, average access and average physical properties.

Table 1. Examples of sampling strata and their numbers

Strata	Size of the stratum	The stratum share (%)
2222	53	18.40
2122	46	15.97
2221	37	12.85
2232	28	9.72
0222	13	4.51
...

Source: Own study.

Based on the prepared strata, 5 learning trials were randomly selected. In the first stratified sampling it was assumed that the probability of drawing a property from a given strata corresponds to the share of the strata. In the following schemes, the sample size was reduced by 20%. In order to ensure the representation of all strata, it was assumed that if the share of a given layer is lower than 1, at least one property must be drawn (cases 8-11 in table 2). The last 12 drawing scheme concerned a non-proportional drawing – each stratum was assigned the same probability of drawing, regardless of its size, and 39 properties were randomly selected - one from each stratum. In each scheme, the drawing was repeated 10 times.

Table 2. *Used sampling schemes*

No.	Sampling scheme	The size of the learning sample
1	simple random sampling – 1.6	117
2	simple random sampling – 1.4	102
3	simple random sampling – 1.2	88
4	simple random sampling – 1	73
5	simple random sampling – 0.8	58
6	simple random sampling – 0.6	44
7	simple random sampling – 0.4	29
8	proportional stratified sampling – 1	97
9	proportional stratified sampling – 0.8	82
10	proportional stratified sampling – 0.6	66
11	proportional stratified sampling – 0.4	57
12	disproportionate stratified sampling	39

Source: *Own study.*

Kendall's partial correlation coefficients were calculated for each sample. In the case when the relationship between the property value and the attribute was negative (although the scaling of the status of the attributes was performed in such a way as to avoid such a situation), it was considered that the attribute did not affect the value and in further calculations the coefficient level was assumed = 0. Based on the partial coefficients Kendall, the influence of individual attribute states on the property value (formula 4) was determined using weights (formula 5). Then, the hypothetical values of the real estate were estimated in the training trials (formula 3). The MAPE_u errors were calculated for the training trials (formula 6). Based on the hypothetical values of real estate, coefficients of market values for individual zones of location attractiveness were calculated (formula 2). Finally, the property value of the SAMWN test sample (formula 1) was determined. MAPE_t errors recalculated, but this time for test trials.

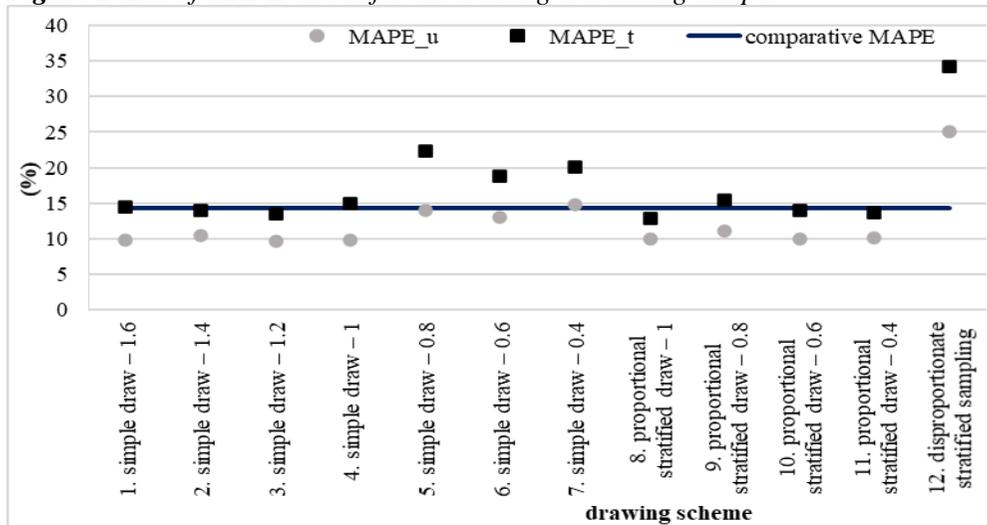
5. The Research Results

The above-described procedure was performed with 120 times for each sampling scheme and the average MAPE_u and MAPE_t error was calculated (Figure 1). The smallest MAPE_u error in the training sample was obtained when a simple sampling scheme was used for the sample size increased by 20 in relation to the necessary sample size determined on the basis of formula (formula 8) – Sampling Scheme 3. The size of the training group was 88 properties. The MAPE_u was at the level of 9.679%. However, the smallest difference between the valuations and the values of the properties in the testing set was obtained by using a layered proportional sampling scheme – Sampling Scheme 8, in which the sample number of the real estate property had 97 observations. The worst results were obtained when using a

disproportionate sampling scheme (Scheme 12). MAPE errors were 25.034% and 34.270% respectively.

Comparing the MAPE_t results obtained with the use of 12 sampling schemes with the error rate in the deliberate selection of the sample (black line in the figure 1), which was MAPE = 14.24% (Gdakowicz and Putek-Szeląg, in print), better results, i.e. smaller MAPE_t errors were obtained for 2, 3, 8, 10 and 11 of the sampling scheme.

Figure 1. List of MAPE errors for the training and testing sample



Source: Own study.

Additionally, when comparing the size of the errors, corresponding to the size of the samples, for simple and proportional drawing, it can be seen that they are smaller when using a stratified sampling. Such a relationship is noticeable for both the training and testing sets.

When the error on the test set increases and the error on the training set decreases, it is usually related to the overfitting phenomenon. In our case we do not deal with such a situation, but the MAPE_t error for the testing group is in each case higher than the MAPE_u error for the training group.

Table 3 presents the average MAPE errors for the training and testing groups, considering the sampling patterns and the Mann Whitney’s *U* test statistics (formula 7) and the test statistics probabilities.

Table 3. MAPE error sizes for the training and testing sample in various sampling schemes and the statistics of the Mann Whitney's U test

No.	Sampling scheme	MAPE _t	MAPE _i	U	p
1	Simple random sampling – 1.6	9.851	14.544	-3.137	0.002
2	Simple random sampling – 1.4	10.484	14.051	-2.457	0.014
3	Simple random sampling – 1.2	9.679	13.483	-3.213	0.001
4	Simple random sampling – 1	9.855	14.958	-3.062	0.002
5	Simple random sampling – 0.8	14.061	22.324	-3.137	0.002
6	Simple random sampling – 0.6	12.954	18.771	-2.759	0.006
7	Simple random sampling – 0.4	14.743	20.072	-3.364	0.001
8	Simple random sampling – 1	9.986	12.820	-3.137	0.002
9	proportional stratified sampling – 0.8	11.158	15.510	-2.986	0.003
10	proportional stratified sampling – 0.8	9.919	13.979	-1.398	0.162
11	proportional stratified sampling – 0.8	10.102	13.608	-2.381	0.017
12	disproportional stratified sampling – 0.8	25.034	34.270	-2.119	0.034

Source: own calculations.

Despite the fact that the smallest MAPE_t error was achieved for the proportional stratified sampling (Scheme 8), an insignificant difference between the MAPE errors for the training and testing sample occurred in the proportional sampling, for which the sample was 66 (less by 31 units).

6. Conclusions

Mass appraisal of real estate in Poland is a topic that still requires in-depth analysis. There is no fixed methodology for the valuation of hundreds of thousands of properties at any one time. One of the proposals to estimate many properties is the Szczecin Mass Real Estate Valuation Algorithm. It allows you to quickly estimate the value of many properties, and additionally considers the specificity of the local real estate market.

An important stage of mass valuation using SAMWN is the selection (drawing) of a sample of real estate, on the basis of which the coefficients of the influence of attributes on the value of the real estate are calibrated. The article presents simulations of the selection and drawing of a sample of real estates, with different sizes and according to different sampling schemes, and their impact on the estimated real estate values. The smallest MAPE_t error for the training sample was obtained for the 8-sampling scheme, which assumed the number = 97 units. However, using Scheme 10, which used stratified sampling with a sample size of 66 observations, the difference between the MAPE errors of the training and testing groups was not significant – there was no overfitting of the model.

The use of strata for sample selection for mass valuation reduces MAPE errors for both the training and testing groups. In all analysed cases, the MAPE error for the testing sample was greater than for the training sample.

When using simple randomization, increasing the sample size to 40% resulted in a decrease in the MAPE error value. On the other hand, reducing the sample size in order to reduce the costs of mass real estate valuation will result in an increase in the error value and model overfitting.

Therefore, it seems that the optimal solution in further procedures of mass valuation of land real estate in the statistical approach, using SAMWN, is to draw the real estate of representatives by proportional stratified sampling (sampling scheme 10). This approach will reduce the costs associated with the initial individual property valuation, and at the same time it will not affect the overfitting of the valuation algorithm.

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