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Assessment of Trajectories of Non-bankrupt and Bankrupt Enterprises

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Abstract:

Purpose: *The aim of this study is to show how long-term trajectories of enterprises can be used to increase the forecasting horizon of bankruptcy prediction models.*
Design/Methodology/Approach: *The author used seven popular forecasting models (two from Europe, two from Asia, two from North America and one from Latin America). These models (five multivariate discriminant analysis models and two logit models) were used to develop 17-year trajectories separately for non-bankrupt enterprises and those at risk of financial failure.*
Findings: *Based on a sample of 200 enterprises, the author evaluated the differences between non-bankrupt and bankrupt firms in development during 17 years of activity. The long-term usability of the models was demonstrated. To date, these models have been used only to forecast bankruptcy risk in the short term (1–3 years' prediction horizon). This paper demonstrates that these models can also serve to evaluate long-term growth and to identify the first symptoms of future bankruptcy risk many years before it actually occurs.*
Practical Implications: *It was proven and specified that long-term developmental differences exist between non-threatened and future insolvent companies. These studies proved that the process of going bankrupt is very long, perhaps even longer than the literature has previously demonstrated.*
Originality/value: *This study is one of the first attempts in the literature globally to assess such long-term enterprise trajectories. Additionally by implementing a dynamic approach to the financial ratios in the risk-forecasting model let visualize the changes occurring in the company.*

Keywords: *Forecasting, trajectory, bankruptcy models, financial crisis.*

JEL classification: *G33, F37.*

Paper Type: *Research study.*

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1. Introduction

In an increasingly competitive global economy, all business threats and opportunities are changing fast. In connection with the permanent (structural) increase in the number of bankruptcies throughout the world, a precise analysis of business failure risk has become even more important today than it was in the past. The key issue in today's business environment is to distinguish areas of risk, current control of the financial situation and effective prediction of the risk of bankruptcy to respond in advance. In addition, a company's financial crisis does not appear suddenly; rather, it is the accumulation of many factors ignored by managers and the symptoms of deteriorating economic situation over a longer period in the firm and its environment. The literature distinguishes between three and five crisis stages, ending in the enterprise's bankruptcy (for example, Fitzpatrick, 1934 is the first author to identify the crisis stages in firms; see also Ooghe and Prijcker, 2006; Richardson *et al.*, 1994).

Bankruptcy risk cannot be eliminated completely. Generally, risk is inherent in making economic decisions. The etymology of the word "risk" has not yet been clearly elucidated. In Persian, "rozi(k)" means a lot, the daily payment, and bread. In Arabic, "risq" means fate, divine retribution. The Spanish "ar-rico" is bravery and danger. The English "risk," a situation that causes danger or the possibility that something bad will happen. The Greek "riza," like the Italian "ris(i)co," means the reef that the ship should avoid; therefore, it is a danger to be avoided. However, most often the word "risk" is derived from the Latin "risicum", meaning a chance or likelihood of occurrence of a positive or negative event, success or failure. Risk is a very broad and interdisciplinary term. The author's intention is to focus on risks from the perspective of assessing risk of corporate bankruptcy.

The most popular model for forecasting bankruptcy risk was developed in 1968 by E. Altman. A pioneer in the use of multivariate discriminant analysis to predict a company's bankruptcy, he estimated a single-function model consisting of five financial ratios (Altman, 1968). Over the past forty years, studies on models predicting a company's collapse have developed intensively, with many articles globally published on this subject (Curtis *et al.*, 2020; Kourtis *et al.*, 2017; 2019). Although these models differ greatly depending on the modeling method, the variables or the sample size used, they share two common characteristics:

1. Most authors of early warning models consider the goal as the advanced recognition of a company's bankruptcy threat, ranging from one to three years. At a horizon of more than one year, their accuracy decreases substantially (Jardin and Severin, 2011). For example, Altman's model accuracy rate decreases from 95% one year before failure to 48% three years before failure (Altman, 1968), and Sharma and Mahajan's model decreases from 91.7% to 73.9% in the same period (Sharma and Mahajan, 1980). According to Jardin and Severin (2011), regardless of the modeling technique

(linear or non-linear, regression or classification), models always have the same drawback of a short forecasting horizon.

2. The financial ratio values used in forecasting models are static by nature. Most financial ratios are calculated based on static values at a given moment (usually at year's end) from the balance sheet and the income statement. Such an analysis lacks a dynamic view of the indicators.

In this study, the author addresses both problems by establishing the following objectives:

1. To determine the 17-year trajectory separately for non-bankrupt and bankrupt enterprises. By using seven popular forecasting models (two from Europe, two from Asia, two from North America and one from Latin America), the author investigates the long-term differences in developing "good" and future "bad" firms. Although as previously mentioned, forecasting models cannot predict horizons longer than two–three years before the failure, an important and still unsolved question in the literature is whether the models can be used to identify significant differences in the "life" trajectories between these two groups of enterprises. Such trajectories could prolong the forecast period by up to 15–20 years before the failure.
2. To implement a dynamic approach integrating financial ratios into forecasting models. The question arises whether changes in indicators relevant predictors of a company's are coming financial crisis because declines or increases in values do not immediately indicate that the company's economic situation is deteriorating. Nevertheless, by observing changes, we can distinguish between a company that has low financial ratios that improve each year and a company that has similarly low ratios that worsen each year. Static models will not detect the difference between such companies. Dynamic models add an element that differentiates companies with a poor financial situation from companies that have a weak financial situation but are improving. To answer this question, the author develops an artificial neural network model using 50% static and 50% dynamic ratios.

This study is one of the first in the literature to analyze such a long-term horizon before the enterprises go bankrupt. One of problems in conducting this research is to create a testing sample consisting of 100 bankrupt firms and a learning sample consisting of 50 bankrupt enterprises with data for as long as 17 years before going into insolvency.

The paper consists of five sections. In the Introduction, the author presents the justification for the topic, the study objectives and the contribution and innovation to the literature. Section 2 presents an overview of the literature on the types of financial ratios most frequently used and on the short characteristics of bankruptcy models. Section 3 introduces this study's assumptions. In Section 4, the author presents 17-year trajectories and the results of the developed dynamic ANN model. Section 5 concludes the paper.

2. Basic Concepts of Financial Failure Forecasting Models

When developing models forecasting bankruptcy, variables must be selected that have high predictive properties. The author of this study reviewed the literature on the financial ratios used in bankruptcy risk forecasting. After studying approximately 600 research papers on this subject, he chose 54 of them based on three criteria: the popularity of the authors and their research in the scientific community (number of citations), the degree of the research's innovation (duplications of studies showing only adaptations of existing models of low importance were avoided), and the diversification of the methods used. Table 1 shows the query results. Table 1 contains the 18 financial ratios that were most frequently used in studies forecasting the financial situations of companies globally.

Table 1. *Overview of the most common financial ratios used in bankruptcy forecasting models*

No.	Financial ratio	Used in studies
1.	Share of working capital in total assets (working capital / total assets)	[1] [8] [9] [10] [11] [12] [13] [14] [15] [16] [17] [20] [23] [27] [29] [30] [31] [33] [40] [45] [46] [47] [50] [52] [53]
2.	Encumbrance of cash surplus with liabilities [(net income + depreciation) / total liabilities or EBIT / total liabilities]	[1] [2] [8] [21]
3.	Quick liquidity [(current assets - inventories) / current liabilities]	[4] [5] [13] [15] [16] [23] [32] [37] [39] [42] [49]
4.	Current liquidity (current assets / current liabilities)	[1] [2] [3] [4] [5] [8] [13] [15] [21] [23] [26] [33] [34] [35] [38] [39] [41] [42] [43] [48] [50]
5.	Cash liquidity [(current assets - inventories - accounts receivables) / current liabilities]	[8] [16] [35] [43] [45] [51]
6.	Return on assets (net income / total assets)	[1] [2] [3] [4] [8] [10] [12] [13] [14] [15] [16] [17] [20] [22] [26] [28] [29] [31] [33] [35] [36] [38] [39] [40] [42] [43] [44] [45] [46] [47] [48] [49] [50] [52] [53]
7.	Relation of equity to total liabilities [equity / total liabilities]	[10] [14] [17] [29] [30] [31] [40] [42] [45] [46] [47] [50] [52] [53]
8.	Period of repayment of short-term liabilities or rotation of liabilities [(current liabilities / operating costs) * 365 days or operating costs / current liabilities]	[13] [16] [22] [25] [26] [32]
9.	Days' inventory or inventory turnover [(inventories / sales) * 365 days or sales / inventories]	[1] [3] [8] [11] [13] [16] [23] [36] [43]

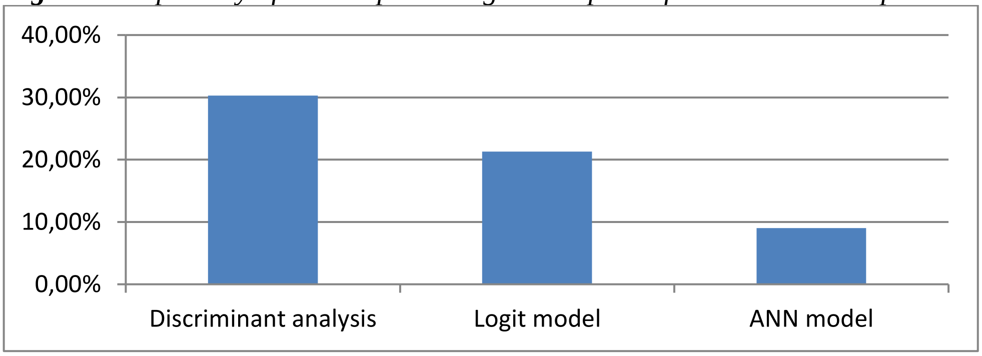
10.	Days' accounts receivable or turnover of short-term receivables [(short-term receivables/sales) * 365 days or sales / short-term receivables]	[11] [23] [34] [41] [43] [49]
11.	Turnover of total assets (sales / total assets)	[2] [3] [6] [9] [10] [12] [13] [14] [15] [16] [17] [22] [26] [27] [29] [30] [34] [35] [36] [37] [40] [43] [45] [46] [50] [52] [53]
12.	Relation of gross income or EBIT to total assets [EBIT / total assets or gross income / total assets]	[3] [4] [7] [9] [10] [12] [14] [15] [17] [30] [31] [38] [40] [45] [46] [50] [52] [53]
13.	Share of total debt in total assets (total liabilities / total assets)	[1] [3] [4] [11] [15] [16] [18] [22] [23] [24] [26] [27] [33] [34] [39] [41] [42] [43] [45] [47] [48] [51] [54]
14.	Share of equity in total assets (equity / total assets)	[1] [15] [19] [21] [28] [32] [34] [36] [37] [41] [49]
15.	Net return on sales (net income / total revenues)	[1] [3] [13] [18] [36] [39] [41] [42]
16.	Operating profit margin or gross profit margin [operating income / sales or gross income / sales]	[2] [11] [15] [16] [19] [22] [49]
17.	Return on equity (net income / equity)	[2] [11] [34] [41] [43] [44] [49]
18.	Coverage of fixed assets with long-term capital or equity [(equity + non-current liabilities) / fixed assets or equity / fixed assets]	[36] [37]

Sources: *[1] – Ahn et al., 2000; [2] – Bian and Mazlack, 2003; [3] – Bryant,1997; [4] – Dimitras et al., 1996; [5] – Fletcher and Goss, 1993; [6] – Andres et al., 2005; [7] –Atiya, 2001; [8] – Back et al., 1996; [9] – Baek and Cho, 2003; [10] – Ignizio and Soltyas, 1996; [11] – Karels and Prakash, 1987; [12] –Lacher et al., 1995; [13] – Lee et al., 1996; [14] – Lee et al., 2005; [15] – Leshno and Spector, 1996; [16] – Lin and McClean, 2001; [17] – Altman, 1993; [18] – Pang-Tien et al., 2008; [19] – Sandin and Porporato, 2007; [20] – Lin and Piesse, 2004; [21] – Maczynska, 2004; [22] – Gajdka and Stos,1996; [23] – Hadasik, 1998; [24] – Gruszczynski, 2003; [25] – Jardin and Severin, 2012; [26] – Hołda, 2001; [27] – Bandyopadhyay, 2006; [28] – Yim and Mitchell, 2004; [29] – Galvao et al., 2004; [30] – Altman et al., 1979; [31] – Ginoglou and Agorastos, 2002; [32] – Emel et al., 2003; [33] – Boritz and Kennedy, 1995; [34] – Kuruppu et al., 2003; [35] – McKee, 2003; [36] – Min and Lee, 2005; [37] – Park and Han, 2002; [38] – Pendharkar and Rodger, 2004; [39] – Piramuthu et al., 1998; [40] – Serrano-Cinca, 1996; [41] – Shah and Murtaza, 2000; [42] – Sikora and Shaw, 1994; [43] – Witkowska, 2002; [44] – Serrano-Cinca, 1997; [45] – Michaluk, 2003; [46] – Sharda and Wilson, 1994; [47] – Zapranis and Ginoglou, 2000; [48] – Anandarajan et al., 2001; [49] – Eklund et al., 2003; [50] – Zhang et al., 1999; [51] – Laitinen and Kankaanpaa, 1999; [52] – Becerra et al., 2005; [53] – Rahimian and Singh, 1993; [54] – Charalambous et al., 2000.*

Economic forecasting methods are now plentiful, including methods originating from different scientific disciplines (such as discriminant analysis models, logit and probit models, decision trees, random forest models, fuzzy sets models, artificial neural networks, genetic algorithms, support vector machines, hazard models, entropy theory models, etc.). Due to size limitations, this study focuses on the three most frequently used types of corporate bankruptcy prediction models; in other words, those models that are popular in scientific and business practice. Figure 1 shows that the most popular type of model is the model of multivariate discriminant analysis (30.3% among all methods) and the logit model (21.3% of all cases). The third most commonly used model is the artificial neural network. However, this model is distinguished by a large difference in the popularity of its use in comparison to the first two models, used in only 9% of studies. In the literature, several types of artificial neural network models are used to forecast a company's financial failure. The most common type is the multilayer perceptron model (74% of cases) and the Kohonen network (5%) (Perez, 2006). Other types of models using different methods of forecasting bankruptcy were used in the marginal range (less than 4–5% of cases).

Based on the query results, each ratio's frequency of use in the 54 aforementioned studies was calculated. Table 1 shows that six financial ratios occurred in at least 30% of the studies: the share of working capital in total assets, current liquidity, net return on total assets, turnover of total assets, return on assets measured by income before taxation and repayment of interest and the share of total debt in total assets. Two of these ratios are liquidity ratios, two are profitability ratios, one is an indicator of debt and one is an indicator of efficiency. The most common (occurring in 63.6% of studies) was net return on total assets. The second and third most-common ratios were the total turnover of assets (50.9%) and the share of working capital in total assets (47.3%).

Figure 1. *Popularity of models predicting bankruptcies for individual companies*



Source: *Aziz and Dar, 2006.*

Multivariate linear discriminant analysis (MDA) allows the classification of enterprises based on many explanatory variables. The method is classified as a pattern (teacher) classification, because the discriminant function's value, determined for the analyzed companies, is compared to a pattern, and thus defines the firms belonging to a particular class. In assessing the risk of a company's financial failure, two populations of businesses are considered: at risk, the "bad;" and not at risk of bankruptcy, the "healthy" companies. Underlying discriminant analysis is the determination whether the populations under consideration differ in terms of the average value of a certain feature, the explanatory variable, which could be used to predict membership in a particular class. Therefore, the basic problem that must be solved by applying discriminant analysis to predict a company's bankruptcy is associated with an appropriate choice of financial ratios: explanatory variables and determination of coefficients (weights) of discriminant function for each ratio so that the difference between the average values of both groups of companies (at risk and not at risk of bankruptcy) is as large as possible. One way of choosing appropriate financial ratios is to use a correlation matrix; only the features should be selected that are poorly correlated with each other and strongly correlated with the grouping variable, representing information about the threat or lack of threat of bankruptcy. This approach provides a selection of such features that do not duplicate information provided by other ratios, while being good representatives of the indicators not selected as diagnostic.

First the linear discriminant function was determined, which is a weighted sum of the analyzed diagnostic variables in the following form:

$$Z = d_0 + d_1x_1 + d_2x_2 + ... + d_nx_n \quad (1)$$

where Z is the dependent variable (explained); x_i is the independent variables (explanatory) (i = 1, 2, ..., n); and d_i is the discriminant weights, the so-called discriminant coefficients (i = 1, 2, ..., n).

Then, the discriminant function's cut-off value is determined based on how the analyzed company is classified. For this purpose, the average model values are specified for both the business populations ("bad" and "healthy"). The threshold is usually determined to occur between the average model values from each company group. Enterprises are classified by comparing their model's calculated value with a set threshold. In a situation where a company's function Z value is less than the threshold, the company is included within the group of companies facing bankruptcy and vice versa. Some authors of bankruptcy forecasting models discriminant analyses assume the existence of an intermediate zone, a so-called "gray area", or an area of uncertainty, in which the tested company cannot be classified. For the purposes of this study, five multivariate linear discriminant analysis models are presented: two from Europe, one from North America, one from Asia and one from Latin America. The first model used here was created by Altman in 1968 based on 66 US enterprises between 1966 and 1968. The model has the following function (Altman, 1993):

$$Z = 1.2 * X1 + 1.4* X2 + 3.3 * X3 + 0.6 * X4 + 0.999 * X5 \quad (2)$$

Where:
X1 = (current assets - current liabilities) / total assets
X2 = net income / total assets
X3 = EBIT / total assets
X4 = market value of equity / total liabilities

X5 = sales / total assets

Altman proposed the use of three decision areas, depending on the Z score's value: if $Z < 1.81$ then the probability of bankruptcy is high, if $1.81<Z<2.99$ then the risk of financial failure cannot be defined ("gray area"), and if $Z>2.99$ then the probability of bankruptcy is low.

The next model was constructed by Taffler for forecasting the financial failure of British enterprises, and it has following function (Agarwal and Taffler, 2007) with a solvency threshold at ZT = 0:

$$ZT = 3.2 + 12.18 * X1 + 2.5 * X2 - 10.68 * X3 + 0.029 * X4 \quad (3)$$

Where:
X1 = income before tax / current liabilities
X2 = current assets / total liabilities
X3 = current liabilities / total assets
X4 = (quick assets - current liabilities) / daily operating expenses with the denominator proxied by (sales - income before taxes - depreciation) / 365.

The second model from the European Union was constructed to forecast the financial situation of enterprises in Central Europe. The model was estimated based on 135 companies from that region. It consists of two discriminant functions, Zban and Znon (Korol, 2013):

$$Zban = -2.95855 + 3.20023 * X1 - 7.73879 * X2 + 0.6318 * X3 + 0.37591 * X4 \quad (4)$$
$$Znon = -6.8088 + 3.17942 * X1 - 5.45035 * X2 + 1.62317 * X3 + 1.51146 * X4 \quad (5)$$

Where:
X1 = (current assets - inventories) / current liabilities
X2 = (net income + depreciation) / total liabilities
X3 = operating costs / current liabilities
X4 = income before tax / current liabilities
If the value of function Zban is larger than the value of function Znon, the enterprise is classified as bankrupt; when the reverse is true, the company is classified as non-bankrupt.

Another popular model in the literature globally is a model created by Yim and Mitchell (2004) based on 70 Japanese enterprises with a cutoff point equal to zero:

$$F = 1.057 - 0.014 * X1 - 0.039 * X2 + 0.32 * X3 \quad (6)$$

Where:
X1 = net income / total assets
X2 = stockholders' equity / total assets
X3 = non-current liabilities / stockholders' equity

The last model of multivariate discriminant analysis used in this study is a model estimated by Sandin and Porporato (2007) for forecasting the financial situation of Latin American enterprises. The model was estimated based on 22 firms from Argentina (the solvency threshold equals zero):

$$F = 15.06 * X1 + 16.11 * X2 – 4.14 \quad (7)$$

Where:
X1 = operational income / sales
X2 = stockholders' equity / total assets

Another popular statistical type of model used to predict bankruptcy risk is the logit model (LOG). The result of the logistic regression function is the likelihood of an event p_i. In estimating a firm's financial failure, it is the probability of an analyzed company belonging to one of two sets: "bankrupt" or "non-bankrupt." In the binomial model, number 1 (e.g., firms at risk of failing) is attributed to one set and number 0 to the second set, the "healthy" companies. The p_i function takes the following form:

$$P(Y=1) = 1 / (1 + exp^{-z}) = exp^{z} / (1 + exp^{z}) \quad (8)$$

Where:
P(Y=1) equals the dependent variable, the probability of adoption by variable Y the value of 1; and
Z equals the value of the linear function Z, where $Z = d_0 + d_1x_1 + d_2x_2 + ... + d_nx_n$ [x_i - explanatory variables (i = 1, 2, ..., n); d_i - weights (i = 1, 2, ..., n)].

The value of indicator P(Y = 1) occurs in the range 0 to 1. Assuming that the number 1 indicates a company at risk of bankruptcy, when the value of P(Y = 1) is greater, the probability of failure is greater. To use the estimated logit model, a certain threshold (P_{cutoff}) of function P(Y=1) must also be adopted, as in the case of discriminant analysis:

$$P(Y=1) \leq P_{cutoff} \text{ then } Y = 0$$
$$P(Y=1) > P_{cutoff} \text{ then } Y = 1$$

On this basis, as the variable Z increases, the P(Y = 1) increases and vice versa.

In this study, the author used two logit models: one from Asia and one from North America. The logit model from North America was estimated by Altman and Sabato (2007) based on 432 enterprises from the USA and Canada:

$$Z = 4.28 + 0.18 * X1 – 0.01 * X2 + 0.08 * X3 + 0.02 * X4 + 0.19 * X5 \quad (9)$$

Where:
X1 = EBIT / total assets
X2 = current liabilities / stockholders' equity
X3 = net income / total assets
X4 = cash / total assets
X5 = EBIT / interest paid

The logit model estimated by Pang-Tien *et al.* (2008) for forecasting the bankruptcy risk of Asian firms (using 116 Taiwanese enterprises) is as follows:

$$Z = - 4.44 + 0.08 * X1 - 0.042 * X2 - 0.021 * X3 \quad (10)$$

Where:
X1 = total liabilities / total assets
X2 = EBIT / interest paid
X3 = operational income / interest paid

Both of these logit models consist of a cutoff point at 0.5. This fact means results above 0.5 indicate a high risk of financial failure (between 50% and 100% probability), and scores below 0.5 indicate a low risk of bankruptcy (between 0% and 50%).

The concept of artificial neural networks is understood as mathematical models composed of networks of computing nodes called neurons and their connections, which simulate the action of biological systems and can effectively solve specific problems. In contrast to multivariate discriminant analysis, for example, the essence of the activity of neural networks is a purely mechanical approach to the analyzed phenomenon, without detection of internal relations and the strength of existing relationships.

The most common type of neural networks in predicting an enterprise's bankruptcy is a feedforward multilayer neural network, in which the signal flows in only one direction, i.e., from the input, where the network takes input data; through the hidden layer, where the main processing of neural signals occurs; to the output, where the network provides a solution. The network is also called a multilayer perceptron (MLP). Determining the number of hidden neurons is not an easy task. Although the literature offers formulas to determine the optimal hidden layer[2], the authors of publications on neural networks postulate not accepting them a priori, but rather designating the number of neurons in each individual case, depending on the problem being solved (Zhang *et al.,* 1999). When providing input into the neural network, independent variables are introduced, consisting of information about the analyzed enterprise, such as financial ratios. Based on data entered at the network inputs, total activation of neuron *e* is calculated, usually as a linear combination of inputs, which can be represented as:

$$e = \sum_{i=1}^{n} w_i x_i \quad (11)$$

Where:
x_i (i=1,2, ..., n) is a vector [n x 1] of input signals,

[2] Formulas to determine the number of hidden neurons: n/2, n, n+1, and 2*n+1, where n equals the number of input neurons.

w_i (i=1,2, ..., n) is a vector [n x 1] of weights, which on the one hand express the degree of validity of the information transmitted via this input and, on the other hand constitute a kind of neuron memory about the relationships between input and output signals.

The output signal of neuron y depends on its total activation (Figure 2):

$$y = \varphi(e) \tag{12}$$

where φ is the so-called neuron activation function.

Output values of neurons in the last layer are output values from the network simultaneously. The literature distinguishes the following most commonly used activation functions (Witkowska, 2002):

- ✓ threshold function:

$$\varphi(e) = \begin{cases} 1 & when \quad e \geq 0 \\ 0 & when \quad e < 0 \end{cases} \tag{13}$$

- ✓ logistic function:

$$\varphi(e) = \frac{1}{1 + \exp(-\beta e)} \tag{14}$$

- ✓ hyperbolic tangent function:

$$\varphi(e) = \tanh(\beta e) = \frac{\exp(\beta e) - \exp(-\beta e)}{\exp(\beta e) + \exp(-\beta e)} \tag{15}$$

- ✓ signum function:

$$\varphi(e) = \begin{cases} 1 & when \quad e > 0 \\ 0 & when \quad e = 0 \\ -1 & when \quad e < 0 \end{cases} \tag{16}$$

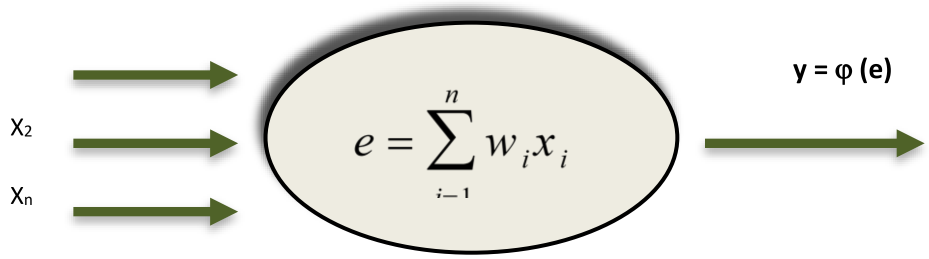
- ✓ Gaussian function:

$$\varphi(e) = \exp\left(\frac{-e^2}{2}\right) \tag{17}$$

- ✓ sinus function:

$$\varphi(e) = \begin{cases} -1 & when \quad e < -\frac{\pi}{2} \\ \sin \beta_e & when \quad -\frac{\pi}{2} \leq e \leq \frac{\pi}{2} \\ 1 & when \quad e > \frac{\pi}{2} \end{cases} \tag{18}$$

Figure 2. *Artificial neuron model*



Source : *Own study.*

3. Research Approach

In the studies, the author used seven popular forecasting models (two from Europe, two from Asia, two from North America and one from Latin America), presented in Section 2. These models (five multivariate discriminant analysis models and two logit models) were used to develop 17-year trajectories separately for non-bankrupt enterprises and those at risk of financial failure. To achieve this study's objectives, two enterprise samples were created:

- ✓ the testing sample consisted of 100 enterprises that were at risk of bankruptcy or were already bankrupt and 100 firms with good economic condition and
- ✓ the learning sample consisted of 50 "healthy" firms and 50 companies at risk of financial failure.

Both samples consisted of enterprises from countries around the world (from such regions as Europe, Far-East Asia, Latin America, North America) from production and service sectors. For all 300 firms, the author calculated 25 different financial ratios (Table 2) for 17 years of their operations. The main difficulty to overcome was to find 150 enterprises at risk of bankruptcy for which there was available data for such a long analytical horizon before these firms went into financial crisis (the information was taken from 1995–2016, depending on the enterprise).

The company's "health" was assumed based on the overall analysis of financial statements (evaluating profitability, liquidity, efficiency and debt ratios). Companies were selected for which there was no doubt they were not at risk of failure. However, the financial data of bankrupt and non-bankrupt firms were used from the previous 1 to 17 years before classifying them as "good" or "bad." Inclusion in the data analysis from the year in which companies were classified would decrease the reliability of estimated trajectory results.

In addition to developing long-term trajectories, the studies include objectives such as evaluating the short term and the long-term effectiveness of the chosen models and identifying which model characterizes the forecast with the smallest decrease of effectiveness along the increasing horizon. The following formula was used to calculate overall effectiveness:

$$S=\{1-[(D1+D2)/(BR+NBR)]\}*100\% \quad (19)$$

where D1 equals the number of bankrupt firms classified by the model as non-bankrupt, D2 equals the number of non-bankrupt enterprises classified by the model as bankrupt, BR equals the number of bankrupt companies in the sample, and NBR equals the number of non-bankrupt companies in the sample.

The final research stage concerns developing the dynamic artificial neural network model that will address the question stated in Section 1: whether changes in financial ratios are relevant predictors of a company's coming financial crisis.

Table 2. *Financial ratios used in the studies*

Symbol of ratio	Calculation formula	Ratio used in the model
X1	total liabilities / total assets	Pang-Tien, Ching-Wen and Hui-Fun 2008
X2	EBIT / interest paid	Altman and Sabato 2007; Pang-Tien, Ching-Wen and Hui-Fun 2008
X3	operational income / interest paid	Pang-Tien, Ching-Wen and Hui-Fun 2008
X4	EBIT / total assets	Altman 1968; Altman and Sabato 2007
X5	current liabilities / stockholders' equity	Altman and Sabato 2007
X6	net income / total assets	Altman 1968; Yim and Mitchell 2004; Altman and Sabato 2007
X7	cash / total assets	Altman and Sabato 2007
X8	(current assets - inventories) / current liabilities	Korol 2013;
X9	(net income + depreciation) / total liabilities	Korol 2013;
X10	operating costs / current liabilities	Korol 2013;
X11	income before tax / current liabilities	Agarwal and Taffler 2007; Korol 2013;
X12	stockholders' equity / total assets	Yim and Mitchell 2004; Sandin and Porporato 2007
X13	non-current liabilities / stockholders' equity	Yim and Mitchell 2004
X14	operational income / sales	Sandin and Porporato 2007
X15	(current assets - current liabilities) / total assets	Altman 1968;
X16	market value of equity / total liabilities	Altman 1968;
X17	sales / total assets	Altman 1968;
X18	current assets / total liabilities	Agarwal and Taffler 2007;
X19	current liabilities / total assets	Agarwal and Taffler 2007;

X20	(quick assets - current liabilities) / daily operating expenses with the denominator proxied by (sales - income before taxes - depreciation) / 365	Agarwal and Taffler 2007;
X21	inventories / sales	author’s dynamic model of ANN
X22	stockholders’ equity / total liabilities	author’s dynamic model of ANN
X23	(stockholders’ equity + noncurrent liabilities) / fixed assets	author’s dynamic model of ANN
X24	current assets / current liabilities	author’s dynamic model of ANN
X25	income before tax / sales	author’s dynamic model of ANN

Source: *Own study.*

4. Results and Discussion

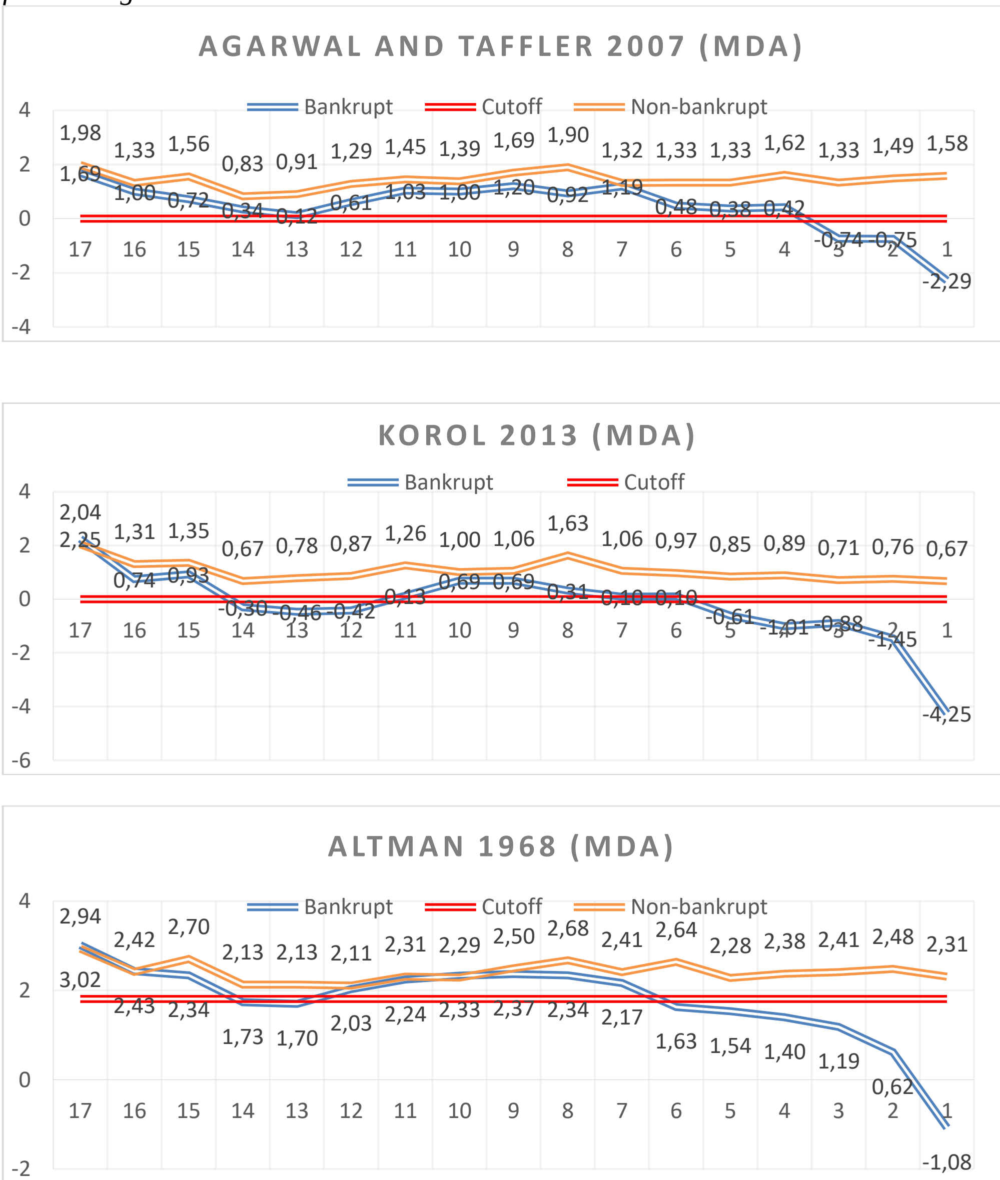
The first step in developing the trajectories of the “lives” of the firms was to calculate the results of seven forecasting models for all enterprises from the testing sample for each analytical year in the 17-year horizon (23 800 results). In the next step, the testing sample was divided into 100 bankrupt and 100 non-bankrupt enterprises. Then, the median of values generated by the forecasting models was calculated separately for these two types of companies for all the years (Figure 3). In the last stage, the author calculated the overall effectiveness for each model for all the years (Table 3).

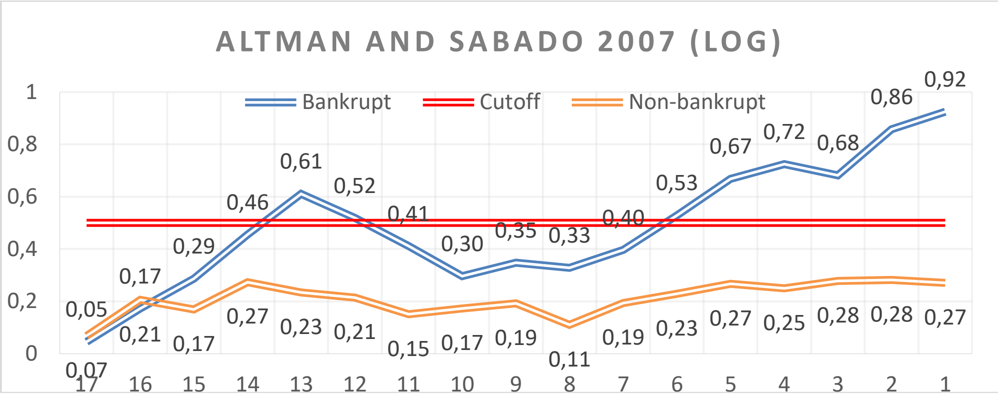
Looking at the trajectories of non-bankrupt and bankrupt enterprises (Figure 3) based on the seven different forecasting models (different enterprise regions on which each model was estimated, different ratio type implemented in the models, and often different forecasting techniques: the multivariate analysis model versus the logit model, the one-function model versus the two-function model, etc.), we draw two important conclusions:

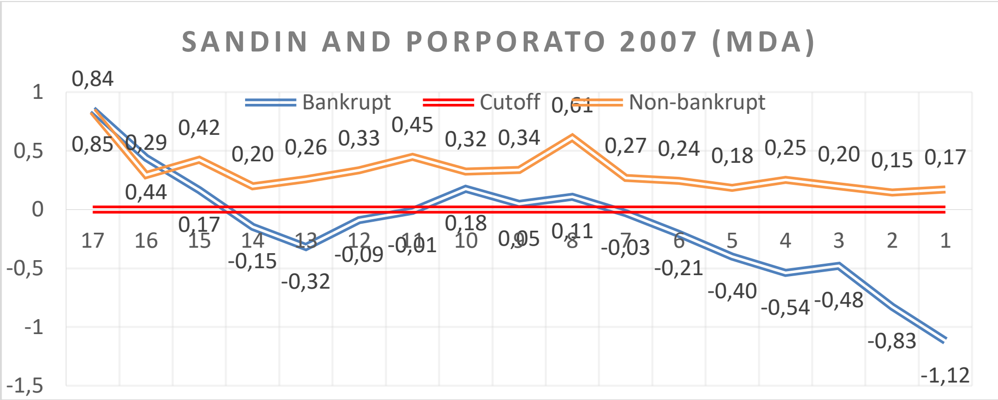
1. Clear, large differences exist in the development between non-bankrupt enterprises and future bankrupt firms. Most of those firms were not in danger of bankruptcy 10 or 15 years before such risk occurred. However, this research proved that the process of going bankrupt is exceptionally long, perhaps even longer than was previously understood in the relevant literature. Using these models and trajectories, analysts can and should identify the symptoms of going bankrupt long before the real bankruptcy risk occurs.
2. The forecasting models have been underestimated. The models characterize with high effectiveness a horizon forecast of one–two years before the financial failure. Prolonging the forecast period, we can observe that effectiveness of all seven models (Table 3) very much decreases. This is a drawback in the literature. However, these studies showed that although the trajectory of bankrupt enterprises for periods longer than four–five years before bankruptcy is above the

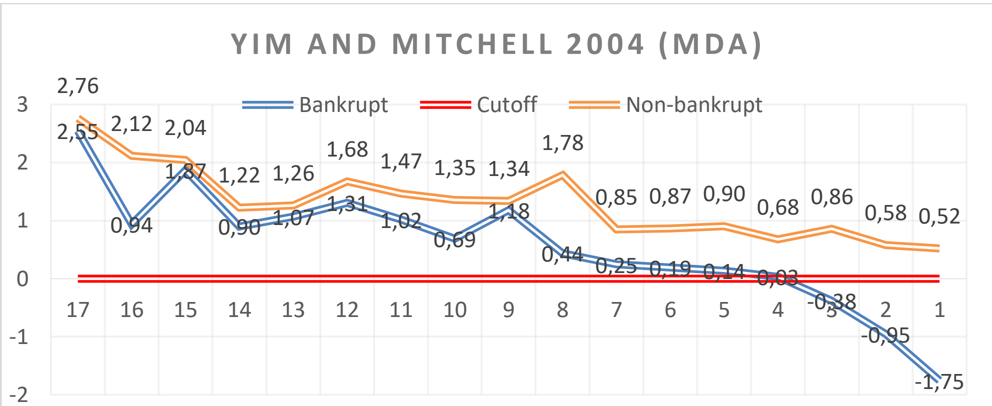
cutoff point for the MDA models and below the cutoff point for the LOG models (meaning the models have a low effectiveness) still, the models are efficient at differentiating good enterprises from firms at risk of financial failure. In addition, they show it perfectly in the entire horizon of 17 years of analysis.

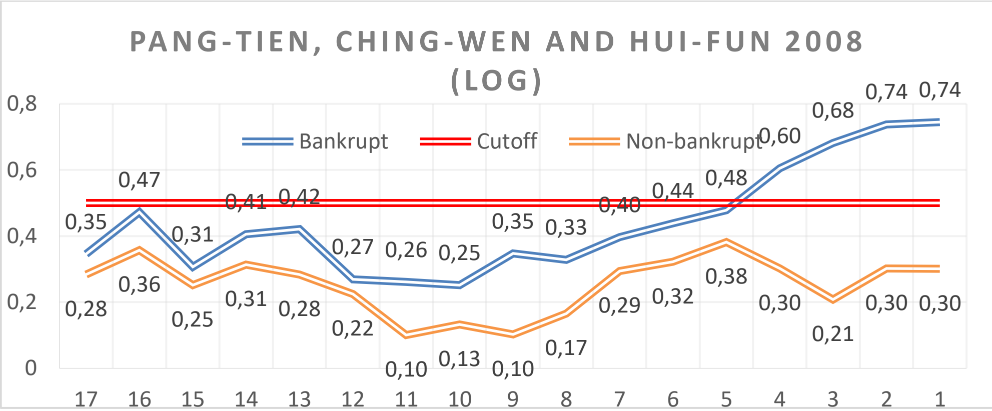
Figure 3. *Trajectories of non-bankrupt and bankrupt enterprises based on the seven forecasting models*




ALTMAN AND SABADO 2007 (LOG)
Bankrupt
Cutoff
Non-bankrupt


SANDIN AND PORPORATO 2007 (MDA)
Bankrupt
Cutoff
Non-bankrupt


YIM AND MITCHELL 2004 (MDA)
Bankrupt
Cutoff
Non-bankrupt



Looking more closely at the effectiveness of the models (Table 3) all of them stand out with good results in the forecasting horizon of one–two years, with an effectiveness above 80%. The highest effectiveness (above 90%) was found in three models: Altman 1968, Korol 2013 and Altman and Sabado 2007. In a horizon longer than 5 years before bankruptcy most models generated an effectiveness smaller than 70%. Generally, effectiveness below 70% is recognized as low.

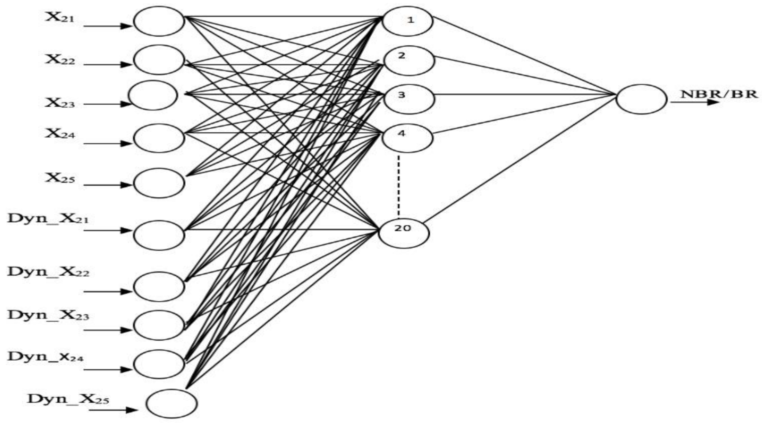
Table 3. *Effectiveness of the forecasting models*

Type of model / Years before	Agarwal and Taffler 2007 (MDA)	Korol 2013 (MDA)	Altman 1968 (MDA)	Altman and Sabado 2007 (LOG)	Sandin and Porporato 2007 (MDA)	Yim and Mitchell 2004 (MDA)	Pang-Tien, Ching-Wen and Hui-Fun 2008 (LOG)
1	89.5%	90.5%	92.0%	90.0%	85.0%	83.5%	87.0%
2	86.0%	87.5%	87.5%	86.0%	83.0%	82.5%	85.5%
3	77.5%	79.5%	75.5%	76.5%	79.5%	74.0%	80.0%
4	71.0%	75.0%	75.0%	74.0%	78.5%	69.5%	70.5%
5	69.5%	75.5%	74.5%	73.0%	72.5%	69.0%	69.5%
6	67.5%	68.0%	73.0%	67.5%	69.0%	67.5%	68.5%
7	65.5%	65.5%	65.0%	64.5%	64.0%	69.0%	65.0%
8	68.0%	68.5%	61.5%	61.5%	62.5%	61.0%	63.0%
9	64.5%	65.0%	60.5%	62.5%	59.5%	59.0%	64.0%
10	63.5%	67.5%	59.0%	61.0%	58.0%	58.5%	62.0%
11	66.5%	66.0%	56.5%	58.5%	69.0%	60.5%	61.5%
12	67.5%	69.5%	61.5%	57.0%	69.5%	62.0%	61.5%
13	70.0%	69.0%	63.0%	72.5%	69.0%	61.0%	62.5%
14	68.0%	69.5%	62.5%	71.5%	64.0%	57.5%	57.0%
15	64.5%	61.5%	55.5%	54.5%	55.5%	56.5%	57.5%
16	64.5%	61.0%	56.0%	55.5%	58.0%	57.0%	56.5%
17	63.5%	59.0%	55.0%	56.5%	57.0%	55.5%	56.0%

Source: *Own study.*

The author addressed the problem of the model's short forecasting horizon, proving their usefulness in long-term analysis too. This section's objective is to address the second research problem too: most of the forecasting models are of a static nature. To verify the effectiveness of ratios and their dynamic form in the model, the author developed the artificial neural network model (multilayer perceptron) using the learning sample. The model consists of 10 entry neurons, 20 hidden neurons and 1 output neuron (NBR: non-bankrupt and BR: bankrupt). The model's architecture is presented in Figure 4. Five entry neurons are in the form of static ratios: X21, X22, X23, X24, X25 (Table 2) and five represent the change dynamics of these five ratios.

Figure 4. *Architecture for the dynamic artificial neural network model*



Source: *Own study.*

The developed trajectories based on the median of the dynamic ANN generated results in the testing sample (Figure 5) show positive influence in both the forecasting horizon and the distance between two different trajectories. Using the ANN model, the bankrupt trajectory for up to 7 years of analysis is below the cutoff point of 0.5 (when the value is lower, the bankruptcy risk is higher). In addition, the difference between the non-bankrupt and bankrupt trajectory is higher than it is in the case of static models (Figure 3). The values of the non-bankrupt trajectory for the entire forecasting horizon of 16 years[3] are 0.8 or bigger, while the values of the bankrupt trajectory are less than 0.62.

Another positive influence of using both static and dynamic information of financial ratios in the model is its overall effectiveness (Table 4). The dynamic ANN model characterizes an effectiveness higher than 80% until 4 years of analysis, while in the case of the static models the horizon was 2 years (Table 3). In the one-year forecasting period, the dynamic model is 2 percentage points better than it is in the best static model (94.0% versus 92.0%).

[3]*There are 16 years of analysis because the author has the financial data for 17 years; because of the dynamic ratio calculations, the forecasting period had to be decreased by 1 year.*

Figure 5. *Trajectory of non-bankrupt and bankrupt enterprises based on the dynamic artificial neural network*

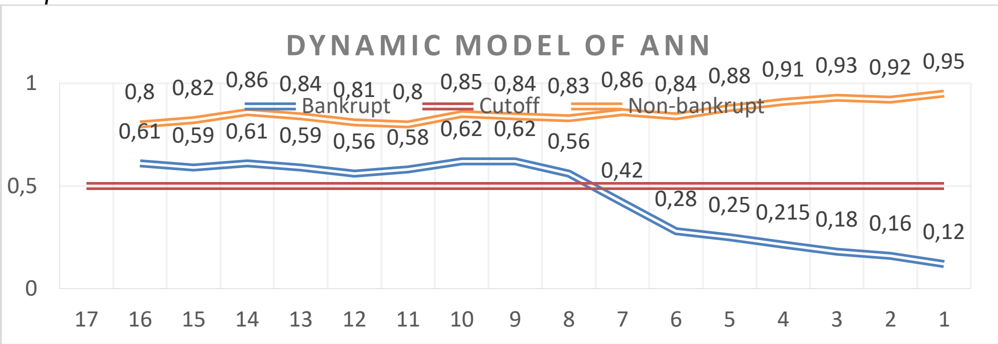


Table 4. *Effectiveness of the author's dynamic model of artificial neural network*

Years before	Effectiveness of ANN	Years before	Effectiveness of ANN
1	94.0%	9	65.5%
2	93.0%	10	63.0%
3	89.5%	11	64.0%
4	82.5%	12	69.5%
5	77.5%	13	67.5%
6	76.5%	14	65.5%
7	75.5%	15	68.0%
8	67.5%	16	61.5%

Source: *Own study.*

5. Conclusions

This study's multifaceted research goals allowed the development of original and novel conclusions. First, 17-year business trajectories were developed. Second, it was proven and specified that long-term developmental differences exist between non-threatened and future insolvent companies. These studies proved that the process of going bankrupt is very long, perhaps even longer than the literature has previously demonstrated. Third, the long-term usability of the models was demonstrated. To date, these models have been used only to forecast bankruptcy risk in the short term (1–3 years' prediction horizon).

This article demonstrates that these models can also serve to evaluate long-term growth and to identify the first symptoms of future bankruptcy risk many years before it actually occurs. Fourth, the author discussed the differences in the effectiveness of seven popular models from different global regions. The models with the highest effectiveness were indicated, as were the models characterized by the smallest decrease in effectiveness during the forecast period. Fifth, the introduction of a dynamic approach to the financial ratios significantly improves the model's effectiveness.

Finally, artificial neural networks better, more clearly identify the differences in trajectories between non-bankrupt and future bankrupt enterprises.

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