By<br>Ioannis Ananiadis<br>Dimitrios Kousenidis*


#### Abstract

The present paper offers an alternative way of estimating the mean expected cost of raw materials so as to improve the estimation of the expected total productcost without expanding beyond the framework of measuring and controlling the efficiency of the business.

Apart from the estimation methodology proposed, the paper also focuses on the implications of the variance between expected and actual raw materials cost. In this respect, the paper is potentially interesting for managers because it offers new information that can help their decision making process in three different ways: first, it may help managers to redefine the targets of their business; second, it offers managers the insights that could help them take the required corrective actions; and third, it helps managers to better analyze the raw-materials variances in a way that the prevailing estimated cost is both realistic and effective.


Keywords: Raw materials cost, variances analysis, stochastic cost estimation.
JEL classification: M4, M49, C69.

## 1. Introduction

Many management accounting textbooks ${ }^{1}$ place substantial emphasis on the fact that traditional costing systems do not allocate overheads to products properly and provide management with inaccurate and biased information about the costing of products or services. On practical grounds, however, many companies appear not to share this view with full enthusiasm and still prefer using costing systems that allocate overheads to products based on traditional volume-based measures ${ }^{2}$. Viewed in this light, employing traditional allocation rates (i.e. machine hours, direct labor hours) reduces the complexity of using other more sophisticated cost measurement methodologies, such as the theory of constraints (TOC), activity-based costing (ABC), throughput accounting, and target costing. Confronted with a large number of alternatives to traditional costing, managers are often confused and the dilemma of

[^0]accuracy over simplicity unavoidably leads to the question as to whether which of the alternative methodologies provides with the most accurate cost information.

Although, in the academic literature activity-based costing prevails as the most widely advocated alternative ${ }^{3}$, it is not uncommon to find authors that contribute ideas on how to improve traditional costing systems. Balakrishnan and Sivaramakrishnan (2002) and Banker and Hansen (2002) offer explanations for the tendency of firms to use full-cost information for pricing decisions and expose the reasons why companies show a continuous preference to traditional costing methods, towards more sophisticated costing methods. Lucas $(1999,2003)$ advocates that the lack of adequate empirical evidence has spread among accountants the misconception that full-cost information sufficiently approximates the required inputs for marginal costing optimal decision-making. Cheatham (1989) Johnsen and Sopariwala (2000), Wing (2000) and Emsley (2001) take another point of view and argue that full-cost information derived by traditional costing methods suffers from the fact that cost variances are either misleadingly calculated or the information concerning cost variances is discarded by senior management. Moreover, Emsley and Wing maintain that if proper attention is given to variance analysis then the information implicit in cost variances could be a relevant input in decision-making and problem solving.

The present paper develops a stochastic model for estimating mean-expected variances. The paper applies estimation procedures on the calculation of the variances of the predetermined direct materials cost, but the proposed model is also applicable on the calculation of other types of cost variances. The analysis is based on truncating cost variances in order to obtain estimates of predetermined cost that do not deviate significantly from the actual cost of a product. For the mathematical proofs of the model, the paper uses continuous analysis. However, the numerical application that follows uses discrete analysis making the model easily understood by both researchers and practitioners.

The proposed model attempts to improve full-cost information and enhances the information content of cost variances in pricing decisions. In doing so it improves the use of traditional costing systems by dissolving potential doubt about misleading information signals or complexity burdens imposed by the changing of cost accounting systems. On the other hand, however, it should be pointed out that the proposed model does not prevail as a competing methodology to alternative costing systems. On the contrary, with few modifications, the model can prove to be helpful under any costing system that uses historical data to calculate predetermined-allocation-rate variances.

The remainder of the paper is organized as follows. Section 2 discusses the assumptions required, and presents the development of a stochastic model for the estimation of the direct materials cost variance. Section 3 applies the model to a numerical example and exposes its practical usefulness. Section 4 summarizes conclusions and implications for further research, while the appendix at the end of the paper relaxes some distributional assumptions of the model.

[^1]
## 2. Development of the model

## Assumptions

It is well known that full-production cost is composed by the following three main categories of costs:
A. Direct materials
B. Direct labor
C. Factory overhead

Having this in mind, the following simple assumptions are necessary for the development of the model.

1) Each period's expected actual cost is estimated (forecasted) at the end of the previous fiscal period.
2) The predetermined cost is estimated with statistical-quantitative methods and is considered to remain constant (fixed) thereafter.
3) The distributions of the random variables (actual quantity and price of direct \materials) are known in advance. This assumption does not necessarily imply that probabilities are determined from a theoretical probability distribution. Problems on practical grounds usually arise because the theoretical probability distribution can never be known with certainty. Instead, this assumption implies that a curve-fitting method is used to estimate a theoretical probability distribution underlying a given frequency distribution ${ }^{4}$.
4) The random variables, used in the model (actual quantity and price of direct materials) may be either independent or dependent, and determine the form of the model to be followed.
5) The total expected variance (be it either positive or negative) is considered to be satisfactory when it does not exceed or fall short of the total expected average actual cost more than $2 \%$. This truncation procedure assumes that the fixed predetermined cost is adjusted by adding or subtracting the expected variance depending on whether the variance is positive or negative. The resulting predetermined cost is considered as the new predetermined cost. The same procedure is repeated until the difference of the total variance does not exceed or fall short of the average actual cost more than the above-mentioned percentage. This procedure is applied in order to minimize possible omissions or mistakes.

## Estimation of the direct materials variance

The total variance of the direct materials is the combined result of two secondary variances
a. The price variance of direct materials
b. The quantity variance of direct materials

[^2]Thus, the variance of the actual price of direct materials results from the equation: Variance of the price of direct material:

$$
\begin{equation*}
V_{p}=\left(\mathrm{P}_{\mathrm{A}}-\mathrm{P}_{\mathrm{E}}\right) \mathrm{Q}_{\mathrm{A}} \tag{1}
\end{equation*}
$$

where,
$\mathrm{P}_{\mathrm{A}}=$ Actual Price
$\mathrm{P}_{\mathrm{E}}=$ Predetermined Price
$\mathrm{Q}_{\mathrm{A}}=$ Actual Quantity
On the other hand, the quantity variance of direct materials is equal to

$$
\begin{equation*}
V_{Q}=\left(\mathrm{Q}_{\mathrm{A}}-\mathrm{Q}_{\mathrm{E}}\right) \mathrm{P}_{\mathrm{E}} \tag{2}
\end{equation*}
$$

where:
$\mathrm{Q}_{\mathrm{A}}=$ Actual Quantity
$\mathrm{Q}_{\mathrm{E}}=$ Predetermined Quantity
$\mathrm{P}_{\mathrm{E}}=$ Predetermined Price
It is clear from equations (1) and (2) that the price variance of direct materials $\left(\mathrm{P}_{\mathrm{A}}-\mathrm{P}_{\mathrm{E}}\right) \mathrm{Q}_{\mathrm{A}}$ is positive or favorable when $\mathrm{P}_{\mathrm{A}}<\mathrm{P}_{\mathrm{E}}$ and negative or unfavorable when $P_{A}>P_{E}$.

Accordingly, the quantities variance of direct materials $\left(\mathrm{Q}_{\mathrm{A}}-\mathrm{Q}_{\mathrm{E}}\right) \mathrm{P}_{\mathrm{E}}$ is positive or favorable when $Q_{A}<Q_{E}$ and negative or unfavorable when $Q_{A}>Q_{E}$. Therefore, on the assumption that the random variables $\mathrm{Q}_{\mathrm{A}}$ and $\mathrm{P}_{\mathrm{A}}$ are dependent, the expected variance of prices of direct materials results from the common distribution of $Q_{A}$ and $P_{A}$ that is $p_{0}\left(Q_{A}, P_{A}\right)$ with respect to the interval of $Q_{A}$ and $P_{A}$. Thus we have:

Expected actual variance of prices $=E\left(V_{P}\right)=$

$$
\begin{equation*}
\int_{0}^{\infty} \int_{0}^{P_{E}}\left(P_{A}-P_{E}\right) Q_{A} p\left(Q_{A,} P_{A}\right) d Q_{A} d P_{A}+\int_{0}^{\infty} \int_{P_{E}}^{\infty}\left(P_{A}-P_{E}\right) Q_{A} p\left(Q_{A}, P_{A}\right) d Q_{A} d P_{A} \tag{3}
\end{equation*}
$$

Expected actual variance of quantities $=E\left(V_{Q}\right)=$

$$
\begin{equation*}
\int_{0}^{Q_{E}}\left(Q_{A}-Q_{E}\right) P_{E} p_{0}\left(Q_{A}\right) d Q_{A}+\int_{Q_{E}}^{\infty}\left(Q_{A}-Q_{E}\right) P_{E} p_{0}\left(Q_{A}\right) d Q_{A} \tag{4}
\end{equation*}
$$

the total variance is equal to the sum of the two secondary variances that is:

Total variance $=E(V)=E\left(V_{P}\right)+E\left(V_{Q}\right)=$
$\int_{0}^{\infty} \int_{0}^{P_{E}}\left(P_{A}-P_{E}\right) Q_{A} p\left(Q_{A} P_{A}\right) d Q_{A} d P_{A}+\int_{0}^{\infty} \int_{P_{E}}^{\infty}\left(P_{A}-P_{E}\right) Q_{A} p\left(Q_{A}, P_{A}\right) d Q_{A} d P_{A}+$
$\int_{0}^{Q_{E}}\left(Q_{A}-Q_{E}\right) P_{E} p_{0}\left(Q_{A}\right) d Q_{A}+\int_{Q_{E}}^{\infty}\left(Q_{A}-Q_{E}\right) P_{E} p_{0}\left(Q_{A}\right) d Q_{A}$

In the case where the random variables $\mathrm{Q}_{\mathrm{A}}, \mathrm{P}_{\mathrm{A}}$ are independent and the distribution of their probability is $p_{0}\left(\mathrm{Q}_{\mathrm{A}}\right)$ and $\mathrm{p}_{0}\left(\mathrm{P}_{\mathrm{A}}\right)$ with respect to the interval of $\mathrm{Q}_{\mathrm{A}}$ and $\mathrm{P}_{\mathrm{A}}$, then the secondary variances are as follows:

Expected actual variance of prices of direct material $=E\left(V_{P}\right)=$

$$
\begin{equation*}
\int_{0}^{\infty} \int_{0}^{P_{E}}\left(P_{A}-P_{E}\right) Q_{A} p_{0}\left(Q_{A}\right) p_{0}\left(P_{A}\right) d Q_{A} d P_{A}+\int_{0}^{\infty} \int_{P_{E}}^{\infty}\left(P_{A}-P_{E}\right) Q_{A} p_{0}\left(Q_{A}\right) p_{0}\left(P_{A}\right) d Q_{A} d P_{A} \tag{6}
\end{equation*}
$$

Expected variance of quantity of direct material $=E\left(V_{Q}\right)=$

$$
\begin{equation*}
\int_{0}^{Q_{E}}\left(Q_{A}-Q_{E}\right) P_{E} p_{0}\left(Q_{A}\right) d Q_{A}+\int_{Q_{E}}^{\infty}\left(Q_{A}-Q_{E}\right) P_{E} p_{0}\left(Q_{A}\right) d Q_{A} \tag{7}
\end{equation*}
$$

Having calculating the secondary variances the total expected variance is estimated as the sum of $E\left(V_{P}\right)$ and $E\left(V_{Q}\right)$. It follows from equations (6) and (7) that:

$$
\begin{align*}
& \mathbf{E}(\mathbf{V})=\int_{0}^{\infty} \int_{0}^{P_{E}}\left(P_{A}-P_{E}\right) Q_{A} p_{0}\left(Q_{A}\right) p_{0}\left(P_{A}\right) d P_{A} d Q_{A}+ \\
& \quad+\int_{0}^{\infty} \int_{P_{E}}^{\infty}\left(P_{A}-P_{E}\right) Q_{A} p_{0}\left(Q_{A}\right) p_{0}\left(P_{A}\right) d P_{A} d Q_{A}+\int_{0}^{Q_{E}}\left(Q_{A}-Q_{E}\right) P_{E} p_{0}\left(Q_{A}\right) d Q_{A}+ \\
& \quad+\int_{Q_{E}}^{\infty}\left(Q_{A}-Q_{E}\right) P_{E} p_{0}\left(Q_{A}\right) d Q_{A} \tag{8}
\end{align*}
$$

which after rearranging terms yields:

$$
\begin{align*}
& \int_{0}^{Q_{E}} \int_{0}^{P_{E}}\left(P_{A}-P_{E}\right) Q_{A} p_{0}\left(Q_{A}\right) p_{0}\left(P_{A}\right) d P_{A} d Q_{A}+ \\
+ & \int_{Q_{E}}^{\infty} \int_{0}^{P_{E}}\left(P_{A}-P_{E}\right) Q_{A} p_{0}\left(Q_{A}\right) p_{0}\left(P_{A}\right) d P_{A} d Q_{A}+ \\
+ & \int_{0}^{Q_{E}} \int_{P_{E}}^{\infty}\left(P_{A}-P_{E}\right) Q_{A} p_{0}\left(Q_{A}\right) p_{0}\left(P_{A}\right) d P_{A} d Q_{A}+ \\
+ & \int_{Q_{E}}^{\infty} \int_{P_{E}}^{\infty}\left(P_{A}-P_{E}\right) Q_{A} p_{0}\left(Q_{A}\right) p_{0}\left(P_{A}\right) d P_{A} d Q_{A} \quad+\int_{0}^{Q_{E}}\left(Q_{A}-Q_{E}\right) P_{E} p_{0}\left(Q_{A}\right) d Q_{A}+ \\
+ & \int_{Q_{E}}^{\infty}\left(Q_{A}-Q_{E}\right) P_{E} p_{0}\left(Q_{A}\right) d Q_{A} \tag{9}
\end{align*}
$$

or equivalently

$$
\begin{align*}
& \int_{0}^{Q_{E}}\left[\left(Q_{A}-Q_{E}\right) P_{E}+\int_{0}^{P_{E}}\left(P_{A}-P_{E}\right) Q_{A} p_{0}\left(P_{A}\right) d P_{A}\right] p_{0}\left(Q_{A}\right) d Q_{A} \\
+ & \int_{Q_{E}}^{\infty}\left[\left(Q_{A}-Q_{E}\right) P_{E}+\int_{0}^{P_{E}}\left(P_{A}-P_{E}\right) Q_{A} P_{0}\left(P_{A}\right) d P_{A}\right] p_{0}\left(Q_{A}\right) d Q_{A}+ \\
+ & \int_{0}^{Q_{E}} \int_{P_{E}}^{\infty}\left(P_{A}-P_{E}\right) Q_{A} p_{0}\left(Q_{A}\right) p_{0}\left(P_{A}\right) d P_{A} d Q_{A}+ \\
+ & \int_{Q_{E}}^{\infty} \int_{P_{E}}^{\infty}\left(P_{A}-P_{E}\right) Q_{A} p_{0}\left(Q_{A}\right) p_{0}\left(P_{A}\right) d P_{A} d Q_{A} \tag{10}
\end{align*}
$$

Equations (5) and (10) constitute the two mathematical models for estimating the cost variances of direct material, irrespective of whether the random variables are assumed to be dependent or independent.

## 3. Numerical Application

This section uses a numerical example to show how the model could be used in practical situations. The example simplifies the analysis by using discrete time framework and assumes that the probability distributions of the random variables have been estimated on the basis of historical data.

Let the XYZ Company manufacture T-shirts at several plants locked in different locations. The production department predetermined the direct material cost for the next fiscal year as follows:
a) 3 kgs of thread per batch of hundred T -shirts is needed
b) the thread costs $\$ 350$ per kg

Normal production is set at 1,000 batches and the company's headquarters wish to determine a price that gives a competitive edge. Thus, it is required to predetermine the expected actual cost of the direct material needed for the production. In order to do so, the method of estimating the variance of the actual direct material cost with respect to the predetermined costs is applied.

For simplicity, it is assumed that the actual price and quantity are distinct, random variables, independent from each other. Moreover the probability distribution of the random variables has been estimated as follows:

Table 1

| Max.Predetermined <br> quantity $\mathbf{Q}_{\mathbf{E}}$ | Probability of <br> $\mathbf{Q}_{\mathbf{A}}=\mathbf{Q}_{\mathbf{E}}$ | Cumulative <br> probability of $\mathbf{Q}_{\mathbf{A}}$ | Max. actual <br> quantity $\mathbf{E}\left(\mathbf{Q}_{\mathbf{A}}\right)$ |
| :---: | :---: | :---: | :---: |
| 2 | 0.00 | 0.00 | 0.00 |
| 3 | 0.95 | 0.95 | 2.85 |
| 4 | 0.05 | 1.00 | 0.20 |
|  |  |  | $\mathrm{E}\left(\mathrm{Q}_{\mathrm{A}}\right)=3.05$ |

Table2

| Min.Predetermined <br> quantity $\mathbf{Q}_{\mathbf{E}}$ | Probability of <br> $\mathbf{Q}_{\mathbf{A}}=\mathbf{Q}_{\mathbf{E}}$ | Cumulative <br> probability of $\mathbf{Q}_{\mathbf{A}}$ | Min. actual <br> quantity $\mathbf{E}\left(\mathbf{Q}_{\mathbf{A}}\right)$ |
| :---: | :---: | :---: | :---: |
| 2 | 0.80 | 0.80 | 1.60 |
| 3 | 0.15 | 0.95 | 0.45 |
| 4 | 0.05 | 1.00 | 0.20 |
|  |  |  | $\mathrm{E}\left(\mathrm{Q}_{\mathrm{A}}\right)=2.25$ |

Table3

| Max.Predetermined <br> price $\mathbf{P}_{\mathbf{E}}$ | Probability of <br> $\mathbf{P}_{\mathbf{A}}=\mathbf{P}_{\mathbf{E}}$ | Cumulative <br> probability of $\mathbf{P}_{\mathbf{A}}$ | Max. actual price <br> $\mathbf{E}\left(\mathbf{P}_{\mathbf{A}}\right)$ |
| :---: | :---: | :---: | :---: |
| 250 | 0.05 | 0.05 | 12.50 |
| 350 | 0.90 | 0.95 | 315.00 |
| 400 | 0.05 | 1.00 | 20.00 |
|  |  |  | 347.50 |

Table 4

| Min.Predetermined <br> price $\mathbf{P}_{\mathbf{E}}$ | Probability of <br> $\mathbf{P}_{\mathbf{A}}=\mathbf{P}_{\mathbf{E}}$ | Cumulative <br> probability of $\mathbf{P}_{\mathbf{A}}$ | Min. actual price <br> $\mathbf{E}\left(\mathbf{P}_{\mathbf{A}}\right)$ |
| :---: | :---: | :---: | :---: |
| 200 | 0.60 | 0.60 | 120 |
| 300 | 0.30 | 0.90 | 90 |
| 350 | 0.10 | 1.00 | 33 |
| 400 | 0.00 | 1.00 | 0 |
|  |  |  | 245 |

## Table5

|  | $\mathbf{E}\left(\mathbf{Q}_{\mathbf{A}}\right)$ | $\sum_{0}^{Q_{E}} p_{0}\left(Q_{A}\right)$ | $\sum_{Q_{E+1}}^{\infty} p_{0}\left(Q_{A}\right)$ | Total <br> $\mathbf{Q u a n t i t y}$ <br> $\mathbf{E}\left(\mathbf{Q}_{\mathbf{A}}\right)$ |
| :---: | :---: | :---: | :---: | :---: |
| Maximum | 3.05 | 0.98 |  | 2.989 |
| Minimum | 2.25 |  | 0.02 | 0.045 |
|  |  |  |  | 3.034 |

## Table 6

|  | $\mathbf{E}\left(\mathbf{P}_{\mathbf{A}}\right)$ | $\sum_{0}^{P_{E}} p_{0}\left(P_{A}\right)$ | $\sum_{P_{E+1}}^{\infty} p_{0}\left(P_{A}\right)$ | Total price E <br> $\left(\mathbf{P}_{\mathbf{A}}\right)$ |
| :---: | :---: | :---: | :---: | :---: |
| Maximum | 347.50 | 0.98 |  | 340.55 |
| Minimum | 245.00 |  | 0.02 | 4.90 |
|  |  |  |  | 345.45 |

The total variance is the sum of the variance of price and quantity of the direct material. According to equation 10, (taken in discrete form) it follows that:

$$
\begin{aligned}
& \sum_{0}^{Q_{E}}\left[\left(Q_{A}-Q_{E}\right) P_{E}+\sum_{0}^{P_{E}}\left(P_{A}-P_{E}\right) Q_{A} p_{0}\left(P_{A}\right)\right] p_{0}\left(Q_{A}\right)+\sum_{Q_{E}+1}^{\infty}\left[\left(Q_{A}-Q_{E}\right) P_{E}\right. \\
& \left.+\sum_{0}^{P_{E}}\left(P_{A}-P_{E}\right) Q_{A} p_{0}\left(P_{A}\right)\right] p_{0}\left(Q_{A}\right)+\sum_{0}^{Q_{E}}\left[\sum_{P_{E}+1}^{\infty}\left(P_{A}-P_{E}\right) Q_{A} p_{0}\left(P_{A}\right) p_{0}\left(Q_{A}\right)\right] \\
& +\sum_{Q_{E}+1}^{\infty}\left[\sum_{P_{E}+1}^{\infty}\left(P_{A}-P_{E}\right) Q_{A} p_{0}\left(P_{A}\right) p_{0}\left(Q_{A}\right)\right]
\end{aligned}
$$

Substituting the data given in tables (1) to (6) yields:
$0.98[(3.05-3) \quad 350+0.98(347.50-350) \quad 3.05]+0.02[(2.25-3) \quad 350+0.98(347.50-350)$ $3.05]+0.98[0.02(245-350) 3.05]+0.02[0.02(245-350) 2.25]=\$-1.93$

The absolute value of the variance per unit produced, that has been calculated lies within the preset limit of $2 \%$ since the total actual average cost of the direct material is $3.034 * 345.45=\$ 1048.09$ and the $2 \%$ amount of the actual cost is $\$ 20.962$.

## 4. Concluding Remarks

The present paper provides a stochastic model of estimating the variance between the actual and the predetermined cost of the direct material. The method developed in the paper involves truncating cost variances and results in predetermined costs being very close to actual costs. The model may be useful for both practitioners and academics for four main reasons:

First, when the selling price depends on the direct material costs, the calculation of the cost variance according to the model provides management with sufficient information to set competitive prices in the market.

Second, this method provides the management with the opportunity to decide whether it is more profitable to either buy or manufacture a product. It also gives the means for comparing the production cost with the prevailing prices in the market.

Third, the determination of the direct material costs using the variance method (when the direct labor and factory overhead cost are known), helps the manager to estimate the financing requirements of the product-manufacturing process.

Fourth, with few modifications, the model can also be applied for estimating the variance of direct labor costs. Moreover, the model consists of a way of truncating variances under any costing system. However, this consists of an implication for future research and expands beyond the scope of the present paper.

## Appendix

This appendix applies the results of the paper in the special case where the random variables are assumed to follow some parametric distributions, such as the normal or lognormal distribution:

Normal distribution:

$$
\mathrm{f}(\mathrm{x})=\frac{1}{\sqrt{2 \pi \sigma^{2}}} \exp \left\{-\frac{1}{2 \sigma^{2}}(x-\mu)^{2}\right\} d x
$$

Truncate normal: $\mathrm{A} \leq \mathrm{x} \leq \mathrm{B}$
$\mathrm{E}(\mathrm{x} \mid A \leq x \leq B)=\mu+\frac{f\left(\frac{\mathrm{~A}-\mu}{\sigma}\right)-f\left(\frac{\mathrm{~B}-\mu}{\sigma}\right)}{\Phi\left(\frac{\mathrm{B}-\mu}{\sigma}\right)-\Phi\left(\frac{\mathrm{A}-\mu}{\sigma}\right)} \sigma$
where: $\quad f\left(\frac{\mathrm{~A}-\mu}{\sigma}\right)=\frac{1}{\sqrt{2 \pi}} \exp \left\{-\frac{1}{2 \sigma^{2}}(\mathrm{~A}-\mu)^{2}\right\}$

$$
\begin{aligned}
f\left(\frac{B-\mu}{\sigma}\right) & =\frac{1}{\sqrt{2 \pi}} \exp \left\{-\frac{1}{2 \sigma^{2}}(\mathrm{~B}-\mu)^{2}\right\} \\
\text { if } \quad \mathrm{A} & =-\infty \quad \mathrm{f}(\cdot)=0 \quad \Phi(\cdot)=0 \\
\mathrm{~B} & =+\infty \quad \mathrm{f}(\cdot)=0 \quad \Phi(\cdot)=1
\end{aligned}
$$

$\mu$ : is the unconditional mean of $x$
$\sigma$ : standard deviation of x
$\Phi(\mathrm{z})=\int_{-\infty}^{2} \frac{1}{\sqrt{2 \pi}} \exp \left\{-\frac{1}{2} s^{2}\right\} d s \quad:$ is the area from $-\infty$ to z under the standard normal distribution

This result can be used to evaluate the integral when $Q_{A}$ and $P_{A}$ are stochastically independent. For example,

$$
\int_{0}^{Q_{E}}\left(Q_{A}-Q_{E}\right) P_{E} p_{0}\left(Q_{A}\right) d Q_{A}=P_{E} \int_{0}^{Q_{E}} Q_{A} p_{0}\left(Q_{A}\right) d Q_{A}-P_{E} Q_{E} \int_{0}^{Q_{E}} p_{0}\left(Q_{A}\right) d Q_{A}=
$$

$$
\begin{aligned}
& =P_{E}\left[\Phi_{0}\left(\frac{Q_{E}-\mu}{\sigma}\right)-\Phi_{0}\left(\frac{0-\mu}{\sigma}\right)\right] \frac{\int_{0}^{Q_{E}} Q_{A} p_{0}\left(Q_{A}\right) d Q_{A}}{\left[\Phi_{0}\left(\frac{Q_{E}-\mu}{\sigma}\right)-\Phi_{0}\left(\frac{0-\mu}{\sigma}\right)\right]}-P_{E} Q_{E} \int_{0}^{Q_{E}} p_{0}\left(Q_{A}\right) d Q_{A}= \\
& =P_{E}\left[\Phi_{0}\left(\frac{Q_{E}-\mu}{\sigma}\right)-\Phi_{0}\left(\frac{0-\mu}{\sigma}\right)\right], \mathrm{E}\left(Q_{A} \mid 0 \leq Q_{A} \leq Q_{E}\right) \\
& =P_{E} Q_{E}\left[\Phi_{0}\left(\frac{Q_{E}-\mu}{\sigma}\right)-\Phi_{0}\left(-\frac{\mu}{\sigma}\right)\right]= \\
& =\left[\Phi_{0}\left(\frac{Q_{E}-\mu}{\sigma}\right)-\Phi_{0}\left(-\frac{\mu}{\sigma}\right)\right]\left[P_{E} \quad E\left(Q_{A} \mid 0 \leq Q_{A} \leq Q_{E}\right)-P_{E} Q_{E}\right]
\end{aligned}
$$

where:

$$
E\left(Q_{A} \mid 0 \leq Q_{A} \leq Q_{E}\right)=\mu+\frac{f\left(-\frac{\mu}{\sigma}\right)-f\left(\frac{Q_{E}-\mu}{\sigma}\right)}{\Phi\left(\frac{Q_{E}-\mu}{\sigma}\right)-\Phi\left(-\frac{\mu}{\sigma}\right)} \sigma
$$

Note: The density function of $Q_{A}$, conditional on $Q_{A} \geq 0$, is given by:

$$
f\left(Q_{A} \mid Q_{A} \geq 0\right)=\frac{1}{\sqrt{2 \pi \sigma^{2}}} \exp \left\{-\frac{1}{2 \sigma^{2}}\left(Q_{A}-\mu\right)^{2}\right\} \frac{1}{1-\Phi\left(-\frac{\mu}{\sigma}\right)}
$$

where: $\left.\begin{array}{l}\mu=E\left(Q_{A}\right) \\ \\ \sigma^{2}=\operatorname{Var}\left(Q_{A}\right)\end{array}\right\}$ unconditional

In this case: $\int_{0}^{Q_{E}} Q_{A} f_{0}\left(Q_{A} \mid Q_{A} \geq 0\right) d Q_{A}=$

$$
\begin{aligned}
& =\frac{1}{1-\Phi_{0}\left(-\frac{\mu}{\sigma}\right)} \int_{0}^{Q_{F}} Q_{A} f_{0}\left(Q_{A}\right) d Q_{A}= \\
& =\frac{1}{1-\Phi_{0}\left(-\frac{\mu}{\sigma}\right)}\left[\Phi_{0}\left(\frac{Q_{E}-\mu}{\sigma}\right)-\Phi_{0}\left(-\frac{\mu}{\sigma}\right)\right] \quad \mathrm{E}\left(Q_{A} \mid 0 \leq Q_{A} \leq Q_{E}\right)
\end{aligned}
$$

$=\frac{\left[\Phi_{0}\left(\frac{Q_{E}-\mu}{\sigma}\right)-\Phi_{0}\left(-\frac{\mu}{\sigma}\right)\right]}{1-\Phi_{0}\left(-\frac{\mu}{\sigma}\right)}\left(\mu+\frac{f_{0}\left(-\frac{\mu}{\sigma}\right)-f_{0}\left(\frac{Q_{E}-\mu}{\sigma}\right)}{\Phi_{0}\left(\frac{Q_{E}-\mu}{\sigma}\right)-\Phi_{0}\left(-\frac{\mu}{\sigma}\right)}\right)$
It is quite clear, that the quantity $\int_{Q_{E}}^{\infty} Q_{A} p_{1}\left(Q_{A}\right) d Q_{A}$ can also be expressed in a similar manner.

## References

1. Balakrishnan, R. and Sivaramakrishnan, K.. (2002). 'A critical overview of the use of full-cost data for planning and pricing', Journal of Management Accounting Research, 14, 3-31.
2. Banker,R.D.and. Hansen, S.C. (2002). 'The adequacy of full-cost-based pricing heuristics', Journal of Management Accounting Research, 14, 33-58
3. Bjornenak, T. and Mitchell, F (2002). 'The development of activity-based costing journal literature: 1987-2000', European Accounting Review, 11(3), 481-508.
4. Cheatham, C. (1989) 'Reporting the effects of excess inventories', Journal of Accountancy, November, 131-140
5. Dearman, D.T. and Shields, M. D. (2001). 'Cost knowledge and cost-based judgment performance', Journal of Management Accounting Research, 13, 118.
6. Dickinson, P.J. (1974). 'Cost-volume-profit analysis under uncertainty', Journal of Accounting Research 12(1), 182-187
7. Drury, C. (2000). Management and cost accounting, Fifth Edition, Business PressThomson Learning.
8. Emsley, D. (2001). 'Redesigning variance analysis for problem solving' Management Accounting Research, 12, 21-40.
9. Garrison, R. H. and Noreen E. W. (2003). Managerial accounting, Tenth Edition, McGraw-Hill, Irwin.
10. Hilliard, J. E. and Leitch, R. A. (1995). 'Cost-volume-profit analysis under uncertainty: A log normal approach', Accounting Review, 50, 69-80.
11. Horngren, C., Foster, G. and Datar, S. (1997). Cost accounting: A managerial emphasis, Ninth Edition, Englewood Cliffs, N.J.: Prentice Hall.
12. Innes, J. and Mitchell F. (1995). 'A survey of activity-based costing in the U.K.'s largest companies', Journal of Management Accounting Research, 6, 137-153.
13. Johnsen, D. and Sopariwala, P. (2000). 'Standard costing is alive and well at Parker Brass', Management Accounting Quarterly, 1(2), 1-9.
14. Kaplan, R.S. and Cooper, R. (1998). Cost and effect: Using integrated cost systems to drive profitability and performance, Harvard Business School Press.
15. Krumwiede, K. (1998a). 'The implementation stages of activity-based costing and the impact of contextual organizational factors', Journal of Management Accounting Research, 10, 239-277.
16. $\qquad$ . (1998b). 'ABC: Why it's tried and how it succeeds.' Management Accounting, April, 32-38.
17. Liao, M. (1975). 'Model sampling: A stochastic cost-volume-profit analysis', Accounting Review, 50, 780-790
18. Lucas, M. R. (1999). 'The pricing decision: economists versus accountants', Management Accounting, June, 34-35.
19. . (2003). 'Pricing decisions and the neoclassical theory of the firm', Management Accounting Research, 14, 201-217.
20. Merchant, K. and Shields, M. (1993). 'When and why to measure costs less accurately to improve decision making', Accounting Horizons, 7(2), 76-81.
21. Wing, K. (2000). 'Using enhanced cost models in variance analysis for better control and decision making', Management Accounting Quarterly, 1(2), 1-9.

[^0]:    * Contact Information: Ioannis Ananiadis and/or Dimitrios Kousenidis, Aristotle University of Thessaloniki; Faculty of Law, Economics, and Political Sciences; School of Economics; Department of Business Administration; 54124 Thessaloniki, Greece.
    Tel. $+302310996466,+302310997201$ Fax. +302310996452 e-mail: dkous@econ.auth.gr
    ${ }^{1}$ See for example Kaplan and Cooper (1998), Drury (2000) and Garrison and Noreen (2003)
    ${ }^{2}$ See Horngren et al.(1997) for a review of such studies in the USA, the UK, Ireland, Japan and Sweden.

[^1]:    ${ }^{3}$ For example, Merchant and Shields (1993), Innes and Mitchell (1995) and Krumwiede (1998a, 1998b) reveal that activity based costing (ABC) can eliminate biases in the costing of products with diverse resource consumption. Dearman and Shields (2001) show that managers who make decisions based on traditional cost accounting information exhibit poorer judgment performance than managers who base their decision making process on ABC product-cost information. For an extensive review of the major studies in the area see Bjornenak and Mitchell (2002).

[^2]:    ${ }^{4}$ See for example Dickinson (1974), Hilliard and Leitch (1975),and Liao (1975)

